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Internal Assessment Test 1 – Aug. 2018

Sub:	Applied Hydraulics				Sub Code:	17CV43	Branch:	CIVIL
Date:	06/03/19	Duration:	90 min's	Max Marks: 50	Sem/Sec:	IV A & B	OBE	

Attempt five full questions

		MARKS	CO	RBT
1	a) A uniform body of size 3m long 2m wide and 1 m deep floats in water. What is the weight of the body if depth of immersion is 0.8 m? b) A solid cylinder of diameter 4m has a height of 4 m. Find the meta-centric height of the cylinder when it is floating in water with its axis vertical. Specific gravity of cylinder = 0.6.	5 5	CO1	L2
2	State Buckingham's pi theorem. The pressure difference Δp in a pipe of diameter D and length l due to turbulent flow depends on velocity V, viscosity μ, density ρ and roughness k. Using Buckingham's method, obtain an expression for Δp.	10	CO1	L3
3	a) With a sketch, explain stability of floating bodies? Explain the significance of metacentric height in designing floating bodies. Write expression for meta-centric height. b) What is meant by dimensionally homogeneous equation? Explain with an example.	6 4	CO1	L2
4	What are the types of forces acting in a moving fluid system? Discuss various non-dimensional numbers.	10	CO1	L2
5	Discuss experimental method of determining meta-centric height.	10	CO1	L3

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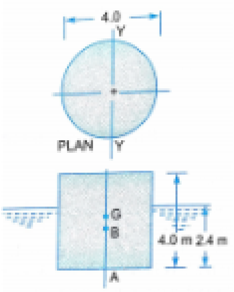
Solution of IAT-1 Question paper:

1) Question 1
   a) Dimention of the body = 3*2*1
      Depth of immersion = 0.8m
      Principle of floatation :
      Weight of the body = weight of water displaced
      = sp. Weight of water * Volume of water displaced
      =ϱg * (3*2*0.8) = 1000*9.81*3*2*0.8 = 47088 N
   b) Principle of floatation: weight of the body = buoyant force
      Or Weight of the body = weight of fluid displaced
      Or $600 \times 9.81 \times \pi(4)^2/4 \times 4 = 1000 \times 9.81 \times \pi(4)^2/4 \times h$.............. (let depth of immersion is h m)
      Or h = 2.4 m
      Distance of centre of buoyancy (B) from base = 2.4/2 = 1.2 m
      Distance of centre of gravity (G) from base = 4/2 = 2 m
      Distance BG = 2-1.2 = 0.8 m
      Metacentric height = GM = BM-BG = (Iyy/V') - 0.8 = $[\pi(D)^4/64] / [\pi(4)^2/4 \times 2.4] - 0.8 = -0.3833$m



2) Buckingham's pi theorem: If there are n variables (independent and dependent) in a physical phenomenon and these variables contain m fundamental dimensions (M,L,T) then the variables are arranged in (n-m) dimensionless terms called π-terms.

Δp is a function of D, l, V, μ, ρ, k

$\therefore$ $\Delta p = f(D, l, V, \mu, \rho, k)$ or $f_1(\Delta p, D, l, V, \mu, \rho, k) = 0$ ...(*i*)

$\therefore$ Total number of variables, $n = 7$.

Writing dimensions of each variable,

Dimension of Δp = Dimension of pressure $= ML^{-1}T^{-2}$

$D = L, l = L, V = LT^{-1}, \mu = ML^{-1}T^{-1}, \rho = ML^{-3}, k = L$

$\therefore$ Number of fundamental dimensions, $m = 3$

Number of π-terms $= n - m = 7 - 3 = 4$.

Now equation (*i*) can be grouped in 4 π-terms as

$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0$ ...(*ii*)

Each π-term contains $m + 1$ or $3 + 1 = 4$ variables. Out of four variables, three are repeating variables. Choosing D, V, ρ as the repeating variables, we have the four π-terms as

$$\pi_1 = D^{a_1} . V^{b_1} . \rho^{c_1} . \Delta p$$

$$\pi_2 = D^{a_2} . V^{b_2} . \rho^{c_2} . l$$

$$\pi_3 = D^{a_3} . V^{b_3} . \rho^{c_3} . \mu$$

$$\pi_4 = D^{a_4} . V^{b_4} . \rho^{c_4} . k$$

First π-term $\pi_1 = D^{a_1} . V^{b_1} . \rho^{c_1} . \Delta p$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} . (LT^{-1})^{b_1} . (ML^{-3})^{c_1} . ML^{-1}T^{-2}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1 + 1$ $\therefore$ $c_1 = -1$

Power of L, $0 = a_1 + b_1 - 3c_1 - 1,$ $\therefore$ $a_1 = -b_1 + 3c_1 + 1 = 2 - 3 + 1 = 0$

Power of T, $0 = -b_1 - 2,$ $\therefore$ $b_1 = -2$

Substituting the values of a_1, b_1 and c_1 in π_1,

$$\pi_1 = D^0 . V^{-2} . \rho^{-1} . \Delta p = \frac{\Delta p}{\rho V^2}$$

Second π-term $\pi_2 = D^{a_2} . V^{b_2} . \rho^{c_2} . l$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} . (LT^{-1})^{b_2} . (ML^{-3})^{c_2} . L$$

Equating powers of M, L, T on both sides,

Power of M, $0 = c_2,$ $\therefore$ $c_2 = 0$

Power of L, $0 = a_2 - b_2 - 3c_2 + 1,$ $\therefore$ $a_2 = b_2 + 3c_2 - 1 = -1$

Power of T, $0 = -b_2,$ $\therefore$ $b_2 = 0$

Substituting the values of a_2, b_2, c_2 in π_2,

$$\pi_2 = D^{-1} . V^0 . \rho^0 . l = \frac{l}{D}$$

Third π-term $\pi_3 = D^{a_3} . V^{b_3} . \rho^{c_3} . \mu$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_3} . (LT^{-1})^{b_3} . (ML^{-3})^{c_3} . ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_3 + 1,$ $\therefore$ $c_3 = -1$

Power of L, $0 = a_3 + b_3 - 3c_3 - 1,$ $\therefore$ $a_3 = -b_3 + 3c_3 + 1 = 1 - 3 + 1 = -1$

Power of T, $0 = -b_3 - 1,$ $\therefore$ $b_3 = -1$

Substituting the values of a_3, b_3 and c_3 in π_3,

$$\pi_3 = D^{-1} . V^{-1} . \rho^{-1} . \mu = \frac{\mu}{DV\rho}.$$

Fourth π-term $\pi_4 = D^{a_4} . V^{b_4} . \rho^{c_4} . k$

or $M^0L^0T^0 = L^{a_4} . (LT^{-1})^{b_4} . (ML^{-3})^{c_4} . L$ [Dimension of $k = L$]

Equating the power of M, L, T on both sides,

Power of M, $0 = c_4,$ $\therefore$ $c_4 = 0$

Power of L, $0 = a_4 - b_4 - 3c_4 + 1,$ $\therefore$ $a_4 = b_4 + 3c_4 - 1 = -1$

Power of T, $0 = -b_4,$ $\therefore$ $b_4 = 0$

Substituting the values of a_4, b_4, c_4 in π_4,

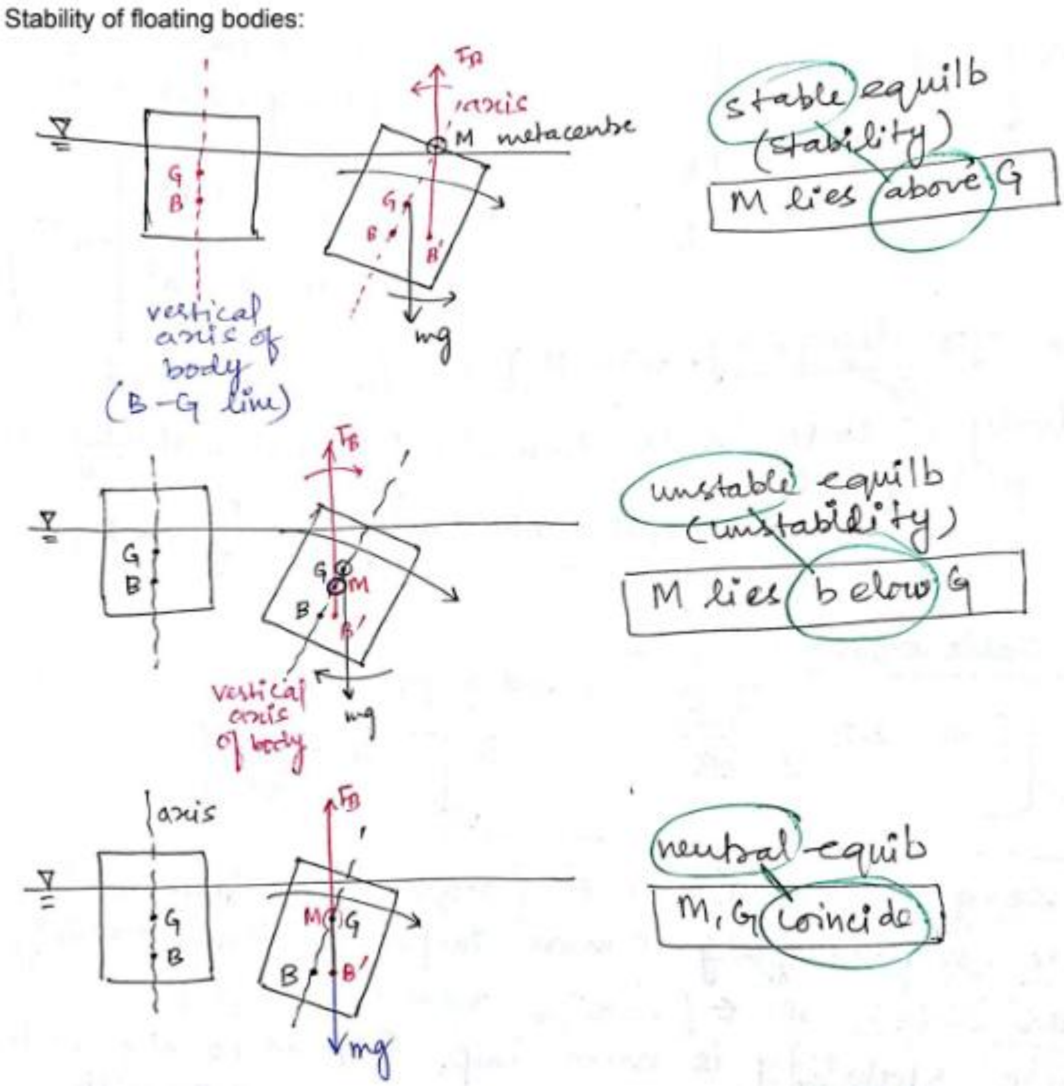
$$\pi_4 = D^{-1} . V^0 . \rho^0 . k = \frac{k}{D}$$

Substituting the values of π_1, π_2, π_3 and π_4 in (*ii*), we get

$$f_1\left(\frac{\Delta p}{\rho V^2}, \frac{l}{D}, \frac{\mu}{DV\rho}, \frac{k}{D}\right) = 0 \quad \text{or} \quad \frac{\Delta p}{\rho V^2} = \phi\left[\frac{\mathbf{l}}{\mathbf{D}}, \frac{\mu}{\mathbf{DV\rho}}, \frac{\mathbf{k}}{\mathbf{D}}\right].$$ **Ans.**

3) Question3

a) Stability of floating bodies: Comparison of M and G



Metacentric height is significant in deciding dimensions of a prototype so that it can be made stable while floating in water. We have to ensure that GM >0.
Thus, Metacentric (GM) height is significant in stability analysis of a floating body like a ship.
a. GM >0 : Stable equilibrium
b. GM<0 : Unstable Equilibrium
c. GM=0 : Neutral equilibrium
Expression of metacentric height: **GM=BM-BG = (I_{yy}/V') – BG** Where B is centre of buoyancy, G is centre of gravity and M is metacentre. I_{yy} is moment of inertia about YY axis and V' is volume of water displaced.

b) Dimensionally homogeneous equation: dimensions of each term in an equation are the the same. The powers of fundamental dimensions on both the sides will be identical for a dim. homogeneous equation.

Let us consider the equation, $V = \sqrt{2gH}$

Dimension of L.H.S. $= V = \frac{L}{T} = LT^{-1}$

Dimension of R.H.S. $= \sqrt{2gH} = \sqrt{\frac{L}{T^2} \times L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} = LT^{-1}$

Dimension of L.H.S. $=$ Dimension of R.H.S. $= LT^{-1}$

$\therefore$ Equation $V = \sqrt{2gH}$ is dimensionally homogeneous. So it can be used in any system of units.

4. Forces in a moving fluid:

Inertia force $(F_I) = m.a$

$$F_I = m \cdot a = \rho L^3 \cdot \frac{V}{t} = \rho L^2 \frac{L}{t} \cdot V$$

$$F_I = \rho L^2 V^2$$

① Viscous force $(F_V) = \tau \cdot A$

$$F_V = \mu \frac{du}{dy} \cdot A = \frac{\mu V}{L} \cdot L^2$$

$$F_V = \mu V L$$

② pressure force (F_P)

$$F_P = \Delta P \cdot A$$

$$F_P = \Delta P \cdot L^2$$

③ Gravity force (F_g)

$$F_g = mg$$

$$F_g = \rho L^3 \cdot g$$

④ Surface tension force (F_σ)

$$F_\sigma = \sigma \cdot L$$

⑤ Compressibility force / Elastic force $(F_{elastic})$

$$F_{elastic} = K \cdot A = K L^2$$

(K = elastic stress)

various non-dimensional numbers :

$$R_e = \frac{F_i}{F_v} = \frac{\rho V l}{\mu}$$

$$F_r = \sqrt{\frac{F_i}{F_g}} = \frac{V}{\sqrt{g l}}$$

$$E_u = \sqrt{\frac{F_i}{F_p}} = \frac{V}{\sqrt{\Delta P/\rho}}$$

$$W_b = \sqrt{\frac{F_i}{F_\sigma}} = \frac{V}{\sqrt{\sigma/\rho l}}$$

$$M_a = \sqrt{\frac{F_i}{F_e}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C}$$

R_e = reynold's no.
F_r = Fronde's no.
E_u = Euler's no.
W_b = Webber's no.
M_a = Mach no.

F_i = inertia force
F_g = gravity force
F_v = viscous force
F_p = pressure force
F_σ = surface tension force
F_e = elastic force

5. Experimental method:

Experimental method of determining metacentric height.

* Consider a ship floating in water.
* Let w be a movable weight placed centrally on the deck of the ship.

  W = total wt. (of ship) including w

* Initially: ship in equilibrium.
  $\Rightarrow$ deck is horiz.

* Now, w moved transversely through a distance 'x' across deck, so that ship tilts thru' small angle θ, and comes to rest in new position of equilibrium.

  $\theta \rightarrow$ measured using long plumbline attached to centre of the deck.

* In new position, G' & B_1 (new C.G.S. & new C.O.B) will be again in vertical line.

  Movement of 'w' through 'x' causes a parallel shift of total centre of gravity from G to G'. We will equate this moment with the moment caused by movement of w.

$$w x = W(G \cdot G')$$

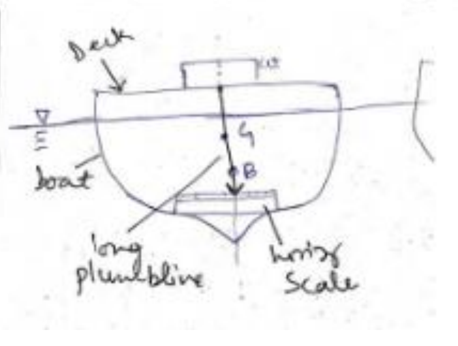
$$w x = W(GM \tan\theta)$$

$$\text{or} \quad GM = \frac{wx}{W \tan\theta}$$

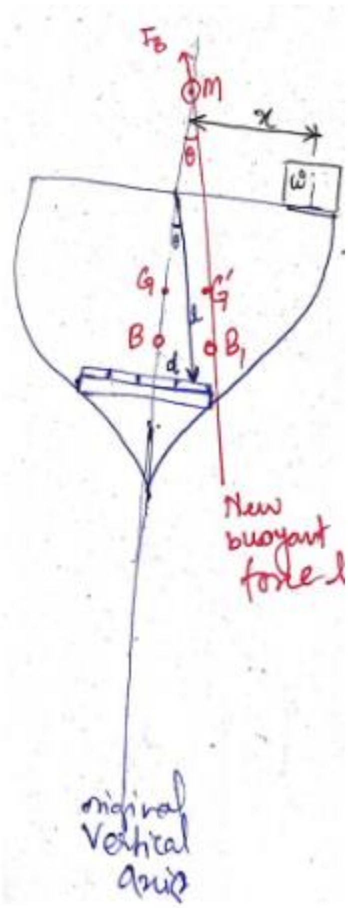
* If l = length of plumbline & d is the distance moved by it on horizontal scale,

  then, $\tan\theta = \frac{d}{l}$

$$\therefore \boxed{GM = \frac{wx}{W} \times \frac{l}{d}}$$



Situation before tilt



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