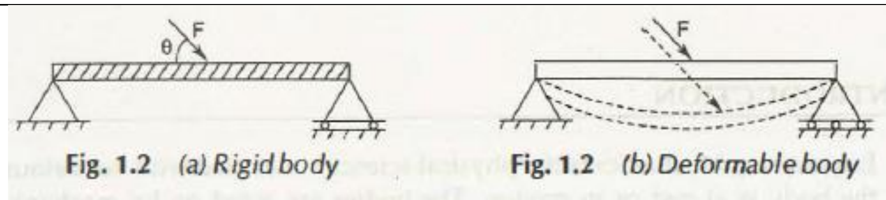
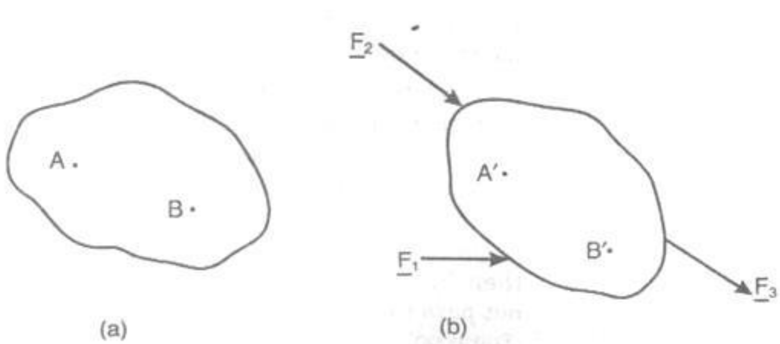
Solution

Internal Assessment Test 1 – April 2019

Sub:	ELEMENTS OF CIVIL ENGINEERING AND MECHANICS							**Code:**	18CIV24
Date:	20/04/2019	Duration:	90mins	Max Marks:	50	**Sem:**	II	**Branch:**	EEE/ECE/ME

Note: Answer Any Five Questions

Question #		**Description**	**Marks Distribution**		**Max Marks**
1	a)	**Explain the role of Civil engineer in infrastructural development of a country.** Civil engineer has a very important role in the development of the following infrastructures Town planning Build suitable structures for the rural and urban areas for various utilities • Build tank, dams to exploit water resources • Purify the water and supply water to needy areas like houses, schools, offices and agriculture field. • Provide good drainage sysem and purification plants • Provide good drainage system and purification plants • provide and maintain communication systems like roads, railways , harbours and airports. • Monitor land, water and air pollution and take measures to control them.		05 M	10 M
	b)	**Explain the basic idealisations made in mechanics** **Particle** An object that has no size but has a mass concentrated at a point, is called a particle. In mathematical sense a particle is a body whose dimensions approach zero so that it may be analyzed as a point mass. **Rigid Body** A body is said to be rigid when the relative movements between its parts are negligible. Actually, every body must deform to a certain degree under the action of forces, but in many cases the deformation is negligible and may not be considered in the analysis. This rigid body concept leads to simplified computations. Refer Fig. 1.2 (a).	4M 1M	05 M	

		 Fig. 1.2 (a) Rigid body Fig. 1.2 (b) Deformable body **Continuum** A body consists of several particles. It is a well known fact that each particle can be sub-divided into molecules, atoms and electrons. It is not feasible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter which will not separate even when various forces considered are acting simultaneously. In other words, we say the body is treated as a continuum. **Rigid Body** As already stated, in Civil Engineering, we treat a body as rigid, when the relative position of any two particles in the body do not change even after the application of a system of forces. For examples, let the body shown in figure (a) move to a position as shown in figure (b) when the system of forces F_2 and F_3 are applied. If the body is treated as a rigid body, the relative position of A to B is the same as A' and B', i.e., AB = A'B'  			
2	a)	**State and prove parallelogram law of forces.**	1 M 3M 1M	05 M	10 M

It states "If two forces P and Q acting at a point be simultaneously represented in magnitude and direction by the two adjacent sides OA and OB of a parallelogram OACB, then the resultant of the two forces R may be represented in magnitude and direction by the diagonal OC, which passes through point O". This implies that a force can be represented by a straight line with an arrow, as vector.

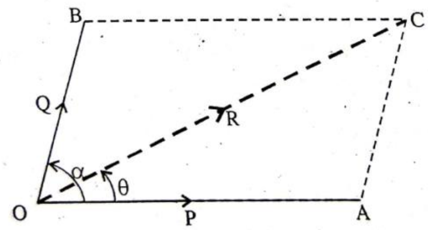


Fig.

Let the forces 'P' and 'Q' simultaneously acting at point 'O' be represented A along the adjacent sides OA and OB respectively of a parallelogram OACB. Let OC be the diagonal. Drop CD to OA extended.

From ΔOCD

$$OC^2 = (OD)^2 + (CD)^2$$
$$= (OA + AD)^2 + (CD)^2$$
$$= (OA)^2 + (AD)^2 + 2(OA)(AD) + (CD)^2 \quad \text{... (1)}$$

From Δle ACD

$$\left.\begin{aligned} \frac{AD}{AC} &= \cos\alpha;\ AD = Q\cos\alpha \\ \frac{CD}{AC} &= \sin\alpha,\ CD = Q\sin\alpha \end{aligned}\right\} \quad \text{... (2)}$$

From (1) and (2)

$$R^2 = P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha$$
$$= P^2 + Q^2(\sin^2\alpha + \cos^2\alpha) + 2PQ\cos\alpha$$
$$= P^2 + Q^2 + 2PQ\cos\alpha \quad (\because\ \sin^2\alpha + \cos^2\alpha = 1)$$
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad \text{... (I) (remember)}$$

Let 'θ' be the angle made by resultant R with 'P' then

$$\tan\theta = \frac{CD}{OD} = \frac{CD}{OA + AD}$$
$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha} \quad \text{... (II) (remember)}$$

	b)	**Determine the force F and its inclination α required so as to lift a block of weight 500 N as shown in Fig 1.**	1 M 2 M 2 M	05 M	

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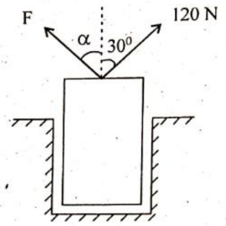

Solution : Since the weight is to be lifted vertically up, the resultant of both forces is also vertically up i.e., $R = \Sigma V$ and $\Sigma H = 0$.

Resolving forces horizontally,

$-F \sin\alpha + 120 \sin 30 = 0$

(lift, away) (right, away)

$F \sin\alpha = 60$ ... (1)

Resolving forces vertically,

$F \cos\alpha + 120 \cos 30 = 500$

$F \cos\alpha = 396.08$ ... (2)

Dividing (1) by (2)

$$\frac{F \sin\alpha}{F \cos\alpha} = \frac{60}{396.08}$$

$$\tan\alpha = 0.161$$

$\therefore \quad \alpha = 8.61^0$ ... (3)

Substituting (3) in (1)

$F \sin 8.61 = 60$

$\therefore \quad F = 400.59$ N

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State and prove Varignon's theorem.

Varignon's Theorem states that the moment of a force about any point is equal to the *algebraic sum* of the moments of its components about that point.

Principal of moments states that the moment of the resultant of a number of forces about any point is equal to the *algebraic sum* of the moments of all the forces of the system about the same point.

Proof of Varignon's Theorem

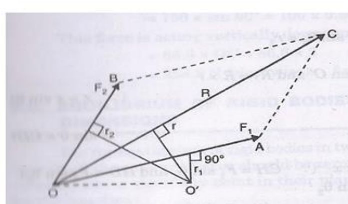

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Fig2.34 *(a)* shows two forces Fj and *F2* acting at point O. These forces are represented in magnitude and direction by *OA* and *OB*. Their resultant R is represented in magnitude and direction by OC which is the diagonal of parallelogram OACB. Let O' is the point in the plane about which moments of F_1, F_2 and R are to be determined. From point *O'*, draw perpendiculars

on OA, OC and OB.

Let r_1= Perpendicular distance between F_1 and *O'*.

r_2= Perpendicular distance between R and *O'*.

r_3= Perpendicular distance between F_2 and *O'*.

Then according to Varignon's principle;

Moment *of R* about O' must be equal to algebraic sum of moments of F_1 *and* F_2 about *O'*.

$R \times r = F_1 \times r_1 + F_2 \times r_2$

Now refer to Fig. 2.34 *(b)*. Join *OO'* and produce it to *D*. From points C, *A* and *B* draw perpendiculars on *OD* meeting at *D*, *E* and *F* respectively. From *A* and *B* also draw perpendiculars on *CD* meeting the line *CD* at G and *H* respectively.

Let θ_1 = Angle made by *F;* with *OD*,

θ = Angle made by *R* with *OD*, and

θ_2 = Angle made by F_2 with *OD*.

In Fig. 2.34 *(b)*, *OA* = *BC* and also *OA* parallel *to BC*, hence the projection *of OA and BC* on the same vertical line *CD* will be equal *i.e.*, *GD* = *CH* as *GD* is the projection of *OA* on *CD* and *CH* is the projection of *BC* on *CD*.

Then from Fig. 2.34 *(b)*, we have

$P_1 \sin \theta_1 = AE = GD = CH$
$F_1 \cos \theta_1 = OE$

$F_2 \sin \theta_1 = BF = HD$
$F2 \cos \theta_2 = OF = ED$

(*OB* = *AC* and also *OB* || *AC*. Hence projections of *OB* and *AC* on the same horizontal line *OD* will be equal *i.e.*, *OF* = *ED)*

$R \sin \theta = CD$
$R \cos \theta = OD$

Let the length $OO' = x$.

Then $x \sin \theta_1 = r$, $x \sin \theta = r$ and $x \sin \theta_2 = r_2$

Now moment of *R* about O'

$= R \times$ (distance between O' and *R*) $= R \times r$

$= R \times x \sin \theta$ ($r = x \sin \theta$)

$= R \times$ (distance between O' and *R*) $= R \times r$

$= R \times x \sin \theta$ ($r = x \sin \theta$)

$= (R \sin \theta) \times x$

$= CD \times x$ ($R \sin \theta = CD$)

$= (CH + HD) \times x$

$= (F_1 \sin \theta_1 + F2 \sin \theta_2) \times x$ ($CH = F_1 \sin \theta_1$ and $HD = F_2 \sin \theta_2$)
$= F_1 \times x \sin \theta_1 + F_2 \times x \sin \theta_2$

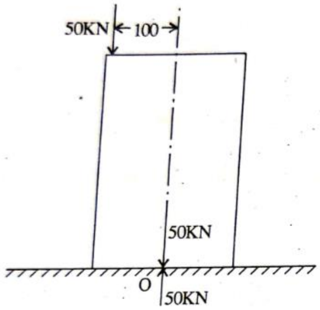
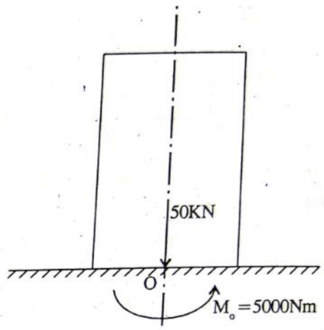
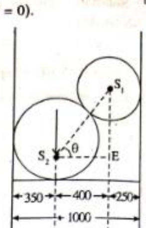
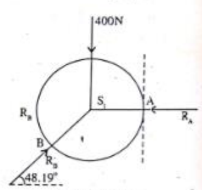
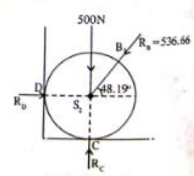
$= F_1 \times r_1 + F_2 \times r_2$ ($x \sin \theta_1 = r_1$ and $x \sin \theta_2 = r_2$)

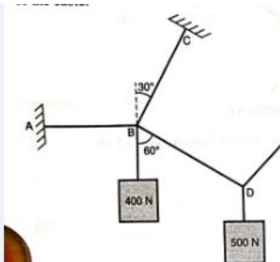
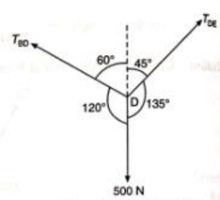
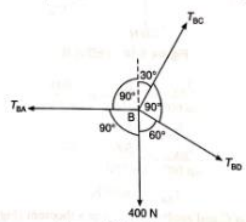
= Moment of F_1 about O' + Moment of F_2 about *O'*.

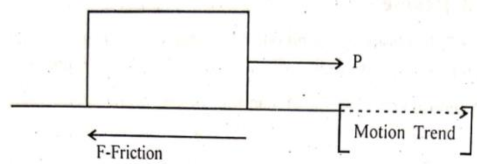
Hence moment of *R* about any point in the algebraic sum of moments of its components *i.e.*, F_1 and F_2) about the same point. Hence Varignon's principle is proved.

The principle of moments (or Varignon's principle) is not restricted to only two concurrent forces but is also applicable to any coplanar force system, *i.e.*, concurrent or non-concurrent or parallel force system.

b) **A concrete column is carrying a force of 50 KN as shown in the Fig 2. Replace the system by force-couple system at the point O.**

		Apply two equal, opposite and collinear force of 50KN at O at in fig. (b). Replace two equal opposite and non-collinear forces by a couple as in figure (c).  Fig. (b)  Equivalent Force Couple at O Fig. (c) Taking Moment Couple at O = $M_O = -50 \times 100$ $= -5000$Nm (anticlockwise)	2 M 2 M	4 M	
4	a)	**A horizontal channel with a inner clearance of 1000 mm carries two spheres of radius 350 mm and 250 mm whose weights are 500 N and 400 N respectively. Find the reactions at all points of contacts. Refer Fig 3.** **Solution :** There are four points of contact A, B, C and D. Let R_A, R_B, R_C and R_D be corresponding reactions. The system is in equilibrium ($\Sigma H = \Sigma V = 0$). Let θ be the angle S_1S_2E (fig. b) $S_2E = 1000 - 350 - 250$ $= 400$mm $S_1S_2 = 350 + 250$ $= 600$mm $\cos\theta = \frac{S_2E}{S_1S_2}$ $= \frac{400}{600}$ $\cos\theta = 0.67$ $\theta = 48.19°$  Fig. (b) **Consider FBD of sphere S_1.** Resolving forces vertically, ($\Sigma V = 0$) $-400 + R_B \sin 48.19 = 0$ $R_B = \frac{400}{\sin 48.19}$ $R_B = 536.66$N Resolving forces horizontally, ($\Sigma H = 0$) $-R_A + R_B \cos 48.19 = 0$ $R_A = 536.66 \cos 48.19$ $R_A = 357.77$N  FBD of body S_1 **Consider F.B.D. of sphere S_2.** Resolving forces horizontally, ($\Sigma H = 0$) $R_D - R_B \cos 48.19 = 0$ $R_D = 536.66 \cos 48.19$ $R_D = 357.77$N Resolving forces vertically, ($\Sigma V = 0$) $R_C - 500 - 536.66 \sin 48.19 = 0$ $R_C = 900$N  FBD of body S_2	3 +3M 2 M 2 M	10 M	10 M
5	a)	**State Lami's theorem**	2 M	2 M	10 M

		Lami's Theorem **Statement :** "If three coplanar forces acting simultaneously a then each force is proportional to the Sine of the Angle between			
	b)	**The system of connected flexible cables is supporting 400 N and 500 N at points B and D respectively. Determine the tension in various segments of the cable. Refer Fig 4.**  **Figure 5.18** Example 5.9.  **Figure 5.19** FBD at D.  **Figure 5.20** FBD at B. *Solution* At D (Figure 5.19): $\frac{500}{\sin 105°} = \frac{T_{DE}}{\sin 120°} = \frac{T_{BD}}{\sin 135°}$ or $T_{BD} = \frac{500}{\sin 105°} \times \sin 135°$ $= 366.025$ N Ans. $T_{DE} = \frac{500}{\sin 105°} \times \sin 120°$ $= 448.287$ N and At B (Figure 5.20): $\Sigma F_x = 0$ $\Sigma F_y = 0$ In this case, it is not possible to apply Lami's theorem, there being 4 forces. $\Sigma F_y = 0$ or $-400 + T_{BC} \sin 60° - T_{BD} \sin 30° = 0$ ∴ $T_{BC} = \frac{400 + 366.025 \sin 30°}{\sin 60°}$ $= 673.204$ N $\Sigma F_x = 0$ or $-T_{BA} + T_{BC} \cos 60° + T_{BD} \cos 30° = 0$ ∴ $T_{BA} = 673.204 \cos 60° + 366.025 \cos 30°$ $= 653.588$ N	2+2 M 4M	8 M	

6	a)	**What are the laws of dry friction?** **Laws of Dry Friction** 1. The Force of Friction always acts in the direction opposite to that in which the body tends to move.  Fig. 2. The magnitude of limiting friction (F) bears a constant ratio to the normal reaction (R) between the two surfaces i.e., $\frac{F}{R} = \mu$ (constant). 3. The magnitude of Force of Friction is exactly equal to the force, which tends the body to move, as long as the body is at rest (i.e., P = F). 4. The force of friction is independent of the area of contact between two surfaces. 5. The force of friction depends upon the roughness of the surfaces in contact.	4M	4 M	10 M
	b)	**Find the resultant magnitude, direction and its point of application from A for the square plate subjected to forces shown in Fig.4.** *Solution* Magnitude of resultant (R): $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$ $\Sigma F_x = 45° \sin 30° + 60° \sin 60° - 75° \cos 45° = 21.428$ $\Sigma F_y = -45° \cos 30° + 60° \cos 60° - 75° \sin 45° = -62.004$ $\therefore$ $R = \sqrt{(21.428)^2 + (-62.004)^2}$ =65.602 kN **Ans.** Direction of resultant (θ): $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$ $\therefore$ $\theta = \tan^{-1}\left(\frac{-62.004}{21.428}\right) = 70.93°$ **Ans.** Position of resultant with respect to point A (d): $\Sigma M_A = -75 \cos 45° \times 3 - 60 \cos 60° \times 2 + 45 \sin 30° \times 3 + 45 \cos 30° \times 2 + 100$ = 26.343 kN-m $x\text{-intercept} = \left\lvert\frac{\Sigma M_A}{\Sigma F_y}\right\rvert = \left\lvert\frac{26.343}{65.602}\right\rvert = 0.425$ m $y\text{-intercept} = \left\lvert\frac{\Sigma M_A}{\Sigma F_x}\right\rvert = \left\lvert\frac{26.343}{21.248}\right\rvert = 1.239$ m **Ans.**	2M 2M 2M	6 M	