USN

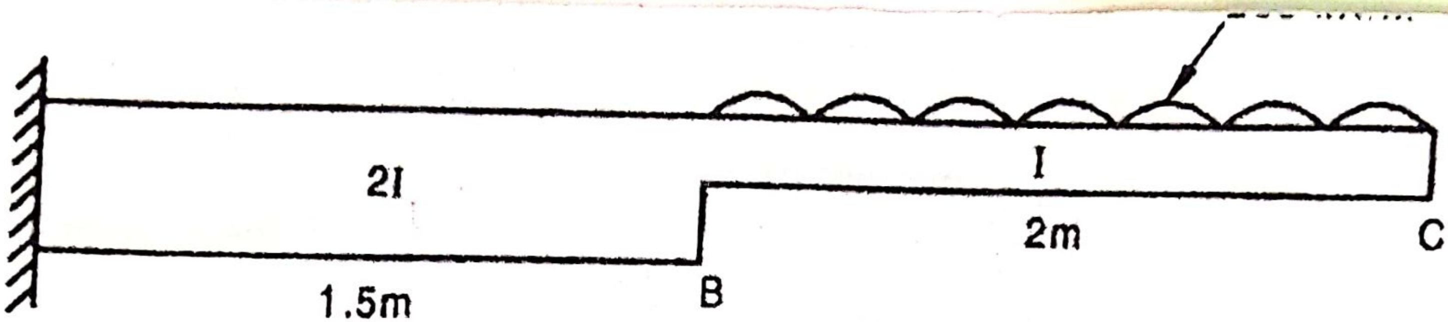
CMRIT

Internal Assessment Test 2

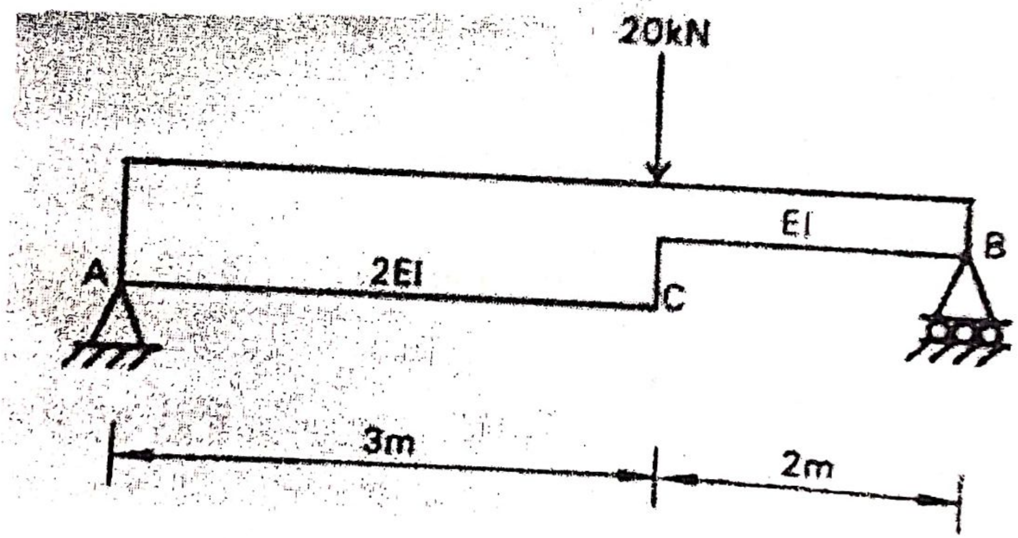
Sub:	Analysis of Determinate Structures			Sub Code:	17CV42	Section:	A&B
Date:	15-04-2019	Duration:	90 min's	Max Marks:	50	Sem	4th

Answer any TWO FULL Questions From Part-A and Part B compulsory question.

		MARKS	OBE CO	RBT
	PART A			
1 (a)	Find Maximum Slope and deflection of Cantilever beam at free end, Loaded with UDL throughout the length of the beam with usual Notations by using Double Integration Method.	[10]	CO2	L2
(b)	State And Prove Moment Area Theorem.	[10]	CO2	L3
2 (a)	A simply supported beam of length 6m Carries two point loads of 48KN and 40KN at a distance of 1m and 3m respectively from Left support. Find Deflection under each load and maximum deflection By using Macaulay's Method. Take E= $2x10^5$ N/mm^2.I=$8.5x10^7mm^4$	[14]	CO2	L2
(b)	Derive the expression for maximum deflection and slope if cantilever beam subjected to Moment at free end. [any Method]	[06]	CO2	L3
3 (a)	State Conjugate beam Theorems .	[4]	CO2	L2
(b)	Calculate slope and deflection at B in fig 1 using Conjugate Beam method. Take E=$210x10^9$ N/m2, I=$120x10^6$ mm4.	[8]		
(c)	Determine Slope and Deflection under Point load in fig.2 using conjugate beam Method.	[8]		
	PART B			
4	A three Hinged parabolic arch has a span of 24m and a central rise of 4m. It carries a concentrated load of 50 KN at 18 m from the left support and UDL of 30KN/m over the half portion. Determine Bending Moment , Normal thrust and Radial Shear at a section 8m from the left support.	[10]	CO3	L2



Fig(1)



Selected Fig(2)

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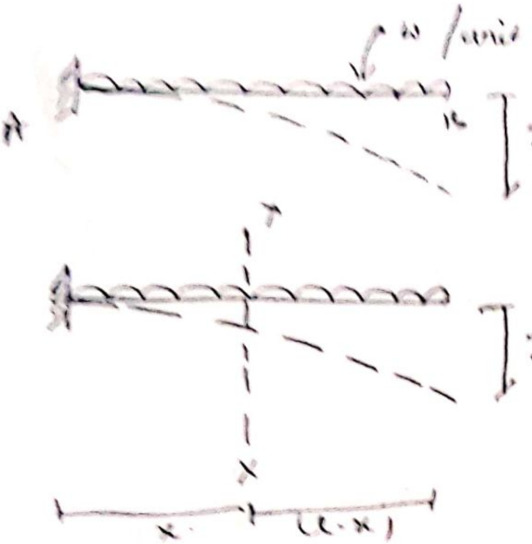
2. Cantilever beam loaded with UDL.



A cantilever AB of length L fixed at point A. Let 'L' be the length & 'w' be UDL load.

consider Moment @ x - x

$$\therefore M_x = -w(L-x)\cdot\frac{(L-x)}{2}$$

$$= -\frac{w(L-x)^2}{2}$$

Now apply diff eqn

$$E.I.\frac{d^2y}{dx^2} = M = -\frac{w(L-x)^2}{2} \quad -(1)$$

Integrating above eqn

$$E.I.\frac{dy}{dx} = -\frac{w(l-x)^3}{6}\cdot(-1) + C_1$$

$$= \frac{w(l-x)^3}{6} + C_1 \quad - (2)$$

Integration on $\frac{(L-x)^2}{2}$,

Let, $u = L-x$.

$\frac{du}{dx} = -1$

$\therefore du = -dx$

$\therefore \int u^2(-du) = \frac{u^3}{3}$.+

$\therefore u = (L-x)$

$\therefore \frac{(L-x)^3}{3}\cdot(-1)$

Integrating eqn (2)

$$EI\cdot y = \frac{w(l-x)^4}{24}\times(-1) + C_1x + C_2$$

$$= \frac{-w(l-x)^4}{24} + C_1x + C_2 \quad - (3)$$

Applying boundary condition

(i) @ $x=0$, slope $\frac{dy}{dx} = 0$

$\therefore$ eqn (2) becomes, $C_1 = -\frac{wl^3}{6}$

$\therefore$ Slope eqⁿ becomes.

$$E.I.\frac{dy}{dx} = \frac{w(l-x)^3}{6} - \frac{wL^3}{6}$$

$$\therefore \frac{dy}{dx} = \frac{1}{EI}\left[\frac{w(l-x)^3}{6} - \frac{wL^3}{6}\right] \quad - (4)$$

ii) @ $x=0$, deflection $y=0$.

$\therefore$ In eqⁿ 3:
$$0 = -\frac{wL^4}{24} + C_2 \qquad \therefore \underline{C_2 = \frac{wL^4}{24}}$$

$\therefore$ eqⁿ (3) becomes.

$$EI\cdot y = \frac{-w(l-x)^4}{24} - \frac{wL^3x}{6} + \frac{wL^4}{24}$$

$$y = \frac{1}{EI}\left[\frac{-w(L-x)^4}{24} - \frac{wL^3x}{6} + \frac{wL^4}{24}\right] \quad - (5)$$

Putting $x=L$ in slope & deflection eqⁿ.

$$\frac{dy}{dx} = \theta_B = \frac{1}{EI}\left[\frac{w(L-L)^3}{6} - \frac{wL^3}{6}\right]$$

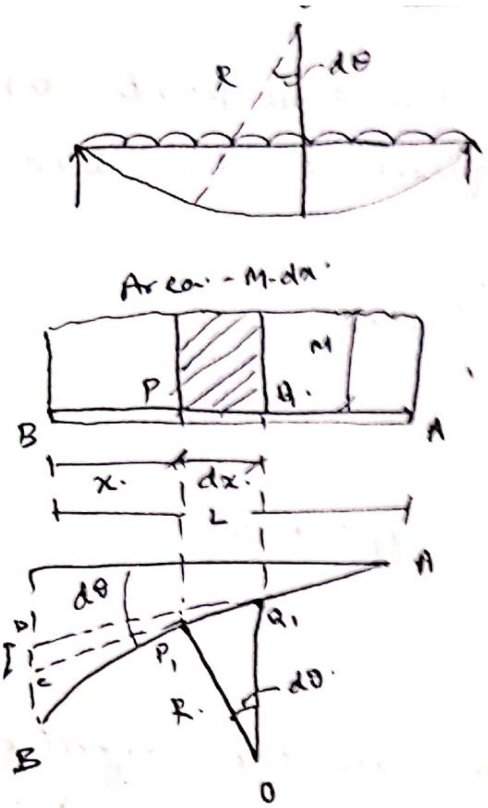
$$\boxed{\theta_B = -\frac{wL^3}{6EI} \quad \text{or} \quad \theta_B = \frac{wL^3}{6EI}}$$

Eqⁿ(5) becomes.

$$y_B = \frac{1}{EI}\left[\frac{-w(L-L)^4}{24} - \frac{wL^3\times L}{6} + \frac{wL^4}{24}\right]$$

$$= \frac{-4wL^4 + wL^4}{24EI} = \frac{-3wL^4}{24EI}$$

$$\boxed{y_B = -\frac{wL^4}{8EI} \quad \text{or} \quad \frac{wL^4}{8EI}}$$



In the above fig beam AB is loaded [Any loading] & hence subjected to bending Moment as shown. Let beam bent to AQ_1P_1B

Let A be be the point of zero slope & zero deflection.

consider of small length dx at a distance x from B.

The corresponding points on the deflected beam are P_1Q_1.

Let R = Radius of curvature of deflected part P_1Q_1

$d\theta$ = Angle subtended by arc P_1Q_1 at the centre 'O'.

M - B.M b/w P & Q.

P_1C = tangent at point P_1

Q_1D = tangent at Point Q_1

~~The mandates~~

The tangent P_1 & Q_1 are cutting the vertical line through B at Point C & D

$$P_1Q_1 = R \cdot d\theta.$$

$$P_1Q_1 = dx$$

Arc of length is given by $l = r \cdot d\theta$

$$\therefore P_1Q_1 = R \cdot d\theta.$$

$$\therefore dx = R \cdot d\theta.$$

$$\therefore d\theta = \frac{dx}{R} \quad -(i)$$

But for loaded beam,

$$\frac{M}{I} = \frac{E}{R}$$

or

$$R = \frac{EI}{M}$$

Sub the values of R in eq (i)

$$d\theta = \frac{dx}{\left(\frac{EI}{M}\right)} = \frac{M \cdot dx}{E \cdot I}.$$

Since the slope at A is assu[med] zero, hence total slope at B is obtained by int[egrat]ing the above eqn with limit

Mohr's theorem

Theorem 1 -

The change in the slope b/w two points on a straight member under flexure is equal to the area of $\frac{M}{EI}$ diagram b/w the two points

$$\theta_{AB} = \theta_B - \theta_A = \frac{1}{EI}\int_A^B M \cdot dx.$$

$$= \frac{1}{EI}\int_A^B M \cdot dx.$$

or
$$\text{slope}, \theta = \int \frac{M}{EI} \cdot dx = \frac{\text{Area of B.M.D}}{EI}.$$

Theorem 2

" Deflection at a point in a beam in the direction perpendicular to its original straight line position measured from the tangent to the elastic curve at another point is given by the Moment of $\frac{M}{EI}$ diagram about the point where deflection required

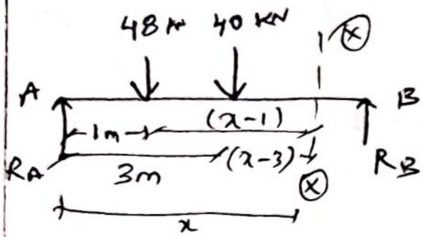
$$y = \int \frac{M\bar{x}}{EI} dx = \frac{\text{Area of BMD}\,\bar{x}}{EI}$$

Note: $1\,N/mm^2 = 10^6\,N/m^2$

$1\,Pa = 1\,N/m^2 = 10^{-6}\,N/mm^2$

2.a A simply supported beam of length 6m is simply supported at its ends & carries two point loads of 48 kN & 40 kN at a distance 1m & 3m resp from left support. Find

(A) Deflection under each load & (B) Maximum deflection.



$E = 2\times10^5 \times 10^6 = 2\times10^{11}\ N/m^2$

$I = 8.5\times10^7 \times 10^{-12} = 8.5\times10^{-5}\ m^4.$

To find Reactions-

$R_A + R_B = 48+40$

$R_A + R_B = 88\ kN$

$\Sigma M_A = 0,$

$R_B \times 6 = (40\times3) + (48\times1)$

$\boxed{\therefore R_B = 28\ kN}$

$\therefore R_A = 60\ kN.$

B.M eqn

$M_x \Rightarrow (R_A \times x) - 48(x-1) - 40(x-3)$

$= 60x - -48(x-1) - 40(x-3)$ — (1)

Now apply diff eqn, we get,

$$EI\cdot\frac{d^2y}{dx^2} = M_x = 60x - 48(x-1) - 40(x-3) \quad (2)$$

Integrate eqn (2)

$$EI\cdot\frac{dy}{dx} = \frac{60x^2}{2} + C_1 - \frac{48(x-1)^2}{2} - \frac{40(x-3)^2}{2}$$

$$= 30x^2 + C_1 - 24(x-1)^2 - 20(x-3)^2 \quad (3)$$

Integrate once agin eqn (2)

$$EI\ y = \frac{30x^3}{3} + C_1x + C_2 - \frac{24(x-1)^3}{3} + \frac{-20(x-3)^3}{3}$$

$$= 10x^3 + C_1x + C_2 - 8(x-1)^3 - 6.67(x-3)^3 \quad (4)$$

2 proble continued.

Boundary condition

(i) @ $x=0$, $y=0$.

$\therefore$ Eq^n (4) becomes $c_2 = 0$

(ii) @ $x = L = 6m$, $y=0$

$\therefore$ eq^n (4) becomes.

$$0 = 10\times(6)^3 + C_1\times 6 - 8(6-1)^3 - 6.667(6-3)^3$$

$$\therefore \quad 6C_1 = -980$$

$$\boxed{C_1 = -163.33}$$

$\therefore$ Deflection eq^n (4) becomes.

$$y = \frac{1}{EI}\left[10x^3 - 163.3x \Big| - (8(x-1)^3 \Big| - 6.667(x-3)^3\right] \quad -(5)$$

(4) To find the deflection under each load.

(i) Deflection under first load, $P = 48$ kN.

$\therefore$ @ $x = 1m$ @ C

$$y_c = \frac{1}{EI}\left[10\times 1^3 - 163.3\times 1\right] = \frac{10 - 163.3}{17\times 10^3} = -0.009$$

(ii) Deflection under second load, $P = 40$ kN. @ $x = 3$ m @ D.

$$y_D = \frac{10\times 3^3 - (163.3\times 3) - 8(3-1)^3}{17\times 10^3}$$

$$= -16.7 \text{ mm} \downarrow$$

To find max deflection

The deflection is likely to be max at a section b/w C & D.

For maximum deflection. $\frac{dy}{dx} = 0$ hence eq the eq^n (3) equal to zero up to second line

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$\therefore$ eqn (3) becomes.

$$0 = 30x^2 - 163.3 - 24(x-1)^2$$

$$6x^2 + 48x - 187.33 = 0.$$

$$\therefore \boxed{x = 2.87m.}$$

$\therefore$ eqn (5) become

$$y = \frac{10 \times (2.87)^3 - 163.33 \times 2.87 - 8(2.87-1)^3}{17 \times 10^3}$$

$$= -16.4795\, m \quad \downarrow.$$

Conjugate Beam Method. 3a

Theorem 1 -

The slope at any section of the given beam or real beam is equal to the shear force at the corresponding section of conjugate beam.

$V = R' =$ shear force. or $\theta = R' = V$

where, $\theta =$ slope
$V =$ shear force.

Theorem 2

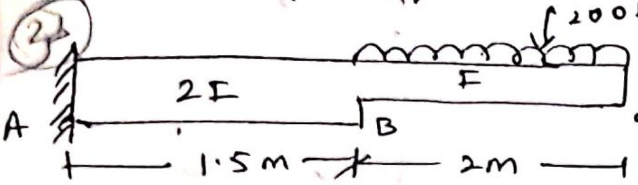
The bending moment at any point on conjugate beam is equal to the deflection at the corresponding point on real beam.

or.

The deflection at any section for the given beam or real beam is equal to the bending moment at the corresponding section of the conjugate beam.

Cal slope & deflection at B & C for the beam show in fig.

(2)



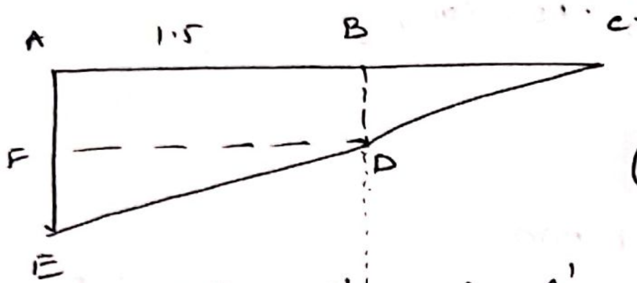
$E = 210 \times 10^{7}\ N/m^2$.

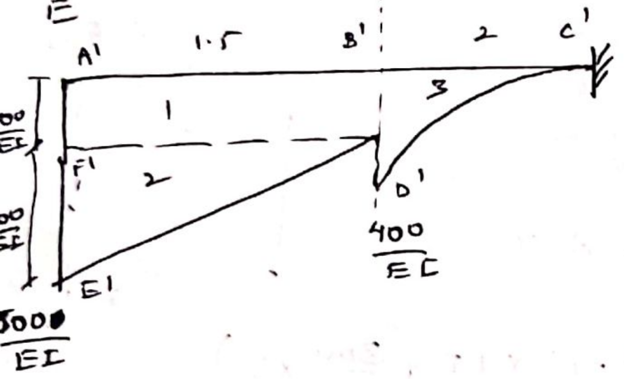
$I = 120 \times 10^{6}\ mm^4$.

$= 120 \times 10^{+6} \times 10^{-12}\ m^4$.

$EI = 2.1 \times 10^{11} \times 120 \times 10^{-6}$

$EI = 25200\ kN.m^2$





① B.M. calculations

$M_C = 0$

$M_B = -200 \times 2 \times \frac{2}{2} = -400\ kN.m$

$M_A = -200 \times 2 \times \left(\frac{2}{2} + 1.5\right)$

$M_A = -1000\ kN.m$.

③ Slope @ B.

$\theta_B = [A_1 + A_2]$

$= \left[-1.5 \times \frac{200}{EI} - \frac{1}{2} \times 1.5 \times \frac{300}{EI}\right]$

$= -\frac{525}{EI} = \frac{525}{25200} = -0.020$ rad

④ Slope @ C [θ_C = S.F @ C for conjugate beam]

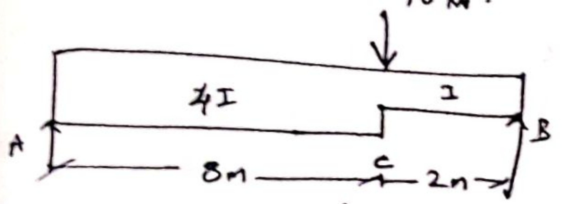
$\theta_C = [A_1 + A_2 + A_3]$

$= \left[-1.5 \times \frac{200}{EI} - \frac{1}{2} \times 1.5 \times \frac{300}{EI} - \frac{1}{3} \times 2 \times \frac{400}{EI}\right.$

$= -\frac{791.66}{EI} = -\frac{791.66}{25200}$

$= -0.031$

Using conjugate beam method, find slope at 'A' & deflection at 10 kN load for the beam shown.

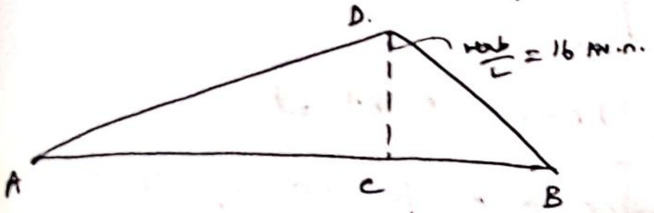


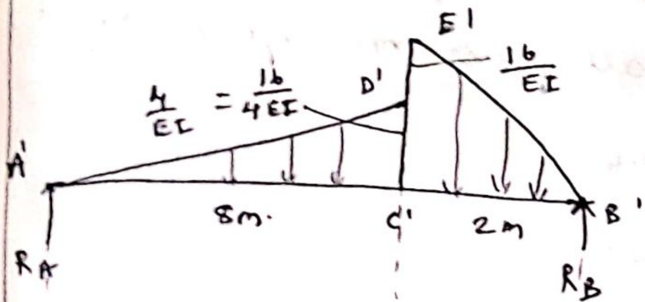
Soln :

Bending Moment calculation.

$M_A = M_B = 0$,

$$M_C = \frac{Wab}{L} = \frac{10 \times 8 \times 2}{10} = 16\ \text{kN.m}$$

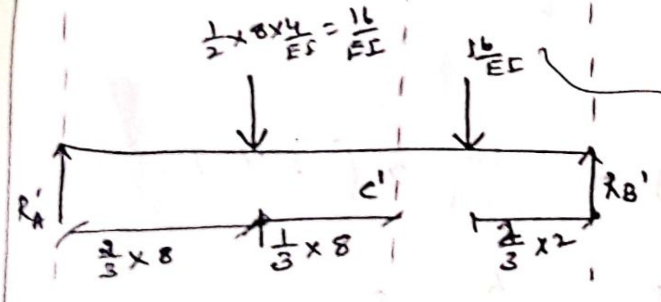




[convert UVL load i.e. $A'C'D'$ to point load] .

i.e. $= \frac{1}{2} \times AC' \times C'D'$

$$= \frac{1}{2} \times 8 \times \frac{4}{EI} = \frac{16}{EI}$$



[convert UVL load to i.e. $C'E'B'$ to point load.

$$= \frac{1}{2} \times C'B' \times C'E'$$

$$= \frac{1}{2} \times 2 \times \frac{16}{EI} = \frac{16}{EI}$$

Find Reaction.

$\Sigma M_B = 0 \Rightarrow R_A' \times 10 - \frac{16}{EI} \times \left[\frac{1}{3} \times 8 + 2\right] - \frac{16}{EI}\left[\frac{2}{3} \times 2\right] = 0$

$$\therefore 10 R_A' = \frac{96.01}{EI}$$

$$\therefore \boxed{R_A' = \frac{9.6}{EI}}$$

$\Sigma V = 0$

$$R_A' + R_B' = \frac{16}{EI} + \frac{16}{EI}$$

$$\therefore \quad R_B' = \frac{32}{EI} - \frac{9.6}{EI}$$

$$\boxed{\therefore \quad R_B' = \frac{22.4}{EI}}$$

According to conjugate beam Method,

Slope @ A = shear force @ A' for conjugate beam

[shear force is sum of all forces on left or right p

$$\theta_A = R_A' = \frac{9.6}{EI}$$

Deflection at 10 kN load.

y_c = B.M. @ c' for conjugate beam.

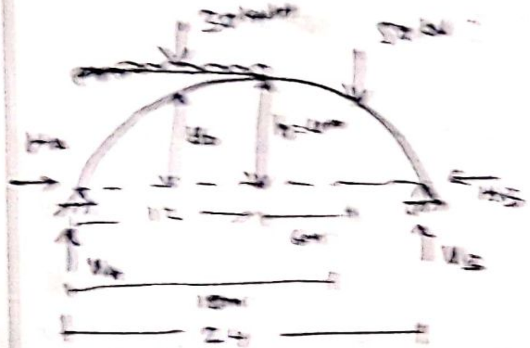
y_c = B.M

$$= (R_A' \times 8) - \frac{16}{EI} \times \left(\frac{1}{3} \times 8\right)$$

$$= \frac{9.6}{EI} \times 8 - \frac{16}{EI} \times \frac{8}{3}$$

$$\boxed{y_c = \frac{34.13}{EI}}$$

4) A three hinged parabolic arch hinged at supports & at the crown has a span of 24 m & central rise of 4m. It carries a concentrated load of 50 kN at 18m from the left support & a UDL of 30 kN/m over the left half portion. Det BM, Normal thrust & radial shear at a section 6m from left support.



i) Vertical reaction.

$\Sigma M_A = 0$,

$$+30 \times 12 \times \frac{12}{2} + 50 \times 18 - V_B \times 24 = 0$$

$$\therefore \underline{V_B = 127.5 \text{ kN}}$$

$\Sigma V = 0$,

$$V_A + V_B = (30 \times 12) + 15$$

$$\therefore \underline{V_A = 282.5 \text{ kN}}$$

Horizontal reaction-

$\Sigma M_C = 0$,

$$-V_B \times 12 + H_B \times 4 + 50 \times 6 = 0 \quad \therefore \quad \underline{H_B = H_A = 307.5 \text{ kN}}$$

ii) Compute vertical ordinate -

$$y_x = \frac{4h x}{L^2}(L - x) = \frac{4 \times 4 \times 6}{24^2}(24 - 6) = \underline{3m}$$

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x) = \frac{4 \times 4}{24^2}(24 - 2 \times 6) =$$

$$\therefore \underline{\theta = 18.43^\circ}$$

B.M calculation:

$$M_D = V_A \times 6 - H_A \times y_D - 30 \times 6 \times \frac{6}{2}$$

$$= 232.5 \text{ kN-m}$$

Normal thrust & radial shear @ 6m from support.

Shear force $V = V_A - 30 \times 6 = 282.5 - 30 \times 6 = 102.5 \text{ kN}$.

Normal thrust $= H \cos\theta + V \sin\theta$.

$$= 307.5 \times \cos 18.43 + 102.5 \times \sin(18.43^\circ)$$

$$\boxed{N = 324 \text{ kN}}$$

Radial shear $= H \sin\theta - V \cos\theta$

$$= 307.5 \times \sin 18.4 - 102.5 \times \cos 18.4$$

$$\boxed{F = -0.027 \text{ kN}}$$