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CMRIT

INTERNAL ASSESSMENT TEST – I

Sub:	DIGITAL COMMUNICATION							Code:	15EC61
Date:	12/ 03 / 2018	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	ECE,TCE

Answer any 5 full questions

		Marks	CO	RBT
1.	Define Hilbert transform. Plot the magnitude response and phase response of an ideal Hilbert transformer. Derive the impulse response of the ideal Hilbert transformer.	[10]	CO1	L3
2(a).	State and prove the properties of ideal Hilbert transformer.	[06]	CO1	L2
2(b)	Determine the Hilbert transform of $m(t)\cos(2\pi f_c t)$ assuming that $m(t)$ is a low pass signal bandlimited to W Hz and $f_c \gg W$.	[04]	CO1	L2
3(a)	Determine the Hilbert transform of the signal $x(t)$ where $x(t) = \begin{cases} A & for\ -\frac{T_b}{2} \le t \le \frac{T_b}{2} \\ 0 & otherwise \end{cases}$	[05]	CO1	L2
3(b)	Determine the Hilbert transform of $x(t) = sinc(t)$.	[05]	CO1	L2
4	Discuss pre-envelope with positive frequencies and complex envelope of bandpass signals with relevant equations and waveforms. Determine the pre-envelope and complex envelope of $x(t) = sinc(t)$.	[10]	CO1	L2
5	Derive an expression for the canonical representation of bandpass signals. Obtain a scheme for extracting in-phase and quadrature components of bandpass signals.	[10]	CO1	L2
6(a)	For the binary data "101101" plot the following waveforms. i) NRZ Unipolar ii) NRZ Polar iii) RZ Polar iv) RZ Unipolar v) NRZ Bipolar vi) Manchester	[06]	CO1	L2
6(b)	Perform differential encoding on the binary data "101101". Show that original binary data can be decoded even in the presence of polarity inversion.	[04]	CO1	L2
7	Derive an expression for power spectral density of NRZ bipolar format assuming equiprobable and statistically independent 0s and 1s. Plot the resulting power spectral density.	[10]	CO1	L3

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INTERNAL ASSESSMENT TEST-I

SOLUTIONS

1. Define Hilbert transform. Plot the magnitude response and phase response of an ideal Hilbert transformer. Derive the impulse response of the ideal Hilbert transformer.

- Hilbert tranform is the process of transforming one time domain signal into an another time domain signal.
- The phase angles of the signal are shifted by $\pm\frac{\pi}{2}$.

$$H(f) = \begin{cases} e^{-j\frac{\pi}{2}}, & f>0 \\ 0, & f=0 \\ e^{+j\pi/2}, & f<0 \end{cases}$$

(or)

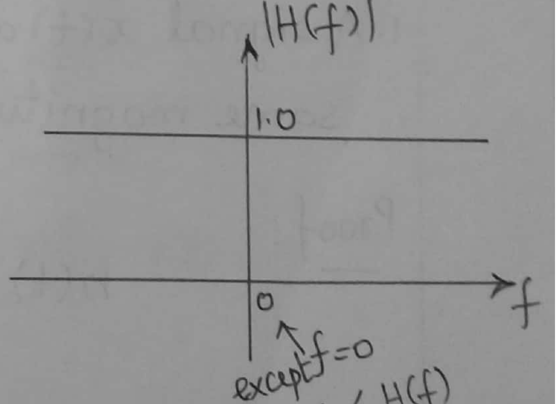
$$H(f) = \begin{cases} -j, & f>0 \\ 0, & f=0 \\ +j, & f<0 \end{cases}$$

- Alternatively,

$$H(f) = -j \cdot \text{sgn}(f)$$

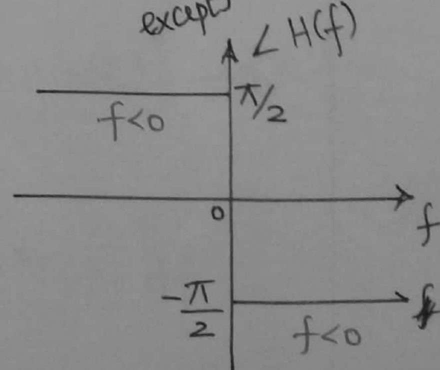
- Magnitude response of $H(f)$,

$$|H(f)| = \begin{cases} 1, & f>0 \\ 0, & f=0 \\ 1, & f<0 \end{cases}$$



- Phase response of $H(f)$,

$$\angle H(f) = \begin{cases} -\pi/2, & f>0 \\ 0, & f=0 \\ +\frac{\pi}{2}, & f<0 \end{cases}$$



$$h(t) = \int_{-\infty}^{\infty} H(f) \cdot e^{+j2\pi ft} \cdot df$$

$$= \int_{-\infty}^{0} j \cdot e^{j2\pi ft} \cdot df + \int_{0}^{\infty} (-j) \cdot e^{j2\pi ft} \cdot df$$

$$= j \int_{-\infty}^{0} \left(\lim_{a \to 0} e^{af} \right) \cdot e^{j2\pi ft} \cdot df - j \int_{0}^{\infty} \left(\lim_{a \to 0} e^{-af} \right) \cdot e^{j2\pi ft} \cdot df$$

$$= j \lim_{a \to 0} \left[\frac{1-0}{a+j2\pi t} \right] - j \lim_{a \to 0} \left[\frac{0-1}{a-j2\pi t} \right]$$

$$h(t) = \frac{j}{j2\pi t} + \frac{j}{j2\pi t}$$

$h(t) = \frac{1}{\pi t}$, impulse response of an ideal Hilbert transformer.

2(a) State and Prove the properties of ideal Hilbert transformer.

1. A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ have the same magnitude spectrum.

Proof:

$h(t) = \frac{1}{\pi t}$ (ideal response of ideal transform) $\rightarrow$ (1)

$H(f) = -j \cdot sgn(f)$ (frequency response) $\rightarrow$ (2)

$|H(f)| = 1$, for all frequencies, except $f=0$

$\hat{x}(t) = x(t) * h(t) \rightarrow$ (3)

Then, $\hat{X}(f) = X(f) \cdot H(f)$

$$|\hat{X}(f)| = |X(f)| \cdot |H(f)|$$

$$= |X(f)| \longrightarrow (4)$$

2. If $\hat{x}(t)$ is the Hilbert transform of $x(t)$, then the Hilbert transform of $\hat{x}(t)$ is $-x(t)$.

Proof:

$$\hat{x}(t) = x(t) * h(t) \rightarrow (1)$$

$$\hat{X}(f) = X(f) \cdot H(f) = -j \cdot sgn(f) \cdot X(f) \rightarrow (2)$$

Hilbert transform of $\hat{x}(t)$

$$\hat{\hat{x}}(t) = \hat{x}(t) * h(t) \rightarrow (3)$$

$$\hat{\hat{X}}(f) = \hat{X}(f) \cdot H(f) = [-j \cdot sgn(f) \cdot X(f)][-j \cdot sgn(f)]$$

$$\hat{\hat{X}}(f) = -X(f) \rightarrow (4)$$

Then, $\hat{\hat{x}}(t) = -x(t) \rightarrow (5)$

3. A signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ are orthogonal over the entire time interval $(-\infty, \infty)$.

Proof:

$$\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t) \cdot dt = 0 \rightarrow (1)$$

$$\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t) \cdot dt = \int_{-\infty}^{\infty} X(f) \cdot \hat{X}^*(f) \cdot df \rightarrow (2)$$

$$\hat{x}(t) = x(t) * h(t) \longrightarrow (3)$$

$$\hat{X}(f) = X(f) \cdot H(f) = X(f)[-j \cdot sgn(f)] \rightarrow (4)$$

$$\hat{X}^*(f) = j \cdot sgn(f) \cdot X^*(f) \rightarrow (5)$$

$$\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t) \cdot dt = \int_{-\infty}^{\infty} X(f) \cdot [j \cdot sgn(f) \cdot X^*(f)] \cdot df$$

$$= j \int_{-\infty}^{\infty} sgn(f) . |X(f)|^2 \, df$$

$$\int_{-\infty}^{\infty} x(t) . \hat{x}(t) . dt = 0 \longrightarrow (6)$$

2(b) Determine the Hilbert transform of $m(t).\cos(2\pi f_c t)$ assuming that $m(t)$ is an low pass signal bandlimited to W Hz and $f_c >> W$.

Solution:

$$x(t) = m(t) . \cos(2\pi f_c t)$$

$$X(f) = M(f) * \frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

$$X(f) = \frac{1}{2}\left[M(f - f_c) + M(f + f_c)\right] \rightarrow (1)$$

$$H(f) = \begin{cases} -j, & f > 0 \\ 0, & f = 0 \\ +j, & f < 0 \end{cases} \rightarrow (2)$$

$$\hat{X}(f) = X(f) . H(f) = \frac{-j}{2} . M(f - f_c) + \frac{j}{2} M(f + f_c) \rightarrow (3)$$

$$\hat{x}(t) = \frac{1}{2}\left[-j . e^{j2\pi f_c t} . m(t) + j . e^{-j2\pi f_c t} . m(t)\right]$$

$$= \frac{-j}{2} m(t)\left[e^{j2\pi f_c t} - e^{-j2\pi f_c t}\right]$$

$$= -j . \frac{m(t)}{2} . 2j . \sin(2\pi f_c t)$$

$$= -j^2 . m(t) . \sin(2\pi f_c t)$$

$$\hat{x}(t) = m(t) . \sin(2\pi f_c t)$$

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3a) Determine the Hilbert transform of the signal $x(t)$ where

$$x(t) = \begin{cases} A, & \text{for } -\frac{T_b}{2} \le t \le \frac{T_b}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Solution:

$$\hat{x}(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau).h(t-\tau).d\tau$$

$$= \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} A.\frac{1}{\pi(t-\tau)}.d\tau = \frac{A}{\pi}\int_{-\frac{T_b}{2}}^{T_b/2} \frac{1}{t-\tau}.d\tau$$

put $u = t-\tau$ when $\tau = -\frac{T_b}{2}$, $u = t + T_b/2$

$du = -d\tau$

$\tau = \frac{T_b}{2}$, $u = t - T_b/2$

$$\hat{x}(t) = \frac{A}{\pi}\int_{t+T_b/2}^{t-T_b/2} \frac{1}{u}(-du) = \frac{A}{\pi}\Big[-\ln(u)\Big]_{t+T_b/2}^{t-T_b/2}$$

$$\hat{x}(t) = \frac{A}{\pi}\left[\ln\frac{(t+T_b/2)}{(t-T_b/2)}\right]$$

3(b) Determine the Hilbert transform of $x(t) = \mathrm{sinc}(t)$.

Solution:

$$x(t) = \mathrm{sinc}(t)$$

$$X(f) = \begin{cases} 1, & -1/2 < f < 1/2 \\ 0, & \text{otherwise} \end{cases}$$

$$\hat{X}(f) = X(f).H(f) = \begin{cases} -j, & 0 < f < 1/2 \\ 0, & f = 0 \\ +j, & -\frac{1}{2} < f < 0 \end{cases}$$

$$\hat{x}(t) = \int_{-1/2}^{0} j.e^{j2\pi ft}.df + \int_{0}^{1/2} (-j).e^{j2\pi ft}.df$$

$$= j\left[\frac{e^{j2\pi ft}}{j2\pi t}\right]_{-1/2}^{0} - j\left[\frac{e^{j2\pi ft}}{j2\pi t}\right]_{0}^{1/2}$$

$$= j\left[\frac{1-e^{j\pi t}}{j2\pi t}\right] - j\left[\frac{e^{j\pi t}-1}{j2\pi t}\right]$$

$$= \frac{j}{j2\pi t}\left[1-e^{j\pi t}-e^{j\pi t}+1\right]$$

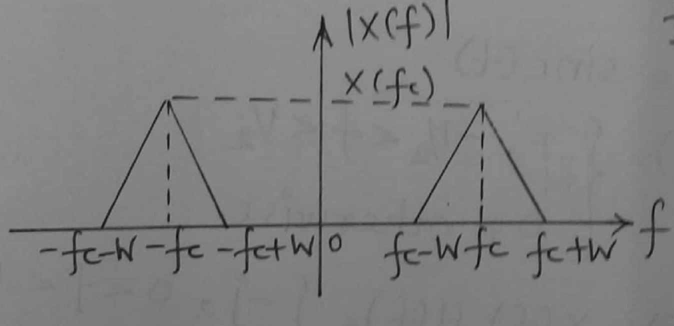
$$= \frac{j}{j2\pi t}\left[2-2\cos(\pi t)\right]$$

$$\hat{x}(t) = \frac{1}{\pi t}\left[1-\cos(\pi t)\right]$$

4) Discuss pre-envelope with positive frequencies and complex envelope of bandpass signals with relevant equations and envelope waveforms. Determine the pre-envelope and complex envelope of $x(t) = \text{sinc}(t)$

Let $X(f)$ be confined to a band of frequencies

$$f_c - W \le |f| \le f_c + W$$



→ Pre-envelope of a signal, $x_+(t) = x(t) + j\hat{x}(t) \rightarrow ①$

$$X_+(f) = X(f) + j\hat{X}(f) \rightarrow ②$$

$$= X(f) + j\left[-j\,\text{sgn}(f).X(f)\right]$$

$$= X(f) + \text{sgn}(f).X(f) \rightarrow ③$$

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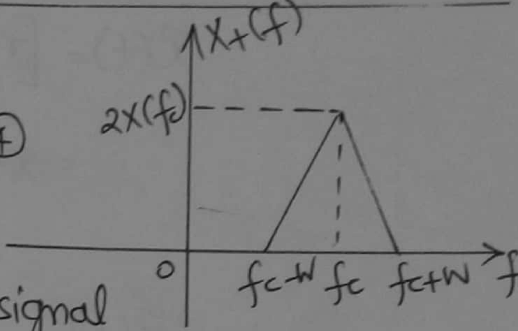
$$X_+(f) = \begin{cases} 2X(f), & f>0 \\ X(0), & f=0 \\ 0, & f<0 \end{cases} \rightarrow ④$$



- Complex envelope of a bandpass signal

$$\tilde{x}(t) = x_+(t) \cdot e^{-j2\pi f_c t} \rightarrow ⑤$$

- $x(t) = \text{sinc}(t)$; $\hat{x}(t) = \dfrac{1-\cos(\pi t)}{\pi t}$ (Refer 3b ques)

$$x_+(t) = x(t) + j\hat{x}(t)$$

$$= \text{sinc}(t) + j\left[\frac{1-\cos(\pi t)}{\pi t}\right]$$

$$x_+(t) = \frac{\sin(\pi t)}{\pi t} + j\left[\frac{1-\cos(\pi t)}{\pi t}\right]$$

$$\tilde{x}(t) = x_+(t) \cdot e^{-j2\pi f_c t}$$

$$= \left\{\frac{\sin(\pi t)}{\pi t} + j\left[\frac{1-\cos(\pi t)}{\pi t}\right]\right\} \cdot e^{-j2\pi f_c t}$$

$$\tilde{x}(t) = \frac{1}{\pi t}\left[\sin(\pi t) + j(1-\cos(\pi t)\right] \cdot e^{-j2\pi f_c t}$$

5. Derive an expression for the canonical representation of bandpass signals. obtain a scheme for extracting in-phase and quadrature components of bandpass signals.

$$\tilde{x}(t) = x_+(t) \cdot e^{-j2\pi f_c t} \rightarrow ①$$

We know that, $x_+(t) = x(t) + j\hat{x}(t) \rightarrow ②$

$$\tilde{x}(t) = \left[x(t) + j \cdot \hat{x}(t)\right] \cdot e^{-j2\pi f_c t}$$

$$\tilde{x}(t) = [x(t) + j\,\hat{x}(t)]\,.[\cos(2\pi f_c t) - j\sin(2\pi f_c t)]$$

$$= \underbrace{x(t).\cos(2\pi f_c t) + \hat{x}(t).\sin(2\pi f_c t)}_{x_I(t)} +$$

$$j\,[\underbrace{\hat{x}(t).\cos(2\pi f_c t) - x(t).\sin(2\pi f_c t)}_{x_Q(t)}]$$

$$\tilde{x}(t) = x_I(t) + j.x_Q(t) \quad \rightarrow (3)$$

$$\therefore\; x_+(t) = \tilde{x}(t).\,e^{j2\pi f_c t} \quad \rightarrow (4)$$

$$= [x_I(t) + j\,x_Q(t)]\,[\cos(2\pi f_c t) + j\sin(2\pi f_c t)]$$

$$= x_I(t).\cos(2\pi f_c t) - x_Q(t).\sin(2\pi f_c t)$$

$$+ j\,[x_Q(t).\cos(2\pi f_c t) + x_I(t).\sin(2\pi f_c t)]$$

Real part of $x_+(t) = x(t) = x_I(t).\cos(2\pi f_c t) - x_Q(t).\sin(2\pi f_c t) \rightarrow (5a)$

$\therefore$ This is the canonical representation of a band-pass signal.

$x_I(t)$: Multiply by $2.\cos(2\pi f_c t)$ in (5),

$$2.\cos(2\pi f_c t).\,x(t) = 2x_I(t).\cos^2(2\pi f_c t) - x_Q(t).\,2.\sin(2\pi f_c t)\cos(2\pi f_c t)$$

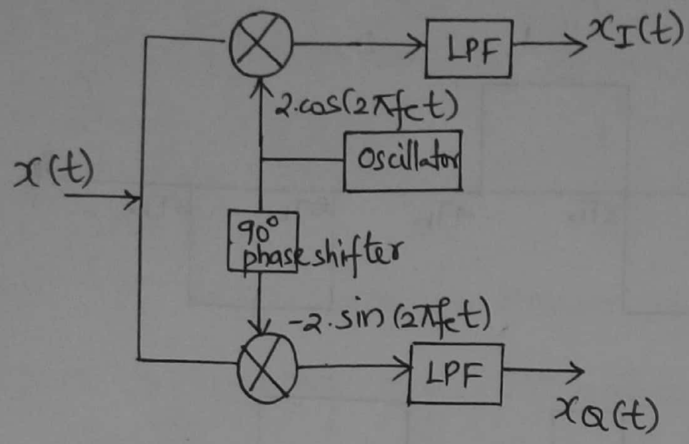
$$= 2.x_I(t)\left[\frac{1+\cos 2(2\pi f_c t)}{2}\right] - x_Q(t).\sin(4\pi f_c t)$$

$$= x_I(t) + \underbrace{\cos 2(2\pi f_c t).\,x_I(t) - x_Q(t).\sin(4\pi f_c t)}_{\text{filtered out}}$$

$$2.\cos(2\pi f_c t).\,x_+(t) = x_I(t)$$

$x_Q(t)$: Multiply by $-2.\sin(2\pi f_c t)$. in (5)

$$-2.\sin(2\pi f_c t).\,x_+(t) = x_Q(t)$$

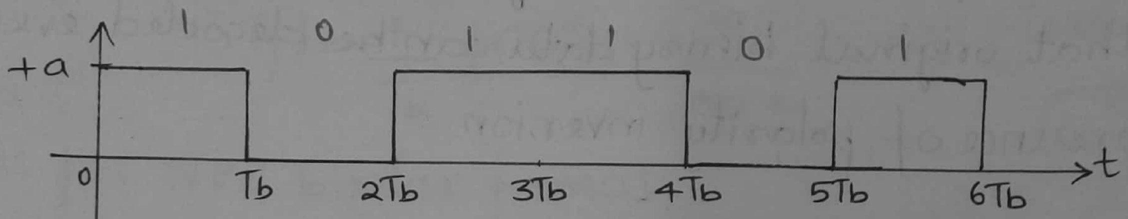


6a) For the binary data "101101" plot the following waveforms.

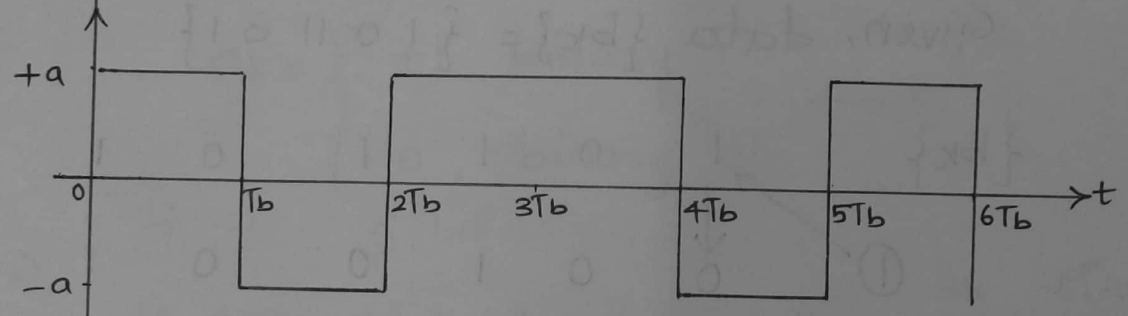
(i) NRZ unipolar ii) NRZ Polar iii) RZ Polar iv) RZ Unipolar

v) NRZ Bipolar vi) Manchester

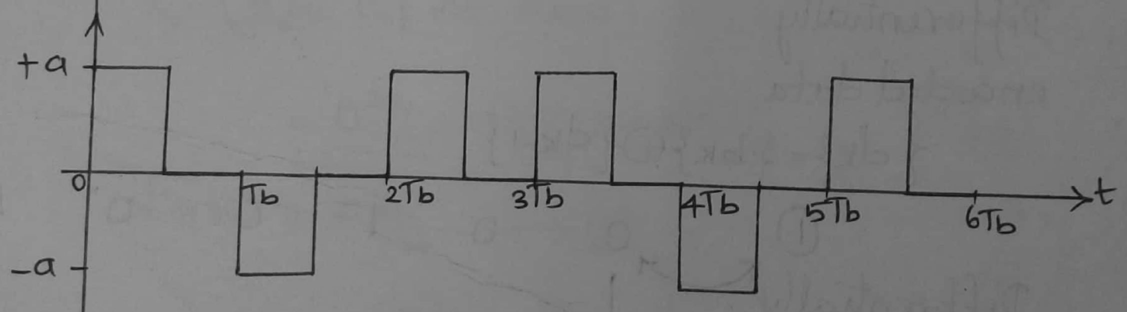
(i)



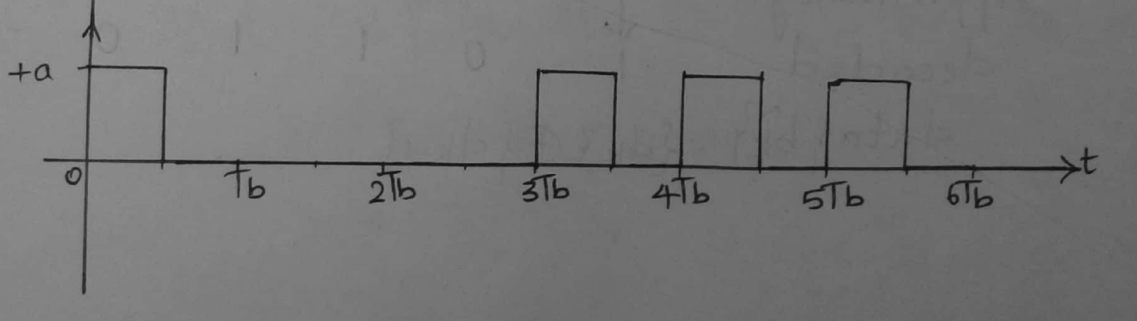
(ii)



(iii)



(iv)



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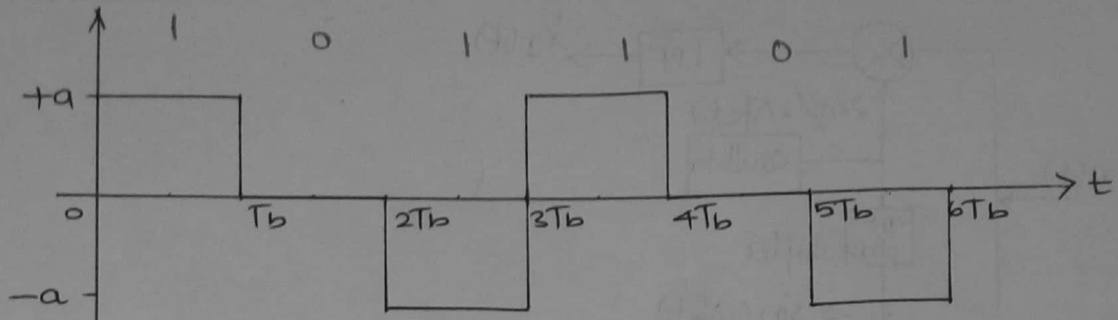
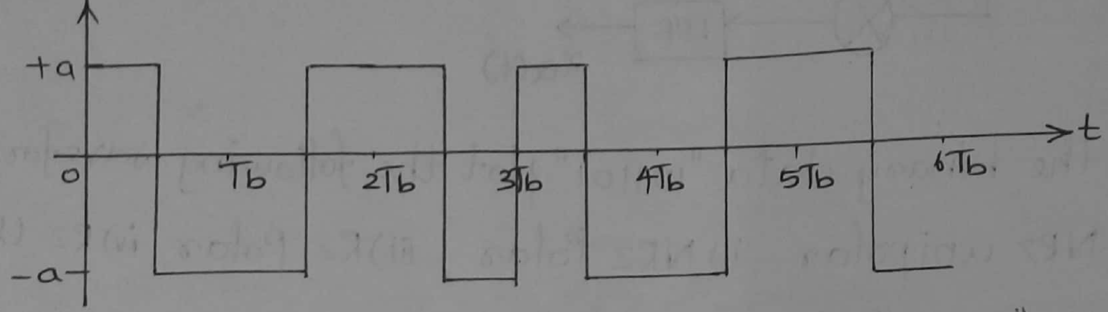
(v)



(vi)



6(b) Perform differential encoding on the binary data "101101". show that original binary data can be decoded even in the presence of polarity inversion.

Given, data $\{b_k\} = \{1\ 0\ 1\ 1\ 0\ 1\}$

$\{b_k\}$		1	0	1	1	0	1
Differentially encoded data $\{d_k\} = \{b_k\} \oplus \{d_{k-1}\}$	1	0	0	1	0	0	1

	1	0	0	1	0	0	1
Differentially decoded data $\{\hat{b}_k\} = \{d_k\} \oplus \{d_{k-1}\}$		1	0	1	1	0	1

	0	1	1	0	1	1	0
polarity inversion							
$\{d_k\}$		1	0	1	1	0	1

7\. Derive an expression for power spectral density of NRZ bipolar format assuming equiprobable and statistically independent 0s and 1s. Plot the resulting Power spectral density.

The power spectral density of a PAM signal

$$S(f) = \frac{1}{T_b} |F(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \cdot e^{-j2\pi f \cdot nT_b} \longrightarrow (1)$$

As per NRZ bipolar format

$$a_k = \begin{cases} \pm 1, & \text{binary '1'} \\ 0, & \text{binary '0'} \end{cases} \longrightarrow (2)$$

$n=0$,

$$R_A(0) = E[A_k \cdot A_{k-0}]$$

$$= 0^2 \cdot \tfrac{1}{2} + a^2 \cdot \tfrac{1}{4} + (-a)^2 \cdot \tfrac{1}{4}$$

$$R_A(0) = a^2/2 \longrightarrow (3)$$

b_k	A_k	$P[A_k]$
0	0	1/2
1	+a	1/4
	-a	1/4

$n=1$,

$$R_A(1) = E[A_k \cdot A_{k-1}]$$

$$= 0^2 \cdot \tfrac{1}{4} + 0^2 \cdot \tfrac{1}{4} + 0^2 \cdot \tfrac{1}{4} + (a)(-a) \cdot \tfrac{1}{4}$$

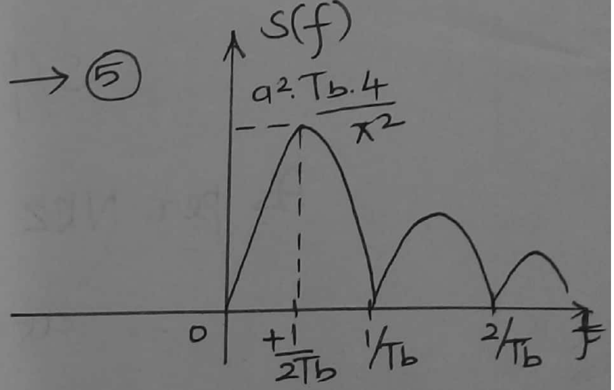
$$R_A(1) = -\frac{a^2}{4} \longrightarrow (4)$$

b_{k-1}	b_k	A_k	A_{k-1}	$P[A_k A_{k-1}]$
0	0	0	0	1/4
0	1	0	±a	1/4
1	0	±a	0	1/4
1	1	±a	∓a	1/4

$n=2,$

b_{k-2}	b_{k-1}	b_k	A_k	A_{k-1}	A_{k-2}	$P[A_k \cdot A_{k-2}]$
0	0	0	0	0	0	1/8
0	0	1	+a	0	0	1/8
0	1	0	0	+a	0	1/8
0	1	1	-a	+a	0	1/8
1	0	0	0	0	+a	1/8
1	0	1	+a	0	-a	1/8
1	1	0	0	-a	+a	1/8
1	1	1	+a	-a	+a	1/8

$$R_A(2) = -\frac{a^2}{8} + \frac{a^2}{8} = 0 \rightarrow (5)$$



$$R_A(n) = \begin{cases} +a^2/2, & n=0 \\ -a^2/4, & n=\pm 1 \\ 0, & n \geq 2 \end{cases}$$

$$F(f) = T_b \cdot \text{sinc}(f \cdot T_b)$$

$$S(f) = \frac{1}{T_b} \cdot T_b^2 \cdot \text{sinc}^2(f \cdot T_b)\left[\frac{a^2}{2} - \frac{a^2}{4} e^{-j2\pi f T_b} - \frac{a^2}{4} \cdot e^{+j2\pi f T_b}\right]$$

$$= T_b \cdot \text{sinc}^2(f \cdot T_b)\left[\frac{a^2}{2} - \frac{a^2}{4}\left(e^{j2\pi f T_b} + e^{-j2\pi f T_b}\right)\right]$$

$$= T_b \cdot \text{sinc}^2(f \cdot T_b)\left[\frac{a^2}{2} - \frac{a^2}{4} \cdot 2 \cdot \cos(2\pi f \cdot T_b)\right]$$

$$= T_b \cdot \text{sinc}^2(f \cdot T_b) \cdot a^2 \left[1 - \cos(2\pi f \cdot T_b)\right]$$

$$S(f) = T_b \cdot a^2 \, \text{sinc}^2(f \cdot T_b) \cdot 2 \cdot \sin^2(\pi f \cdot T_b)$$

When $f=0$, $\sin^2(\pi f \cdot T_b) = 0$

$f = +\frac{1}{2T_b}$

$$S(f) = a^2 \cdot T_b \cdot \text{sinc}^2(1/2) \cdot 2 \cdot \sin^2(\pi/2)$$

$$S(f) = a^2 \cdot T_b \cdot \frac{4}{\pi^2}$$

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