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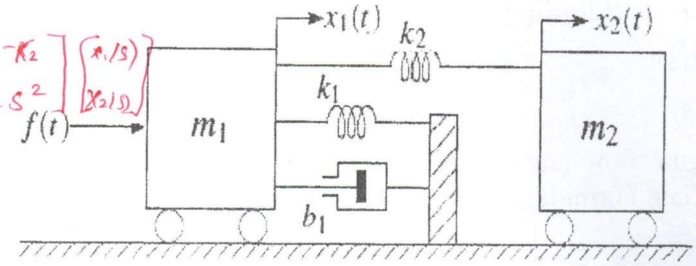
CMR INSTITUTE OF TECHNOLOGY	USN								
InternalTest– I									
Sub:	Control Systems							Code:	15EC43
Date:	13/ 03 / 2018	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	ECE-A ,B
Answer any FIVE FULL Questions									

		Marks	OBE CO	OBE RBT
1 a)	Explain with examples open loop and closed loop control systems. List merits and demerits of both.	[5]	CO2	L4
b)	For a unity feedback control system the open loop transfer function is $G(S) = 10(S+2) / S^2(S+1)$. Determine 1. Static error constants. $K_p = \infty,\ K_v = \infty,\ K_a = 20$ $\quad E(S) = \frac{R(S)}{1+G(S)H(S)}$ 2.Steady state error when the input $R(S) = \frac{3}{S} - \frac{2}{S^2} + \frac{1}{3S^3}$ $= 1/60$	[5]	CO4	L3
2	For the system shown in Figure.2 write mechanical network and obtain its mathematical model and transfer function.	[10]	CO3	L3
3	For the rotational system shown in Figure.3. (i) Draw the mechanical network. (ii) Write the differential equations. (iii) Obtain torque to voltage analogy.	[10]	CO3	L3

$$\begin{bmatrix} F(S) \\ 0 \end{bmatrix} = \begin{bmatrix} M_1S^2+K_1+B_1S+K_2 & -K_2 \\ -K_2 & K_2+M_2S^2 \end{bmatrix} \begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix}$$

$$\frac{X_2(S)}{F(S)} = \frac{K_2}{M_1M_2S^4 + B_1M_2S^3 + S^2(M_1K_2+K_1M_2+M_2K_2) + B_1K_2S + K_1K_2}$$



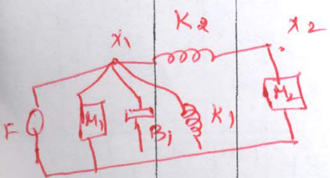
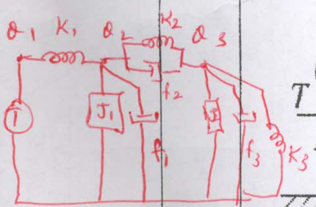
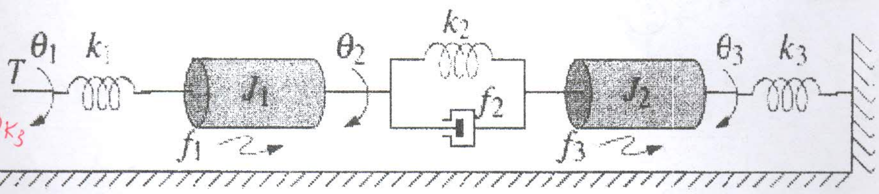
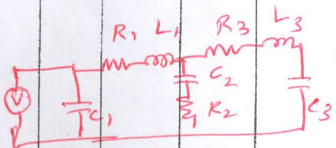


Fig. 2







$T \to V,\ J \to L,\ f \to R,\ K \to 1/C,\ \theta \to q$

Note : f_1, f_2, f_3 are dampers. Fig. 3

		Marks	CO	RBT
4	Q 4. : For the mechanical system shown in Figure.4. (i) Draw the free-body diagram and mechanical network. Write the differential equations describing behaviour of the system. (ii) Draw the electrical network based on force-voltage analogy and write the	[5] [5]	CO3 CO3	L3

analogous electrical equations.

(iii) Draw the electrical network based on force-current analogy and write the analogous electrical equations.

$f(t) = K_1(y - y_1)$

$0 = B_1 \frac{d(y_1 - y_2)}{dt} + K_1(y_1 - y) + M_1 \frac{d^2 y_1}{dt^2}$

$0 = M_2 \frac{d^2 y_2}{dt^2} + B_1 \frac{d(y_2 - y_1)}{dt} + K_2 x_2 + B_2 \frac{dx_2}{dt}$

$F \to V,\ M \to L,\ D \to R,\ K \to 1/C,\ x \to q$

$F \to I,\ M \to C,\ D \to 1/R,\ K \to 1/L,\ x \to \phi$

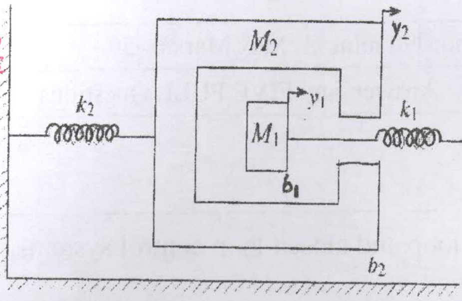
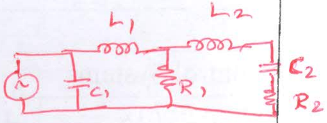
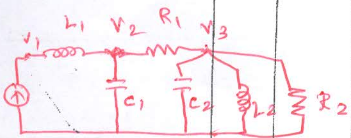


Fig. 4





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5 The performance equations of a controlled system are given by the following set of linear algebraic equations: **[10] CO5 L3**

$E1(s) = R(s) - H3(s)C(s);$

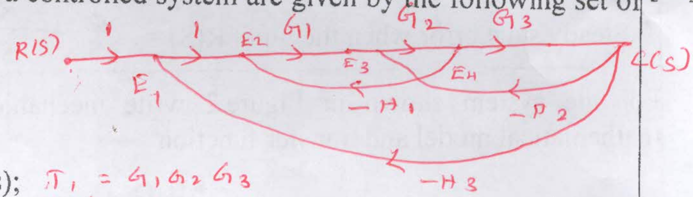
$E2(s) = E1(s) - H1(s)E4(s);$

$E3(s) = G1(s)E2(s) - H2(s)C(s);$

$E4(s) = G2(s)E3(s);$

$C(s) = G3(s)E4(s);$

(i) Draw the signal flow graph. (ii) Find the overall transfer function C(s) / R(s) using Mason's Gain Formula.



$T_1 = G_1G_2G_3$

$L_1 = -G_1G_2G_3H_3$

$L_2 = -H_1G_1G_2$

$L_3 = -H_2G_2G_3$

$\frac{T_1\Delta_1}{\Delta} = \frac{G_1G_2G_3}{1-(L_1+L_2+L_3)}$

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6 Using block diagram reduction techniques, determine the transfer function C1/ R1 of the system shown in Figure.6. **[10] CO5 L3**

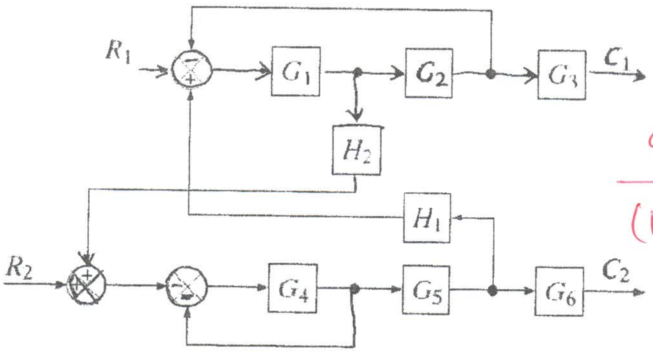

Fig. 6

$\frac{G_1G_4G_5G_6H_2}{(1+G_1G_2)(1+G_4) + G_1G_4G_5H_1H_2}$

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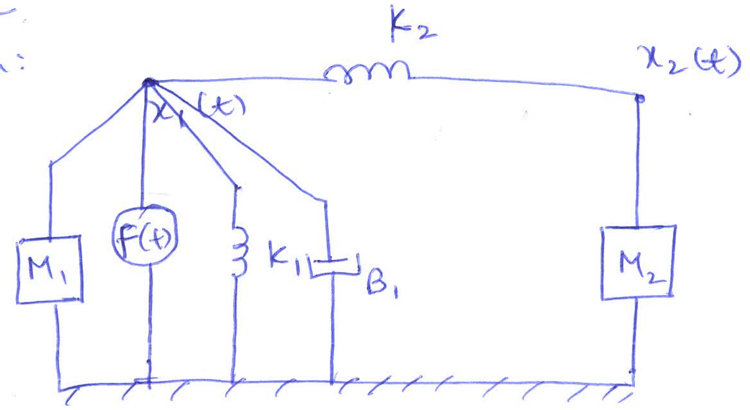
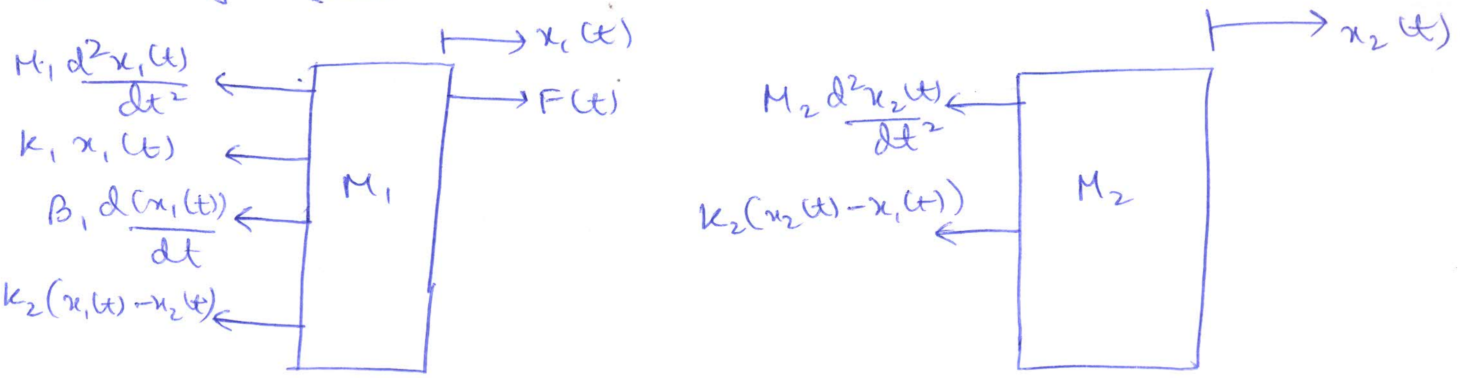
Q 2) Mechanical network.

modal diagram:



Free body diagram:



$$F(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + K_1 x_1(t) + B_1 \frac{d(x_1(t))}{dt} + K_2(x_1(t) - x_2(t \quad \text{—} (1)$$

$$0 = K_2(x_2(t) - x_1(t)) + M_2 \frac{d^2 x_2(t)}{dt^2} \quad \text{—} (2)$$

Laplace transform:

$$F(s) = M_1 s^2 X_1(s) + K_1 X_1(s) + B_1 s X_1(s) + K_2(X_1(s) - X_2(s \quad \text{—} (3)$$

$$0 = K_2(X_2(s) - X_1(s)) + M_2 s^2 X_2(s) \quad \text{—} (4)$$

$$\begin{bmatrix} F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} M_1 s^2 + K_1 + B_1 s + K_2 & -K_2 \\ -K_2 & K_2 + M_2 s^2 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

Transfer function $= \dfrac{X_2(s)}{F(s)}$

$$X_2(s) = \frac{\begin{vmatrix} M_1 s^2 + K_1 + B_1 s + K_2 & F(s) \\ -K_2 & 0 \end{vmatrix}}{\begin{vmatrix} M_1 s^2 + K_1 + B_1 s + K_2 & -K_2 \\ -K_2 & K_2 + M_2 s^2 \end{vmatrix}}$$

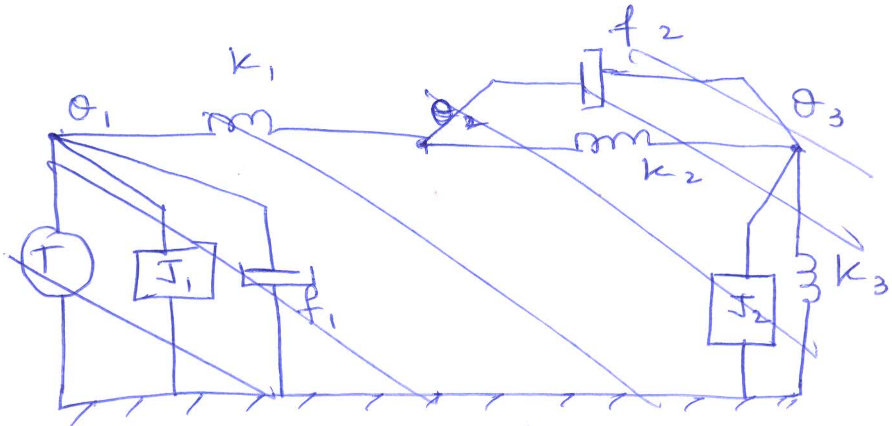
$$X_2(s) = \frac{F(s)\, K_2}{\left[M_1 s^2 K_2 + M_1 M_2 s^4 + K_1 K_2 + K_1 M_2 s^2 + B_1 s K_2 + B_1 s M_2 s^2 + \cancel{K_2^2} + M_2 K_2 s^2 - \cancel{K_2^2} \right]}$$

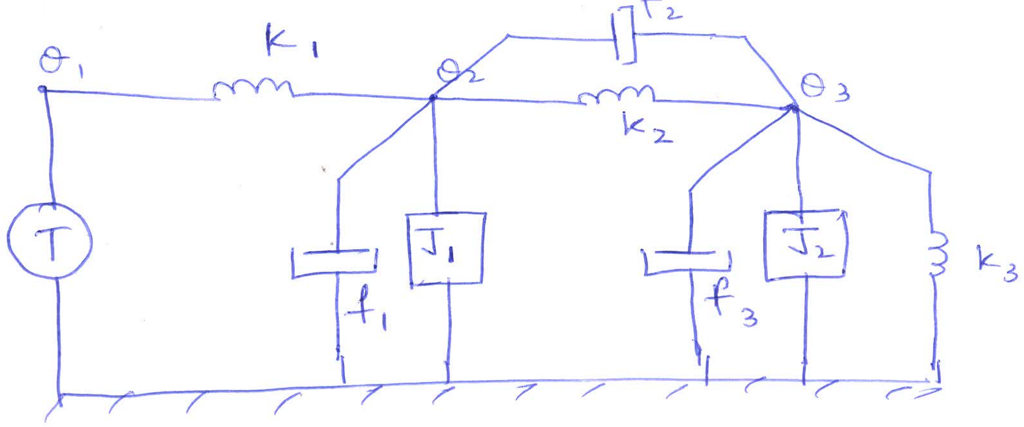
$$\frac{X_2(s)}{F(s)} = \frac{K_2}{\left[M_1 M_2 s^4 + s^2 (M_1 K_2 + K_1 M_2 + M_2 K_2) + B_1 M_2 s^3 + B_1 K_2 s + K_1 K_2 \right]}$$

∴ Transfer function is

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{K_2}{\left[M_1 M_2 s^4 + s^3 B_1 M_2 + s^2 (M_1 K_2 + K_1 M_2 + M_2 K_2) + B_1 K_2 s + K_1 K_2 \right]}}$$

Q 3) i) Mechanical network:





(ii) Differential equation.

$$T = k_1(\theta_1 - \theta_2) \quad — (1)$$

$$0 = J_1 \frac{d^2\theta_2}{dt^2} + f_1 \frac{d\theta_2}{dt} + k_1(\theta_2 - \theta_1) + k_2(\theta_2 - \theta_3) + f_2 \frac{d(\theta_2 - \theta_3)}{dt} \quad — (2)$$

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$$0 = J_2 \frac{d^2\theta_3}{dt^2} + f_3 \frac{d\theta_3}{dt} + k_3(\theta_3) + k_2(\theta_3 - \theta_2) + f_2\left(\frac{d}{dt}(\theta_3 - \theta_2)\right) — (3)$$

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iii) Torque to voltage analogy

$T \to V$, $J \to L$, $\underset{(B/D)}{f} \to R$, $K \to 1/C$, $\theta \to q$

$$V = \frac{1}{C_1}(q_1 - q_2) \quad — \quad (4)$$

$$0 = L_1 \frac{d^2 q_2}{dt^2} + R_1 \frac{dq_2}{dt} + \frac{1}{C_1}(q_2 - q_1) + \frac{1}{C_2}(q_2 - q_3) + R_2 \frac{d}{dt}(q_2 - q_3)$$

$$0 = L_2 \frac{d^2 q_3}{dt^2} + R_3 \frac{dq_3}{dt} + \frac{1}{C_3} q_3 + \frac{1}{C_2}(q_3 - q_2) + R_2 \frac{d}{dt}(q_3 - q_2) \quad — \quad (6)$$

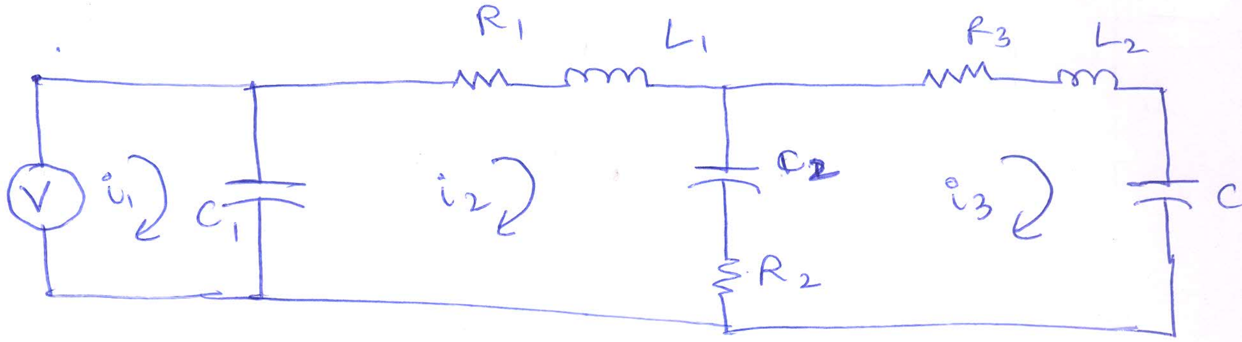
$$i = \frac{dq}{dt}$$

$$\cancel{V = \frac{1}{C_1}\int (q_1 - q_2)\, dt} \qquad V = \frac{1}{C_1}\int i_1 - i_2 \, dt$$

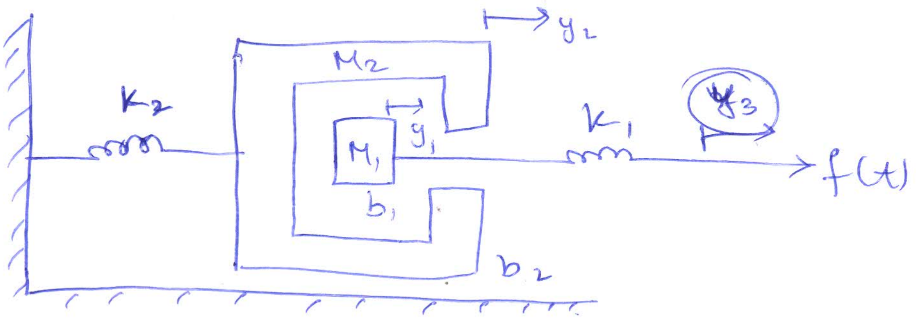
$$0 = L_1 \frac{di_2}{dt} + R_1 i_2 + \frac{1}{C_1}\int i_2 - i_1 \, dt + \frac{1}{C_2}\int (i_2 - i_3)\, dt + R_2 (i_2 - i_3)$$

$$0 = L_2 \frac{di_3}{dt} + R_3 i_3 + \frac{1}{C_3}\int i_3 \, dt + \frac{1}{C_2}\int (i_3 - i_2)\, dt + R_2 (i_3 - i_2) \quad — \quad (9)$$

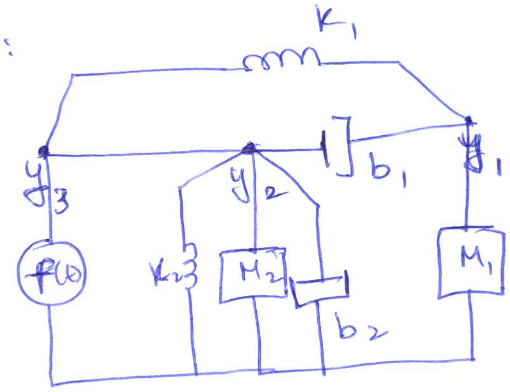
using (7), (8) and (9),



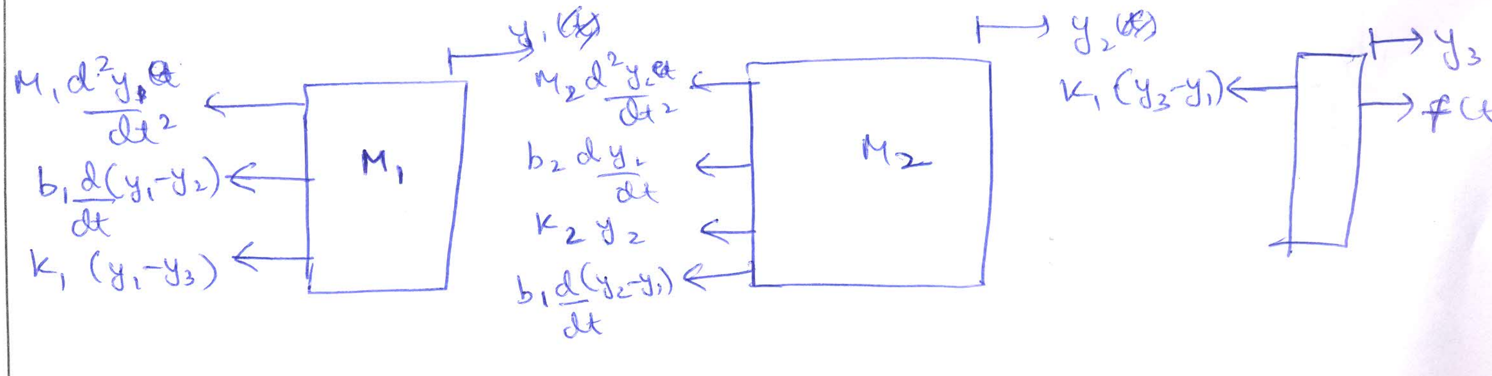
Q 4)



→ i) Nodal diagram:



Free body diagram:



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Differential equation:

$$\cancel{f}(t) = K_1(y_3 - y_1) \quad - (1)$$

$$0 = M_1\frac{d^2y_1}{dt^2} + b_1\frac{d(y_1-y_2)}{dt} + K_1(y_1-y_3) \quad - (2)$$

$$0 = M_2\frac{d^2y_2}{dt^2} + b_1\frac{d(y_2-y_1)}{dt} + K_2(y_2) + b_2\frac{dy_2}{dt} \quad - (3)$$

ii) Force-voltage:

$F \to V$, $M \to L$, $B/D \to R$, $K \to 1/C$, $y \to q$.

$$V(t) = 1/C_1(q_3 - q_1) \quad - (4)$$

$$0 = L_1\frac{d^2q_1}{dt^2} + R_1\frac{d(q_1-q_2)}{dt} + \frac{1}{C_1}(q_1-q_3) \quad - (5)$$

$$0 = L_2\frac{d^2q_2}{dt^2} + R_1\frac{d(q_2-q_1)}{dt} + \frac{1}{C_2}(q_2)$$

$$+ R_2\frac{dq_2}{dt} \quad - (6)$$

$$\boxed{i = dq/dt}$$

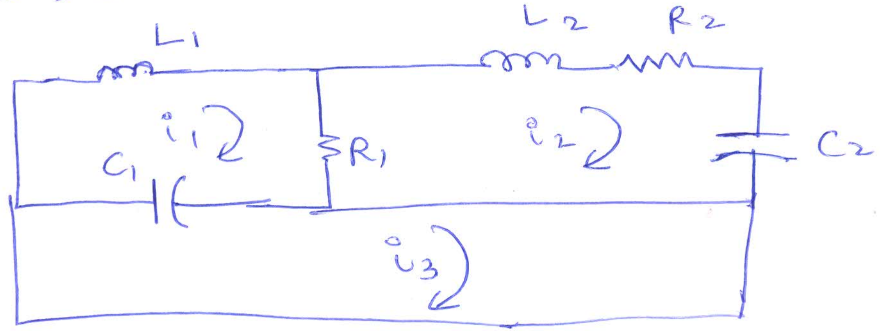
$$V(t) = \frac{1}{C_1}\int(i_3 - i_1)dt \quad - (7)$$

$$0 = L_1\frac{di_1}{dt} + R_1(i_1-i_2) + \frac{1}{C_1}\int(i_1-i_3)\,dt \quad - (8)$$

$$0 = L_2\frac{di_2}{dt} + R_1(i_2-i_1) + \frac{1}{C_2}\int(i_2)\,dt +$$

$$R_2(i_2) \quad - (9)$$

Using (7), (8) and (9)



Force-voltage analogous system.

(iii) Force - current analogy:

$F \rightarrow I$, $M \rightarrow C$, $B/D \rightarrow 1/R$, $K \rightarrow 1/L$, $y \rightarrow \phi$

$$I(t) = \frac{1}{L_1}(\phi_3 - \phi_1) \quad \text{— (10)}$$

$$0 = C_1 \frac{d^2\phi_1}{dt} + \frac{1}{R_1}\frac{d}{dt}(\phi_1 - \phi_2) + \frac{1}{L_1}(\phi_1 - \phi_3) \quad \text{— (11)}$$

$$0 = C_2 \frac{d^2\phi_2}{dt^2} + \frac{1}{R_1}\frac{d}{dt}(\phi_2 - \phi_1) + \frac{1}{L_2}(\phi_2) + \frac{1}{R_2}\frac{d\phi_2}{dt} \quad \text{— (1}$$

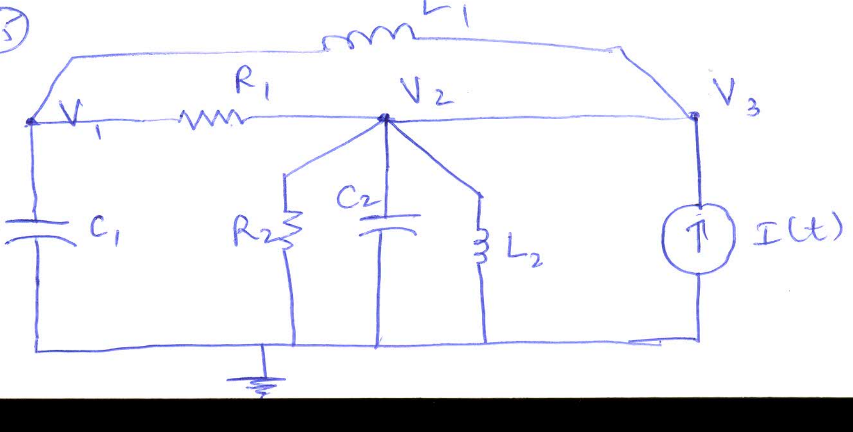
$$\boxed{V = \frac{d\phi}{dt}}$$

$$I(t) = \frac{1}{L_1}\int (V_3 - V_1)\, dt \quad \text{— (13)}$$

$$0 = C_1 \frac{dV_1}{dt} + \frac{1}{R_1}(V_1 - V_2) + \frac{1}{L_1}\int (V_1 - V_3)\, dt \quad \text{— (14)}$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{R_1}(V_2 - V_1) + \frac{1}{L_2}\int (V_2)\, dt + \frac{1}{R_2}(V_2) \quad \text{— (1}$$

Using (13), (14) & (15)



Force-current analogous circuit.

Q 5.)

$$E_1(s) = R(s) - H_3(s)C(s)$$

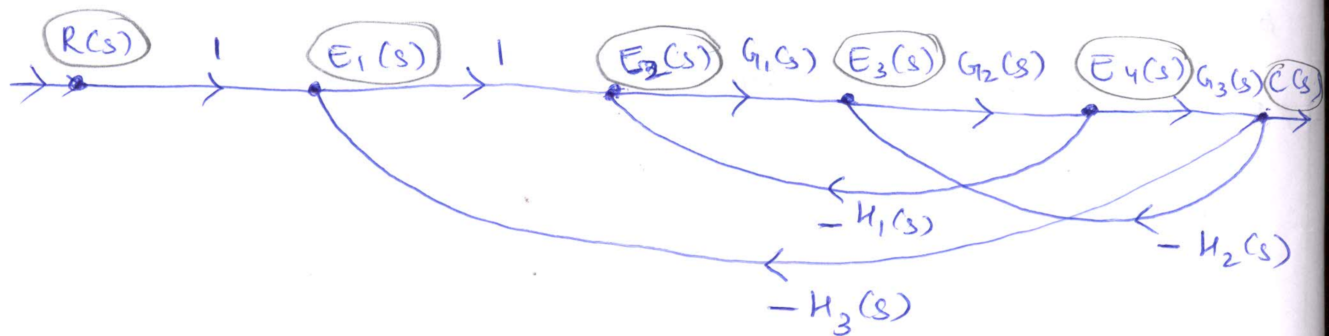
$$E_2(s) = E_1(s) - H_1(s)E_4(s)$$

$$E_3(s) = G_1(s)E_2(s) - H_2(s)C(s)$$

$$E_4(s) = G_2(s)E_3(s)$$

$$C(s) = G_3(s)(E_4(s))$$

i) Signal flow graph:



ii) Overall transfer function:

→ Forward paths:

nodes	Gain
$R(s)-E_1(s)-E_2(s)-E_3(s)-E_4(s)-C(s)$	$T_1 \rightarrow G_1(s)G_2(s)G_3(s)$

→ loops

nodes	gain
$E_1(s)-E_2(s)-E_3(s)-E_4(s)-C(s)-E_1(s)$	$L_1 \rightarrow G_1(s)G_2(s)G_3(s)(-H_3(s))$
$E_2(s)-E_3(s)-E_4(s)-E_2(s)$	$L_2 \rightarrow G_1(s)G_2(s)(-H_1(s))$
$E_3(s)-E_4(s)-C(s)-E_3(s)$	$L_3 \rightarrow G_2(s)G_3(s)(-H_2(s))$

Two non-touching loops : NIL.

$$T.F = \frac{T_k \Delta k}{\Delta}$$

$k = 1$

$$\Delta = 1 - \{L_1 + L_2 + L_3\} + \{0\} - \{0\}$$

$$\Delta = 1 + \{G_1 G_2 G_3 H_3 + G_1 G_2 H_1 + G_2 G_3 H_2\}$$

$$\Delta_1 = 1 - \{0\} = 1$$

$$T.F = \frac{T_1 \Delta_1}{\Delta}$$

$$\boxed{T.F = \frac{[G_1(s) G_2(s) G_3(s)](1)}{1 + \{G_1(s) G_2(s) G_3(s) H_3(s) + G_1(s) G_2(s) H_1(s) + G_2(s) G_3(s) H_2(s)\}}}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1(s) G_2(s) G_3(s)}{1 + \{G_1(s) G_2(s) G_3(s) H_3(s) + G_1(s) G_2(s) H_1(s) + G_2(s) G_3(s) H_2(s)\}}$$

Q 1.)

b.) 1) $G(s) = \frac{10(s+2)}{s^2(s+1)}$ $H(s) = 1$

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$K_p = \lim_{s \to 0} \frac{10(s+2)}{s^2(s+1)} (1)$$

$$\boxed{K_p = \infty}$$

$$K_v = \lim_{s \to 0} s\,G(s)H(s)$$

$$K_v = \lim_{s \to 0} \frac{\cancel{s}\,10(s+2)}{s^{\cancel{2}}(s+1)} (1)$$

$$\boxed{K_v = \infty}$$

$$K_a = \lim_{s \to 0} s^2 G(s)H(s)$$

$$K_a = \lim_{s \to 0} \frac{\cancel{s^2}\,10(s+2)}{\cancel{s^2}(s+1)} (1)$$

$$\boxed{K_a = 20}$$

b) 2) $R(s) = \frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}$

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

~~E(s) =~~

$$\lim_{t\to\infty} e_{ss}(s) = \lim_{s\to 0} s\,E(s)$$

$$e_{ss} = \lim_{s\to 0} s\left(\frac{\frac{3}{s} - \frac{2}{s^2} + \frac{1}{3s^3}}{1 + \frac{10(s+2)}{s^2(s+1)}}\right)$$

$\frac{1}{s^2}\left(\frac{3s}{\cancel{s}}\right)$

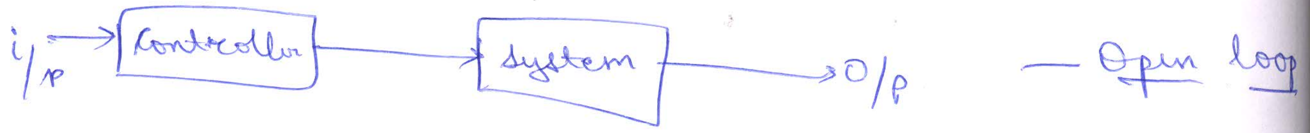
$$e_{ss} = \lim_{s\to 0} s\left(\frac{3s - 2 + \frac{1}{3s}}{s^2 + \frac{10(s+2)}{(s+1)}}\right)$$

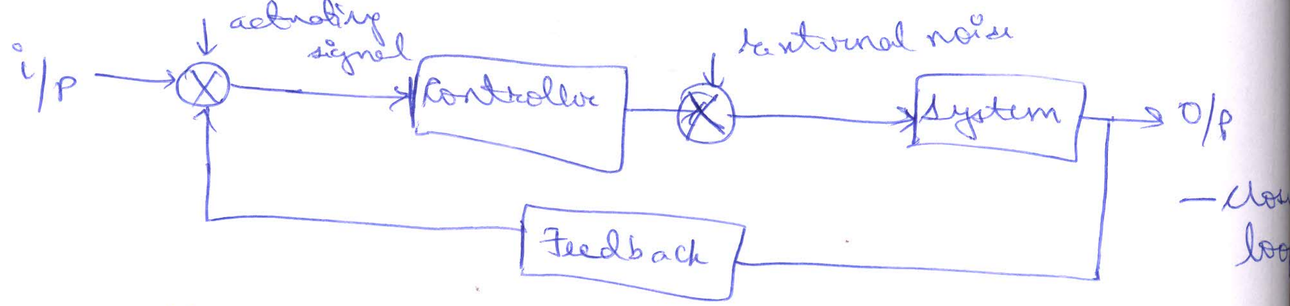
$$e_{ss} = \lim_{s\to 0}\left(\frac{3s^2 - 2s + \frac{1}{3}}{s^2 + \frac{10(s+2)}{s+1}}\right)$$

$$e_{ss} = \frac{\frac{1}{3}}{\frac{10(2)}{1}}$$

$$\boxed{e_{ss} = \frac{1}{60}}$$

Q 11) a) Open loop control system is a system in which is is no feedback whereas a closed loop control system has a feedback.





→ In automatic ~~traffic~~ traffic signals, there is just a timer which controls green, red and yellow signals and it doesn't depend on the amount of traffic. Therefore this is open loop.

→ In an air conditioner, ~~the~~ there is a sensor which senses the temperature and if it increases ~~temp~~ beyond certain levels, then air conditioner switches on. This is closed loop.