

1) Explain basic central office and its linkages.

1.2.2. Basic Central Office Linkages

During the analysis of a digital switching system, it is helpful to define the extent of a central office (CO)¹ and its linkages to other facilities. Figure 1.1 shows a typical central office along with some important facilities. Familiarity with this setup is essential to better understand various operations that may impact the overall reliability of a digital switching system.

The following relate to the basic linkages of a typical central office:

Main distributing frame (MDF). Location where all lines and other related links are cross-connected to a central office switch, also referred to as the *line side of* a switch. The MDF is probably the most labor-extensive part of a CO. All lines from subscribers terminate in the MDF. The MDF has two sides: a vertical and a horizontal. The subscriber cables terminate on the vertical side. The wiring from the digital switching system referred to as *line equipment* terminates on the horizontal side. Based on the assignment of subscribers to line

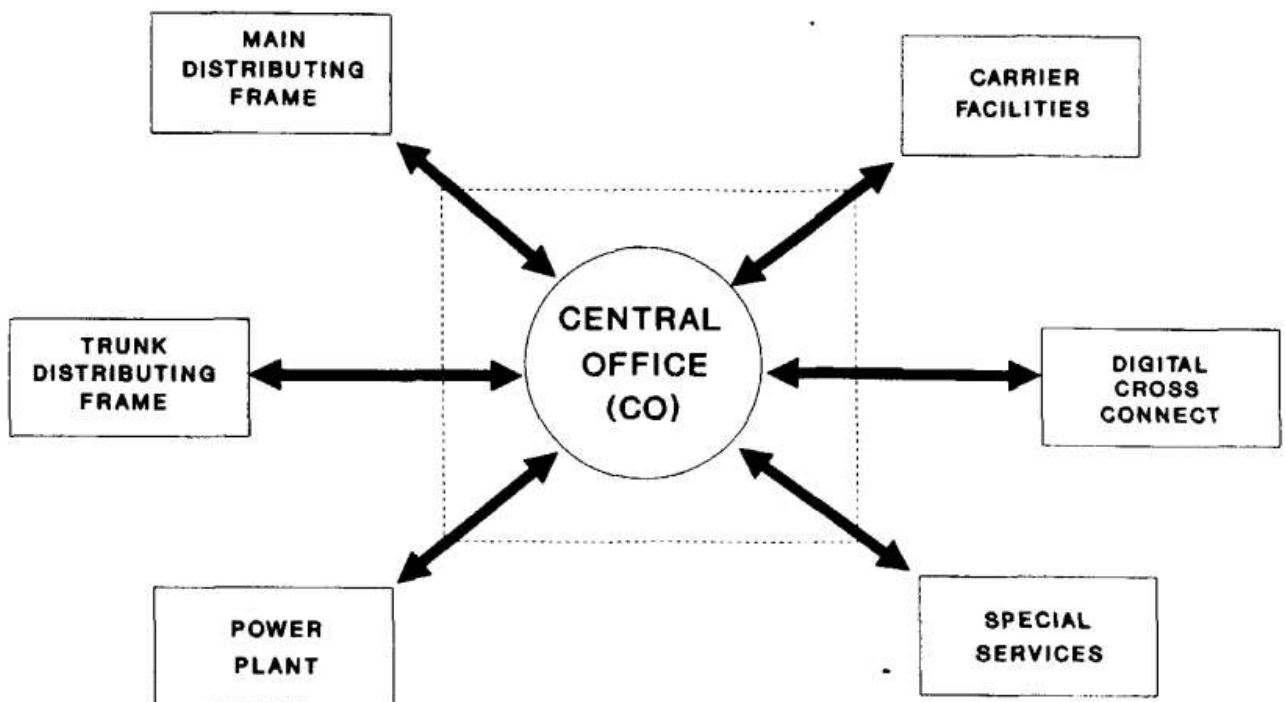


Figure 1.1. Basic central office linkages

Trunk distributing frame (TDF). Location where all trunks and other related links are cross-connected to a central office switch, also referred to as the *trunk side of* a switch. The TDF is usually smaller than the MDF. All trunk cabling from different locations terminates in the TDF. The TDF has two sides: a vertical and a horizontal. The trunk cables terminate on the vertical side. The wiring from the digital switching system, referred to as *trunk equipment*, terminates on the horizontal side. Based on the assignment of cable to trunk equipment, the vertical cable pair are connected to the horizontal trunk equipment pair. The assignment process for trunks to trunk equipment is usually automated.

Power plant. A combination of power converters, battery systems, and emergency power sources which supply the basic -48- and +24-V direct-current (dc) power and protected alternating-current (ac) power to a CO switch or a group of switches. These should not be confused with the power distributing frames in the central offices that provide special voltage conversions and protection for the CO.

Carrier facilities. Facilities which provide carrier or multiplex transmission mode between central offices and with other parts of the telephony network. These facilities typically employ coaxial cables (land or undersea) and radio and satellite systems. The carrier facilities usually terminate on the TDF for cross-connection to the digital switching system.

Digital X-connect. Digital cross-connect provides automatic assignments and cross-connection of trunks to digital switching systems. It can be considered a small switching system for trunks.

2) Explain all types of basic call processing techniques.

Basic Call Processing

This section describes some *basic* types of calls that are usually processed through a digital switching system:

- Intra-LM calls
- Inter-LM calls
- Incoming calls
- Outgoing calls

Intra-LM Calls.

- When a customer dials from a telephone that is connected to a specific line module and calls another customer who is also connected to the same line module, this type of call is classified as an intra-LM call.
- A call path for this type of call is shown in Fig. 1.6a. The off-hook (line origination request) condition is detected by the line module, and service circuits are attached to supply a dial tone to the calling customer.
- Many other functions are performed before a dial tone is given to a calling customer; The line module's request for a path through the switching fabric is processed by the interface controller, which in turn works with the network control processor to make a path assignment.
- Consequently, a path is established through the switching fabric for the called line, and a service circuit is attached to ring the line. Again, many other functions are performed before ringing is applied to the called customer; Since this is an intra LM call, the same line module will be involved in controlling the origination and termination of a call.
- This very simplified explanation is offered here for introductory purposes only. Later chapters go into far greater detail in explaining various functions such as digit reception, digit translation, and tests that are performed before a call is completed.

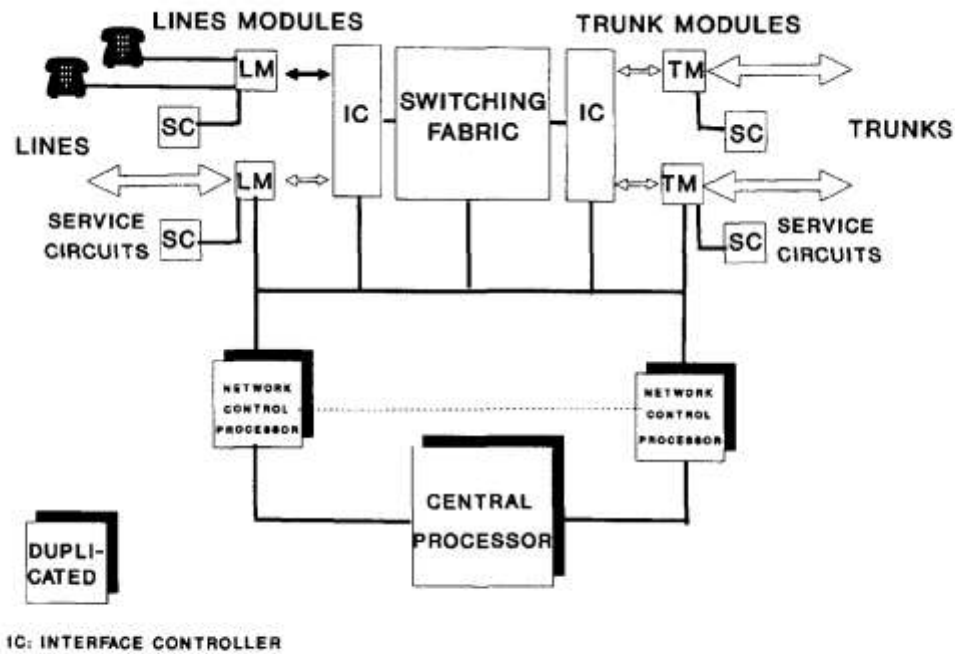


Figure 1.6a. Calls within a line module

Inter-LM Calls.

- The workings of an inter-LM call are similar to those of an intra-LM call, except that the terminating line equipment is located in another line module. Figure 1.6 & shows interconnections for such a call. There are some subtle differences in how an inter-LM call is handled versus an intra-LM call.

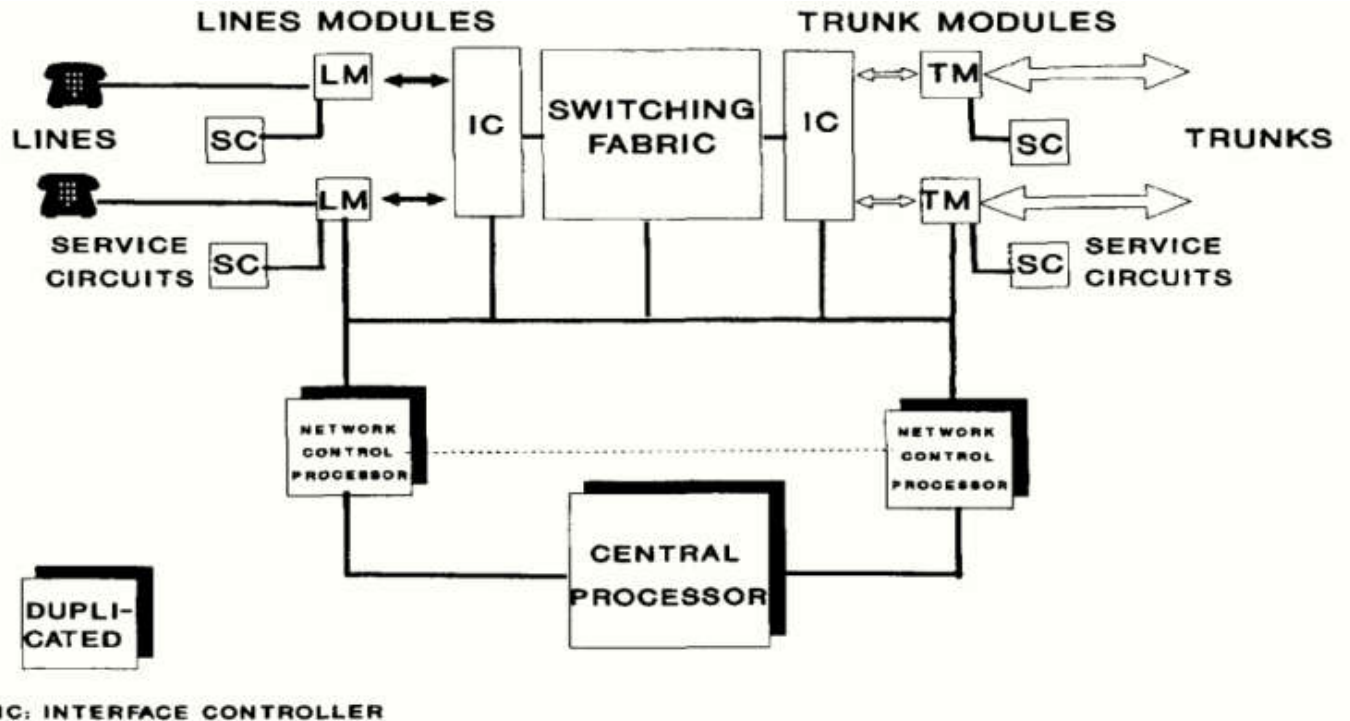


Figure 2.6b. Calls outside a line module

Incoming Calls.

- When a TM detects an incoming call, it attaches service circuits to control the call and requests a path through the switching fabric from the interface controller and network control processor.
- Once a path is found through the switching fabric to a LM that has the terminating line/ service circuits are attached to ring the called telephone. This also provides functions such as audible ringing to the calling line. Use Fig. 1.6c to visualize this simple connection of an incoming call.

Outgoing Calls.

- When a LM processes a call which has terminating equipment outside the CQ the LM requests a path through the switching fabric to a trunk module via the interface controller.
- The interface controller works with the network control processor to establish a path to an outgoing trunk. Once a path is established through the switching fabric, the TM connects a service circuit for controlling the call to the called CO or a tandem office.
- Functions such as out pulsing and multi frequency (MF) signaling are provided by the trunk service circuits. An outgoing call from an originating office is an incoming call to a terminating office. Figure 1.6c shows the paths of incoming and outgoing calls.

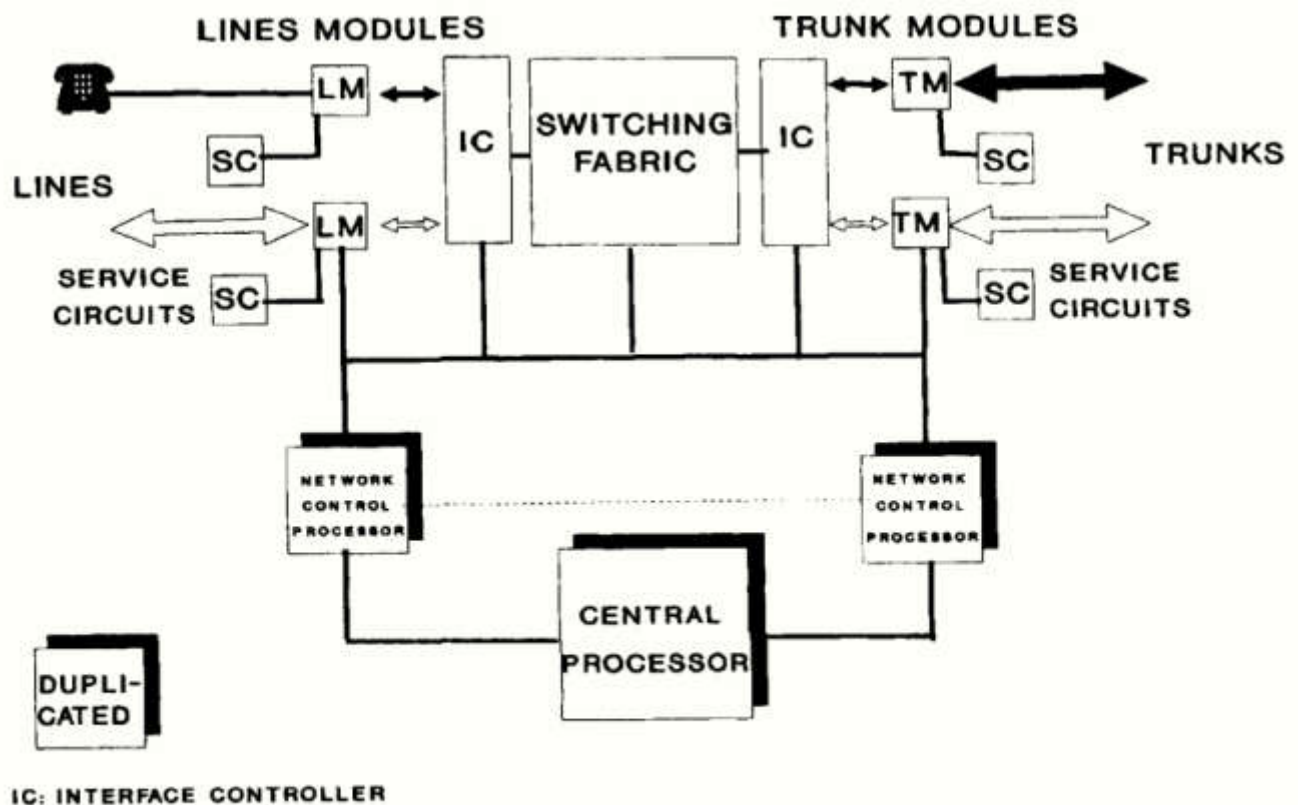


Figure 2.6c. Incoming/outgoing trunk call

3) Derive an expression for First Erlang's Distribution Formula?

Lost call systems:

Our aim is to find the GOS (B) of a lost call system that is offered traffic of A erlangs and has N outgoing trunks. Erlang worked out a solution to this problem based on the following assumptions:

- i. Pure chance traffic – implies call arrivals and call terminations are independent random events.
- ii. Statistical equilibrium - implies probabilities remain unchanged for the period being considered.
- iii. Full availability – all incoming calls can be connected to any outgoing trunk that is free.
- iv. Calls that encounter congestion are lost which is the basis on which lost call systems are classified.

Lost Call System (OR) 1st Erlang Distribution

Erlang determined the grade of service, nothing but lost probability for N-trunks & it depends on pure chance, traffic, statistical equilibrium, full availability, call with encounter congestion loss. (limited availability)

If x calls are in progress,

$$P(x) = \frac{A^x}{x!} P(0), \quad 0 \leq x \leq N \rightarrow (1)$$

We can represent probability as,

$$\sum_{x=0}^N P(x) = 1$$
$$\sum_{x=0}^N \frac{A^x}{x!} P(0) = 1$$
$$P(0) = \frac{1}{\sum_{x=0}^N \frac{A^x}{x!}} \rightarrow (2)$$

Substitute eqⁿ (2) in (1)

$$P(x) = \frac{A^x}{x!} \times \frac{1}{\sum_{k=0}^n \frac{A^k}{k!}}$$

$$P(x) = \frac{A^x/x!}{\sum_{k=0}^n A^k/k!} \rightarrow (3)$$

eqⁿ (3) is erlang loss formula (OR) 1st formula.

Its nothing but grade of service $E_{1,N}(A)$. Loss probability for a full availability group or no trunk offered traffic Erlang.

$$B = E_{1,N}(A) = \frac{A^N/N!}{\sum_{k=0}^n \frac{A^k}{k!}} \rightarrow (4)$$

In terms of N-trunk eqⁿ (4) can be written as →

$$E_{1,N-1}(A) = \frac{A^{N-1}/(N-1)!}{\sum_{k=0}^{N-1} \frac{A^k}{k!}}$$

$$\sum_{k=0}^N \frac{A^k}{k!} = \sum_{k=0}^{N-1} \frac{A^k}{k!} + \frac{A^N}{N!} \rightarrow (5)$$

from (4)

$$\sum_{k=0}^N \frac{A^k}{k!} = \frac{A^N/N!}{E_{1,N}(A)} \rightarrow (6)$$

Similarly,

$$\sum_{k=0}^{N-1} \frac{A^k}{k!} = \frac{A^{N-1}/(N-1)!}{E_{1,N-1}(A)} \rightarrow (7)$$

Substituting (6) & (7) in (5)

$$\frac{A^N/N!}{E_{1,N}(A)} = \frac{A^{N-1}/(N-1)!}{E_{1,N-1}(A)} + \frac{A^N}{N!}$$

$$E_{1,N}(A) = \frac{A^N/N!}{\frac{A^{N-1}/(N-1)!}{E_{1,N-1}(A)} + \frac{A^N}{N!}}$$

we have,

$$E_{1,N}(A) = \frac{A^{N-1}}{(N-1)!} \times \frac{A}{N}$$
$$\frac{\frac{A^{N-1}}{(N-1)!}}{E_{1,N-1}(A)} + \frac{A^{N-1}}{(N-1)!} \cdot \frac{A}{N}$$

$$E_{1,N}(A) = \frac{A E_{1,N-1}(A)}{N + A E_{1,N-1}(A)} = \frac{\frac{A}{N}}{\frac{1}{E_{1,N-1}(A)} + \frac{A}{N}} = \frac{\frac{A}{N}}{\frac{N + A E_{1,N-1}(A)}{E_{1,N-1}(A) N}}$$

$A E_{1,N-1}(A)$

4) Derive an expression for Second Erlang's Distribution Formula?

In queuing systems, the trunks present in the systems are called *servers*. Erlang's solution to queuing systems is based on the following assumptions:

- i. Pure chance traffic
- ii. Statistical equilibrium
- iii. Full availability
- iv. Calls that encounter congestion are stored in a queue until a server becomes free.

Such a system is also called a M/M/N system.

Queuing System (OR) 2nd erlang distribution
The erlang solution depends on following assumption -
Erlang determine the grade of service i.e., Nothing but lost probability of N-trunks and it depends on

- pure chance traffic
- Statistical equilibrium
- full availability
- calls encounter congestion will be stored and forwarded.

If $A > N$ calls are entering at greater than they leave.
If $A \leq N$ → "corresponds to 'lost call'"
Let us consider,
'x' be the total number of calls
If $x < N$ → No. of calls will be served is x
N = trunk
x = No. of calls
If $x > N$ → No. of calls served is $(x - N)$ calls in Queue.

Case i) $x \leq N$

$$P(x) = \frac{A^x}{x!} P(0) \rightarrow (1)$$

Case ii) $x > N$, st

$$P(x) = \frac{A^x}{x!} \rightarrow (2)$$

$$\begin{aligned} P(x-1 \rightarrow x) &= P(x-1) P(x) \\ &= P(x-1) \frac{A^x}{x!} \rightarrow (3) \end{aligned}$$

$$\begin{aligned} P(x \rightarrow x-1) &= P(x) - P(x) \\ &= P(x) N \frac{A^x}{x!} \rightarrow (4) \end{aligned}$$

Equating eqⁿ (3) & (4)

$$P(x-1) \frac{A^x}{x!} = P(x) N \frac{A^x}{x!}$$

$$P(x) = \frac{A}{N} P(x-1) \rightarrow (5)$$

$$P(N+1) = \frac{A}{N} P(N) \rightarrow (6)$$

at, $P(N) = \frac{A^N}{N!} P(0) \rightarrow (7)$

Substitute (7) in (6)

$$\frac{A^{N+1}}{N \cdot N!} P(0)$$

$$P(N+1) = \frac{A^{N+1}}{N \cdot N!} P(0) \rightarrow (8)$$

In general eqⁿ (9) can be written as,

$$P(x) = \frac{A^x}{N^{x-N} N!} P(0) \quad \text{for } x > N$$

$$= \frac{N^N}{N!} \left(\frac{A}{N}\right)^x P(0) \rightarrow (10)$$

Let $0 \leq x \leq \infty$

$$\sum_{x=0}^{\infty} P(x) = 1$$

Adding eqⁿ (1) & (10)

$$P(x) = \underbrace{\frac{A^x}{x!} P(0)}_{x \leq N} + \underbrace{\frac{N^N}{N!} \left(\frac{A}{N}\right)^x P(0)}_{x > N}$$

$$= \sum_{x=0}^{N-1} \frac{A^x}{x!} P(0) + \sum_{x=N}^{\infty} \frac{N^N}{N!} \left(\frac{A}{N}\right)^x P(0)$$

$$\frac{1}{P(0)} = \left[\sum_{x=0}^{N-1} \frac{A^x}{x!} + \sum_{x=N}^{\infty} \frac{N^N}{N!} \left(\frac{A}{N}\right)^x \right] \frac{1}{\sum_{x=0}^{\infty} P(x)}$$

Let $k = x - N$

When $x = N$, $k = 0$

& $x = \infty$, $k = \infty$

$$\frac{1}{P(0)} = \sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{N^N}{N!} \sum_{k=0}^{\infty} \left(\frac{A}{N}\right)^{k+N}$$

$$= \sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{N^N}{N!} \frac{A^N}{N^N} \sum_{k=0}^{\infty} \left(\frac{A}{N}\right)^k$$

$$\frac{1}{P(0)} = \sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{A^N}{N!} \sum_{k=0}^{\infty} \left(\frac{A}{N}\right)^k$$

Wkt,

$$\sum_{k=0}^{\infty} \left(\frac{A}{N}\right)^k = \left(1 - \frac{A}{N}\right)^{-1} = \left(\frac{N-A}{N}\right)^{-1} = \frac{N}{N-A}$$

where $\left(1 - \frac{A}{N}\right)^{-1}$ known as "Maclaurian serie" which is given as follows:

$$\left(1 - \frac{A}{N}\right)^{-1} = 1 + \frac{A}{N} + \left(\frac{A}{N}\right)^2 + \left(\frac{A}{N}\right)^3 + \dots$$

$$\therefore \frac{1}{P(0)} = \sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{A^N}{N!} \left(\frac{N}{N-A}\right)$$

5)

a. Define the unit of traffic, write the expression for the traffic carried.

The teletraffic intensity or simply the traffic is defined as the average no. of calls in progress. The unit of traffic is erlang (named after the Danish pioneer in teletraffic A.K.Erlang). It is a dimensionless quantity.

On a group of trunks, the average number of calls in progress depends on both the no. of calls which arrive and their duration. The duration of a call is called its holding time because it holds the trunk for that time.

Consider a holding time T for a group of 3 trunks:

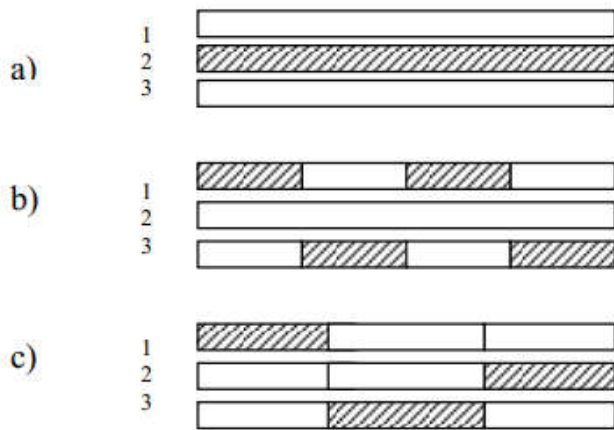


Fig. 2
Example of 1 erlang of traffic carried on 3 trunks

Figure 2(a) shows 1 erlang of traffic resulting from one trunk being busy for the holding time T. Figure 2(b) shows 1 erlang of traffic resulting from two trunks with each trunk being busy for 50% of the time T. Figure 2(c) shows 1 erlang of traffic being carried by three trunks with each of the trunks being busy for 33.33% of the time T.

Sometimes the traffic is also expressed in terms of hundreds of call seconds per hour (CCS).

$$1 \text{ erlang} = 36 \text{ CCS}$$

Mathematically traffic can be represented by the following equation:

$$A = Ch/T \tag{1}$$

where A=traffic in erlangs

C=average number of calls arriving during time T

h=average holding time

From eqn 1, if $T=h$, $A=C$. Thus traffic in erlangs can be defined as mean number of calls arriving during a period equal to the mean duration of the calls (average holding time).

A single trunk cannot carry more than one call, therefore $A \leq 1$ for a single trunk. This is called the occupancy of the trunk. The occupancy of the trunk is also the probability of finding the trunk busy.

b. Define the following terms (any three):

i. Traffic intensity

Traffic intensity more often called simply traffic, is defined as the average number of calls in progress. Although this is dimensionless quantity, a name has been given which is Erlang(E).

ii. Grade of service (GOS)

GOS-Grade of Service (B):

The proportion of calls lost or delayed due to congestion is a measure of the service provided. For a lost call system the grade of service is given by,

$$B = \frac{\text{number of calls lost}}{\text{number of calls offered}} = \frac{\text{traffic lost}}{\text{traffic offered}}$$

The traffic carried by a lost call system will always be less than the traffic offered.

Traffic offered = A erlangs

Traffic lost = AB erlangs

Traffic carried = A(1-B) erlangs

(2)

Note: Larger the Grade of service worse is the service given. Ideally B=0.

iii. Busy hour

The maximum number of calls occurs between 8:00 and 10:00 am for this particular exchange. This hour which corresponds to the peak traffic of the exchange is called the busy hour.



iv. Blocking probability

Blocking probability definition as probability that all Trunks on busy network system. When all Trunks are busy, then the system is no more to process incoming traffic. In this situation incoming traffic is believed to experience blocking.

v. Blocking network

When all Trunks are busy, then the system is no more to process incoming traffic. In this situation incoming traffic is believed to experience blocking. This type of Network is blocking network.

6)

a. A group of 20 trunks is offered 2E of traffic:

Find

- i) The grade of service.
- ii) The probability that only one trunk is busy.
- iii) The probability that only one trunk is free.
- iv) The probability that at least one trunk is free.

Detailed solutions for problems:

6a

$$E_{1,N}(A) = \frac{A \cdot E_{1,N-1}(A)}{N + A \cdot E_{1,N-1}(A)}$$

20 Trunks offered 2E of traffic

$$B = E_{1,N}(A) = \frac{A^N/N!}{\sum_{K=0}^N A^K/K!}$$

If 5 Trunks
(RED is Ans)

$$E_{1,5}(2) = 0.037$$

$$B = E_{1,20}(2) = \frac{2^{20}/20!}{\sum_{K=0}^{20} 2^K/K!} = \frac{4.3099 \times 10^{-13}}{7.39} = 5.83 \times 10^{-14}$$

$$\begin{aligned} \sum_{K=0}^{20} \frac{2^K}{K!} &= \frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \frac{2^5}{5!} + \frac{2^6}{6!} + \frac{2^7}{7!} + \frac{2^8}{8!} + \frac{2^9}{9!} + \frac{2^{10}}{10!} + \dots + \frac{2^{20}}{20!} \\ &= 1 + \frac{2}{1} + \frac{4}{2} + \frac{8}{6} + \frac{16}{24} + \frac{32}{120} + \frac{64}{720} + \frac{128}{5040} + \frac{256}{40320} + \frac{512}{362880} + \frac{1024}{3628800} + \dots \\ &\approx 7.380 + \frac{256}{40320} + \frac{512}{362880} + \frac{1024}{3628800} \\ &\approx 7.388 \\ &\approx 7.39 \end{aligned}$$

(ii)

$$P(x) = \frac{A^x/x!}{\sum_{K=0}^N A^K/K!}$$

$$P(1) = \frac{2^1/1!}{\sum_{K=0}^5 2^K/K!} = \frac{2}{7.266} = 0.275$$

$$P(1) = \frac{2^1/1!}{\sum_{K=0}^{20} 2^K/K!} = \frac{2}{7.39} = 0.270$$

$$(iii) P(19) = \frac{2^{19}/19!}{\sum_{k=0}^{20} 2^k/k!} = \frac{4.30 \times 10^{-12}}{7.39} = 5.83 \times 10^{-13}$$

$$P(4) = 0.0917$$

$$(iv) P(x < 20) = 1 - P(20)$$

$$= 1 - B$$

$$= 1 - (5.83 \times 10^{-14}) = 1$$

~~$$= 1 - 10^{14} \cdot 10^{-14} = 1 - 5.83 \times 10^{-14}$$~~

$$P(x < 5) = 1 - P(5) = 0.963$$

~~$$= 10^{-14} (10^{14} - 5.83)$$~~

b) A group of 20 trunks provides a grade of services of 0.01 when offered 12E of Traffic.

i). How much is the GOS improved if one extra trunk is added to the group

ii). How much does the GOS deteriorate if one trunk is out of service.

6
B

$$E_{1,N}(A) = \frac{A \cdot E_{1,N-1}(A)}{N + A \cdot E_{1,N-1}(A)}$$

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$$(i) E_{1,21}(12) = 0.0057$$

$$(ii) E_{1,19}(12) = 0.017$$

7) Explain the digital switching system hierarchy and its components

Switching System Hierarchy

Figure 2.2 also shows the different classes of switching system in the North American network:

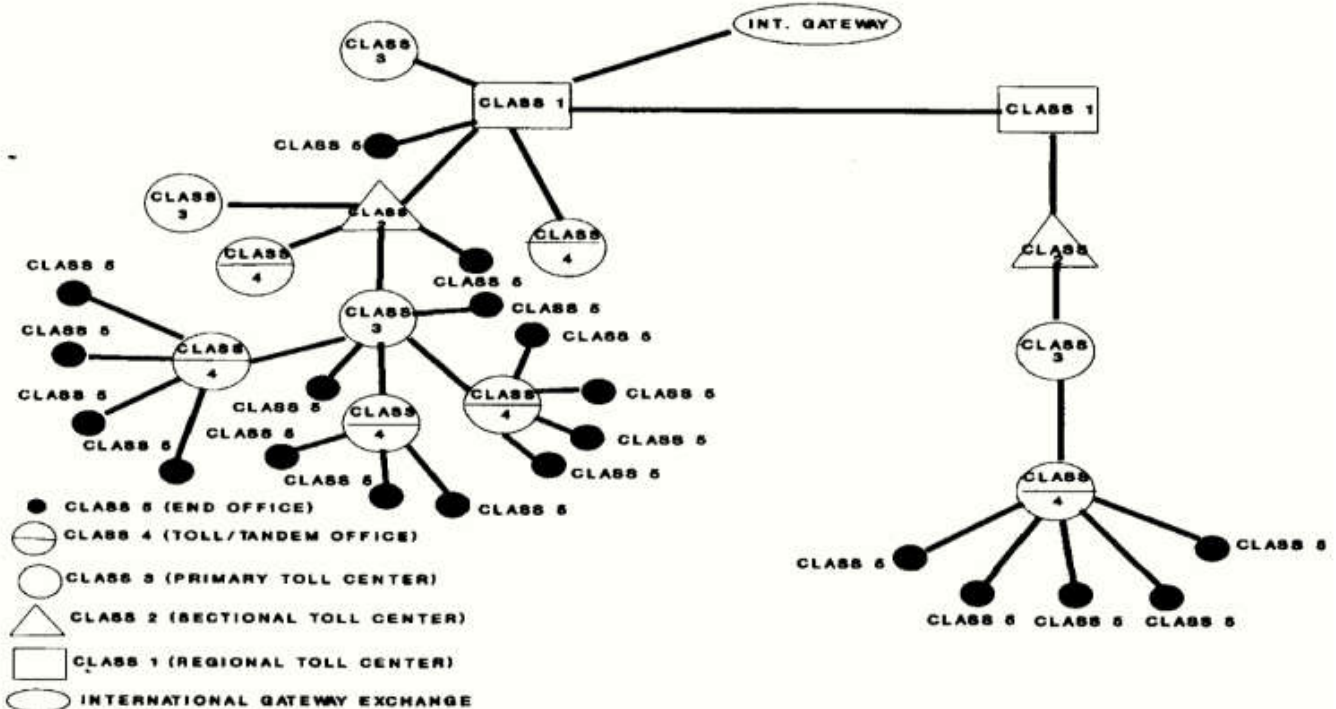


Figure 2.2. Switching system hierarchy

- Local exchange (class 5).

It is also referred to as the *end office (EO)*. It interfaces with subscribers directly and connects to toll centers via trunks. It records subscriber billing information.

- Tandem and toll office (class 4).

Most class 5 COs interface with the tandem offices. The tandem offices primarily switch trunk traffic between class 5 offices; they also interface with higher-level toll offices. Toll operator services can be provided by these offices.

- Primary toll center (class 3).

The class 3 toll center can be directly served by class 4 or class 5 offices, depending upon the trunk deployment. In other words, if the normal number of trunks in these offices are exhausted, then, traffic from lower-hierarchy offices can home into a class 3 office. Class 3 offices have the capability of storing, modifying, prefixing, translating, or code-converting received digits as well as finding the most efficient routing to higher-level toll offices.

- Sectional toll center (class 2).

It functions as a toll center and can home into class 1 offices.

- Regional toll center (class 1).

It functions as a toll center and can home into international gateway offices.

- *International gateway.*

These offices have direct access to international gateway offices in other countries. They also provide international operator assistance.

- The advantage of the hierarchical network is that it provides an efficient way of searching for a path through the network.
- The disadvantage is that if the primary sectional, or regional toll center goes down, then large areas of North America can become inaccessible.

- This switching hierarchy and the classification of offices are covered here to emphasize that just analyzing the reliability of a digital switching system may not solve the problem of overall network reliability.

- However, to the analyst who understands the interconnection of digital switching systems, it is clear that every part of a switching network must be analyzed to fully appreciate the impact of network reliability.