

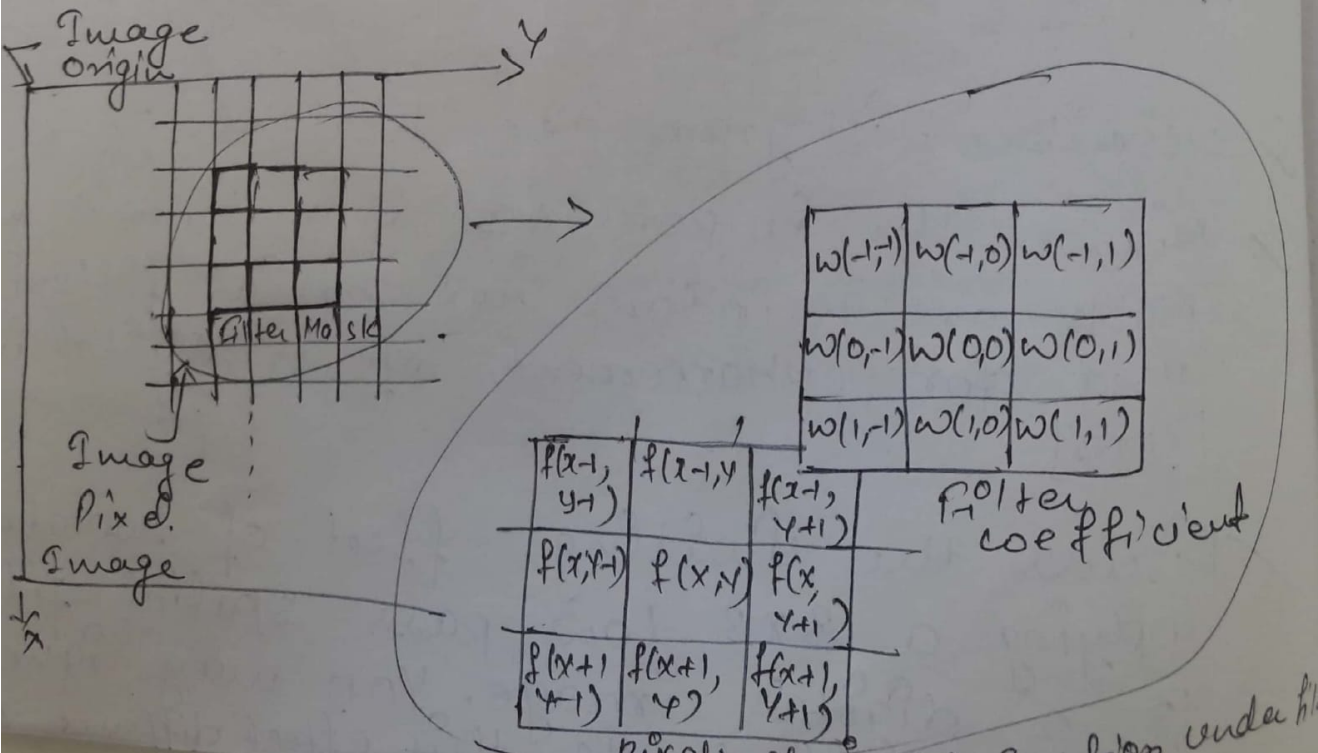
apply

Answer

The basic concepts of spatial filtering in image enhancement are as follows

The mechanics of Spatial Filtering consists of a neighborhood and a predefined operation that is performed on the image pixels encompassed by the neighborhood. Filtering creates a new pixel with coordinates equal to the coordinates of the center of the neighborhood, and whose value is the result of the filtering operations.

If the operation performed on the image pixel is linear, then the filter is called a linear spatial filter. Otherwise the filter is non-linear.



- Figure illustrates the mechanics of linear spatial filtering using a 3×3 neighborhood. At any point (x, y) in the image, the response, $g(x, y)$, of the filter is the sum of products (SOP) of the filter coefficients and the image pixels encompassed by the filter:

$$g(x, y) = w(-1, -1) f(x-1, y-1) + w(-1, 0) f(x-1, y) + \dots + w(0, 0) f(x, y) + \dots + w(1, 1) f(x+1, y+1)$$

- Linear spatial filtering of an image of size $M \times N$ with a filter of size $m \times n$ is given by the expression:

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x+s, y+t)$$

where, x & y are varied so that each pixel in w visits every pixel in f .

Spatial Correlation and Convolution

- Correlation is the process of moving a filter mask over the image and computing the sum of products at each location.
- Convolution is the same, except that the filter is first rotated by 180° .

Correlation

c
 A: \downarrow 00010000 1 2 3 2 8
 B: 0 0 0 1 0 0 0 0

1 2 3 2 8
 ↪ starting position alignment

c) Zero-padding

0000 000 10000 0000
 1 2 3 2 8

d) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
 1 2 3 2 8

↪ position after 1 shift

e) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
 1 2 3 2 8

↪ after 4 shifts

f) 0000 000 1 0 0 0 0 0 0 0
 1 2 3 2 8

↪ final position

0 0 0 8 2 3 2 1 0 0 0 0 [Full correlation result]

0 8 2 3 2 1 0 0 [cropped correlation result]

convolution

i) 00010000

rotated to 180°
82321

j) 00010000

82321

k) 0000000100000000
82321

l) 0000000100000000
82321

m) 0000000100000000
82321

n) 0000000100000000
~~82321~~ 82321

o) Full convolution result
000123280000

p) Cropped convolution result
01232800

- * correlation is a function of displacement of filter.
- Using correlation and convolution to perform SP filtering is a matter of preference.

Q Describe Histogram

- Histogram equalization automatically determines a transformation function that produces an uniform histogram.

In Histogram matching this method is used to generate a processed image that has a specified histogram and this process is called Histogram matching or Histogram specification.

$$s = T(r) \\ = (L-1) \int_0^r P_x(w) dw$$

where, w is dummy variable for integration
 s is the continuous version of Histogram equalization.

Let x be continuous random variables
consider, z to be continuous random variable with property

$$G(z) = (L-1) \int_0^z P_z(t) dt$$

where,

$P_z(t)$ is probability density function
 t is dummy variable for integration

so, $G(z) = s$

or, $G(z) = T(r)$

or, $z = G^{-1}[T(r)]$

$$= G^{-1}[s]$$

Consider 2-bit image of size 5×5

0	0	1	1	2
1	2	3	0	1
3	3	2	2	0
2	3	1	0	0
1	1	3	2	2

$$L = 4$$

intensity level $[0, 3]$

~~Histogram~~ of 25

Total pixels = 25

Histogram component

$$P(r_0) = \frac{6}{25} = 0.24$$

$$P(r_1) = \frac{7}{25} = 0.28$$

$$P(r_2) = 0.28$$

$$P(r_3) = 0.20$$

Ang. value of intensities in the image.

$$m = \sum_{i=0}^3 p_i(r_i) r_i$$

$$= 0(0.24)$$

$$+ 1(0.28)$$

$$+ 2(0.28)$$

$$+ 3(0.20)$$

$$= 1.44$$

In addition to gamma correction power law transformations are useful for general purpose.

Histogram:-

1	1	1	5	0	7
1	1	1	4	8	8
2	1	1	3	8	8
3	2	2	2	2	6
4	3	2	2	2	6
2	5	3	3	7	8
2	1	4	3	7	1

$p(r_k)$
norm $h(r_k)$

0.023
0.238
0.238
0.142
0.0714
0.0716
0.0716
0.0714
0.119

no. of pixels
1
10
10
6
3
2
2
3
5

Probab.
 $\frac{1}{42}$
 $\frac{10}{42}$
 $\frac{10}{42}$
 $\frac{6}{42}$
 $\frac{3}{42}$
 $\frac{2}{42}$
 $\frac{2}{42}$
 $\frac{3}{42}$
 $\frac{5}{42}$

Intensity
0
1
2
3
4
5
6
7
8

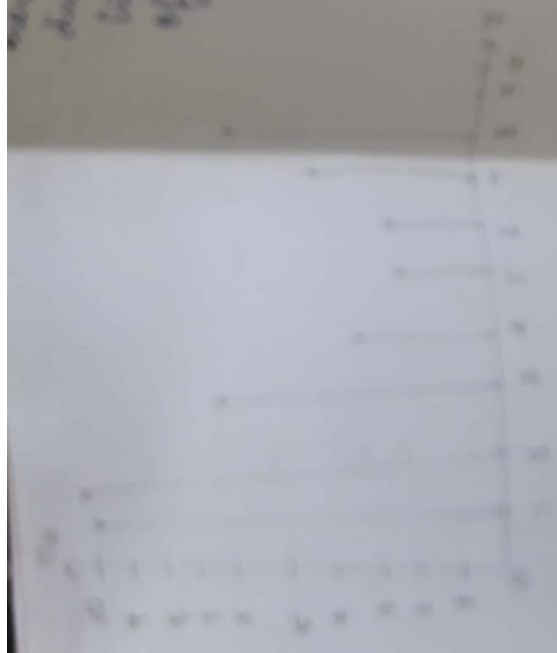
histogram of a digital image with intensity levels in range $[0, L-1]$ is a discrete function $h(r_k) = n_k$ where r_k is the k th intensity level and n_k is the no. of pixels in the image with intensity r_k

Normalizing a histogram -
 divide each component of
 histogram by total number
 of pixels in the image

The normalized histogram

$$P(r_k) = \frac{n_k}{N}$$

- N - no. of pixels
- n_k - no. of pixels at r_k



Otsu's algorithm may be summarized as follows:

1. Compute the normalized histogram of the input image. Denote the components of the histogram by p_i , $i = 0, 1, 2, \dots, L - 1$.
2. Compute the cumulative sums, $P_1(k)$, for $k = 0, 1, 2, \dots, L - 1$, using Eq. (10.3-4).
3. Compute the cumulative means, $m(k)$, for $k = 0, 1, 2, \dots, L - 1$, using Eq. (10.3-8).
4. Compute the global intensity mean, m_G , using (10.3-9).
5. Compute the between-class variance, $\sigma_B^2(k)$, for $k = 0, 1, 2, \dots, L - 1$, using Eq. (10.3-17).
6. Obtain the Otsu threshold, k^* , as the value of k for which $\sigma_B^2(k)$ is maximum. If the maximum is not unique, obtain k^* by averaging the values of k corresponding to the various maxima detected.
7. Obtain the separability measure, η^* , by evaluating Eq. (10.3-16) at $k = k^*$.

The following example illustrates the preceding concepts.

■ Figure 10.39(a) shows an optical microscope image of polymersome cells, and the objective of this example is to segment the image.

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Region-Based Segmentation

discussed in Section 10.1, the objective of segmentation is to partition an image into regions. In Section 10.2, we approached this problem by attempting to find boundaries between regions based on discontinuities in intensity levels. In Section 10.3, segmentation was accomplished via thresholds based on the distribution of pixel properties, such as intensity values or color. In this section, we discuss segmentation techniques that are based on finding the regions directly.

4.1 Region Growing

As its name implies, *region growing* is a procedure that groups pixels or subregions into larger regions based on predefined criteria for growth. The basic approach is to start with a set of "seed" points and from these grow regions by appending to each seed those neighboring pixels that have predefined properties similar to the seed (such as specific ranges of intensity or color).

Selecting a set of one or more starting points often can be based on the nature of the problem, as shown later in Example 10.23. When a priori information is not available, the procedure is to compute at every pixel the same set of properties that ultimately will be used to assign pixels to regions during the growing process. If the result of these computations shows clusters of values, the pixels whose properties place them near the centroid of these clusters can be used as seeds.

The selection of similarity criteria depends not only on the problem under consideration, but also on the type of image data available. For example, the analysis of land-use satellite imagery depends heavily on the use of color. This problem would be significantly more difficult, or even impossible, to solve without the inherent information available in color images. When the images are monochrome, region analysis must be carried out with a set of descriptors based on intensity, level, and spatial properties (such as moments or texture). We discuss the characterization in Chapter 11.

Descriptors alone can yield misleading results if connectivity properties are not used in the region-growing process. For example, visualize a random arrangement of pixels with only three distinct intensity values. Grouping pixels with the same intensity level to form a "region" without paying attention to connectivity would yield a segmentation result that is meaningless in the context of this discussion.

Another problem in region growing is the formulation of a stopping criterion. Region growth should stop when no more pixels satisfy the criteria for inclusion in that region. Criteria such as intensity values, texture, and color are local in nature and do not take into account the "history" of region growth. Additional criteria that increase the power of a region-growing algorithm include the concept of size, likeness between a candidate pixel and the pixels grown far from it (such as a comparison of the intensity of a candidate and the average intensity of the grown region), and the shape of the region being grown. The choice of these types of descriptors is based on the assumption that a model of the expected results is at least partially available.

Let: $f(x, y)$ denote an input image array; $S(x, y)$ denote a seed array containing 1s at the locations of seed points and 0s elsewhere; and Q denote a predicate to be applied at each location (x, y) . Arrays f and S are assumed to be of the same size. A basic region-growing algorithm based on 8-connectivity may be stated as follows.

1. Find all connected components in $S(x, y)$ and erode each connected component to one pixel; label all such pixels found as 1. All other pixels are labeled 0.
2. Form an image f_Q such that, at a pair of coordinates (x, y) , let $f_Q(x, y)$ be 1 if the input image satisfies the given predicate, Q , at those coordinates; otherwise, let $f_Q(x, y) = 0$.
3. Let g be an image formed by appending to each seed point in S all 1-valued points in f_Q that are 8-connected to that seed point.
4. Label each connected component in g with a different region label (1, 2, 3, ...). This is the segmented image obtained by region growing.

10.4.2 Region Splitting and Merging

The procedure discussed in the last section grows regions from a set of seed points. An alternative is to subdivide an image initially into a set of arbitrary disjoint regions and then merge and/or split the regions in an attempt to satisfy the conditions of segmentation stated in Section 10.1. The basics of splitting and merging are discussed next.

...each ...
 ...quadrant regions so that, for any region R_i , $Q(R_i) = \text{FALSE}$. If $Q(R) = \text{FALSE}$, we divide the image into smaller and smaller regions. If $Q(R) = \text{TRUE}$, we start with the entire image. This particular splitting technique has a convenient representation in the form of so-called *quadtrees*, that is, trees in which each node represents a quadtree sometimes are called *quadregions* or *quadimages*. Note that the root of the tree corresponds to the entire image and that each node corresponds to the subdivision of a node into four descendant nodes. In this case, only R_4 was subdivided further.

If only splitting is used, the final partition normally contains adjacent regions with identical properties. This drawback can be remedied by allowing merging as well as splitting. Satisfying the constraints of segmentation outlined in Section 10.1 requires merging only adjacent regions whose combined pixels satisfy the predicate Q . That is, two adjacent regions R_j and R_k are merged if $Q(R_j \cup R_k) = \text{TRUE}$.

The preceding discussion can be summarized by the following procedure in which, at any step, we

1. Split into four disjoint quadrants any region R_i for which $Q(R_i) = \text{FALSE}$.
2. When no further splitting is possible, merge any adjacent regions R_j and R_k for which $Q(R_j \cup R_k) = \text{TRUE}$.
3. Stop when no further merging is possible.

It is customary to specify a minimum quadregion size beyond which no further splitting is carried out.

Numerous variations of the preceding basic theme are possible. For example, significant simplification results if in Step 2 we allow merging of any two adjacent regions R_i and R_j if each one satisfies the predicate individually. This results in a much simpler (and faster) algorithm, because testing of the predicate is limited to individual quadregions. As the following example shows, this simplification is still capable of yielding good segmentation results.

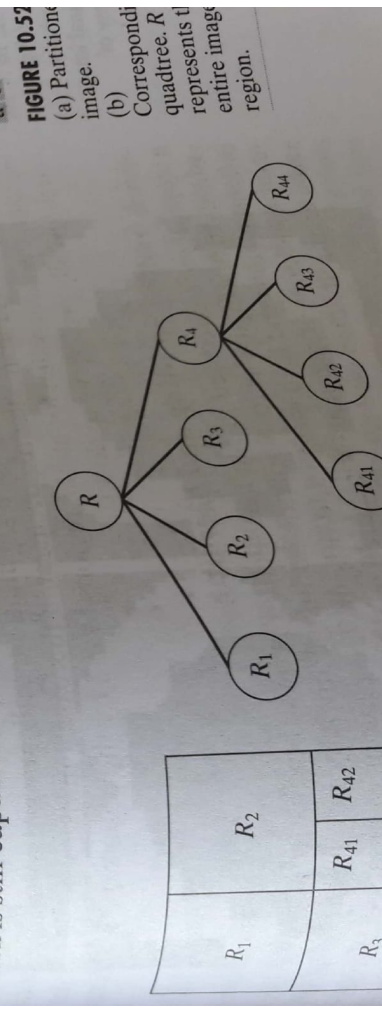


FIGURE 10.52
 (a) Partitioned image.
 (b) Corresponding quadtree. R represents the entire image region.

See Section 2.5.2 regarding region adjacency.

11.1.1.2 Chain Codes

Chain codes are used to represent a boundary by a connected sequence of straight-line segments of specified length and direction. Typically, this representation is based on 4- or 8-connectivity of the segments. The direction of each segment is coded by using a numbering scheme, as in Fig. 11.3. A boundary code formed as a sequence of such directional numbers is referred to as a Freeman chain code.

Digital images usually are acquired and processed in a grid format with equal spacing in the x - and y -directions, so a chain code can be generated by following a boundary in, say, a clockwise direction and assigning a direction to the segments connecting every pair of pixels. This method generally is unacceptable for two principal reasons: (1) The resulting chain tends to be quite long and (2) any small disturbances along the boundary due to noise or imperfect segmentation cause changes in the code that may not be related to the principal shape features of the boundary.

An approach frequently used to circumvent these problems is to resample the boundary by selecting a larger grid spacing, as Fig. 11.4(a) shows. Then, as the boundary is traversed, a boundary point is assigned to each node of the large grid, depending on the proximity of the original boundary to that node, as in Fig. 11.4(b). The resampled boundary obtained in this way then can be represented by a 4- or 8-code. Figure 11.4(c) shows the coarser boundary points represented by an 8-directional chain code. It is a simple matter to convert from an 8-code to a 4-code, and vice versa (see Problems 2.12 and 2.13). The starting point in Fig. 11.4(c) is (arbitrarily) at the topmost, leftmost point of the boundary, which gives the chain code 0766...12. As might be expected, the accuracy of the resulting code representation depends on the spacing of the sampling grid.

The chain code of a boundary depends on the starting point. However, the code can be normalized with respect to the starting point by a straightforward procedure. We simply treat the chain code as a circular sequence of direction numbers and redefine the starting point so that the resulting sequence of numbers forms an integer of minimum magnitude. We can normalize also for rotation (in angles that are integer multiples of the directions in Fig. 11.3) by using the first difference of the chain code instead of the code



FIG. 11.3 Direction numbers for 8-directional code and 4-directional code.