IAT 2 SOLUTIONS – PCS(15EC45)

1) Derive the time domain expression for a wideband FM wave.

Sol:

Wide band frequency modulation

→ Equ for FM wave,

$$s(t) = A_c \cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right) = \mathrm{Re}\{A_c \exp(j2\pi f_c t + j\beta \sin 2\pi f_m t)\}$$

$$= \mathrm{Re}\{\tilde{s}(t) \exp(j2\pi f_c t)\}$$

where $\tilde{s}(t)$ = complex envelope of FM

$$= A_c \exp(j\beta \sin(2\pi f_m t))$$

→ $\tilde{s}(t)$ = Periodic fn of t with fundamental freq f_m.

→ Expanding $\tilde{s}(t)$ in the form of complex Fourier series

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t) \quad \text{—(B)}$$

where

$$c_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp(-j2\pi n f_m t)\,dt$$

$$= f_m A_c \int_{-1/2f_m}^{1/2f_m} \exp\left[j\beta \sin(2\pi f_m t) - j2\pi f_m n t\right] dt \quad \text{—(A)}$$

Define a new variable,

$$\boxed{x = 2\pi f_m t}$$

$dx = 2\pi f_m \, dt$

$dt = \dfrac{dx}{2\pi f_m}$

$dt : -1/2f_m \rightarrow 1/2f_m$

$dx : -\pi \rightarrow \pi$

eqn (A) $\Rightarrow C_n = \dfrac{A_c}{2\pi} \displaystyle\int_{-\pi}^{\pi} \exp\{j(\beta \sin x - nx)\} dx$

$$= A_c \cdot J_n(\beta) \quad \text{——} \; (C)$$

where $J_n(\beta)$ = n^{th} order Bessel fn of 1^{st} kind & argument β

$$\boxed{J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\{j(\beta \sin x - nx)\} dx}$$

Sbtg (C) in to (B) $\Rightarrow$

$\therefore \tilde{s}(t) = A_c \displaystyle\sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j 2\pi n f_m t)$

$s(t) = A_c \cdot \mathrm{Re}\left\{ \displaystyle\sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j 2\pi (f_c + n f_m) t] \right\}$

$$\boxed{s(t) = A_c \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi (f_c + n f_m) t]}$$

Fourier series representation of Single-tone FM

$$\boxed{S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)]}$$

2a) Explain the direct method of generating FM waves.

Sol:

Direct method (using VCO)

> Inst. freq. of carrier is varied directly in accordance with msg s/l by means of a device known as voltage controlled oscillator (VCO)

> ctrl of Sinusoidal oscillator is done by symmetrical incremental variation of reactive components of this n/w

Implementation of VCO - by using a sinusoidal osc having a highly freq. selective freq determining resonant n/w.

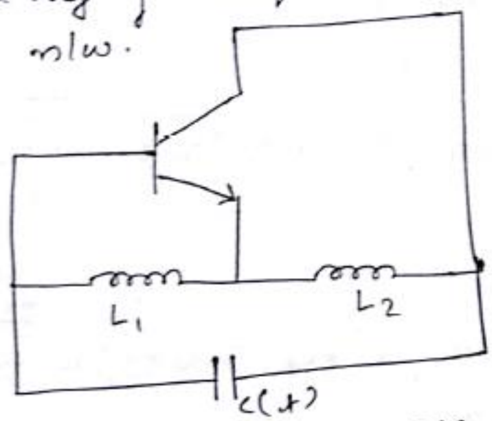


Fig 4.10 Hartley osc. (eg. of direct method)

Capacitor = fixed cap || voltage variable cap = C(t)

↓ Varicap or varactor

Varactor - capacitance depends on the voltage appl across the electrodes

Varactor - can be obtained by using P.N jn in rev. biase (rev.bias ↑ ⇒ cap ↓es)

→ Freq. of osc. of Hartley ,

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}} \quad \text{— (1)}$$

L_1, L_2 - inductance

$C(t)$ = fixed ‖ variable cap cap

→ For a sinusoidal modulating wave of freq f_m,

$$C(t) = C_0 + \Delta C \cos(2\pi f_m t) \quad \text{— (2)}$$

C_0 - total cap in the absence of modulation

ΔC - change in cap.

→ Sbtg. (2) in (1) ⟹

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0+\Delta C\cos 2\pi f_m t)}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left(1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right)}}$$

$$= \frac{f_0}{\sqrt{\left(1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right)}}$$

where $f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$ (unmodulated freq. of osc)

Assuming $\Delta C \lll C_0 \Rightarrow$

$$f_i(t) = f_0\left(1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right)^{-1/2}$$

$(1+x)^n = (1+nx)$ for $x<1$

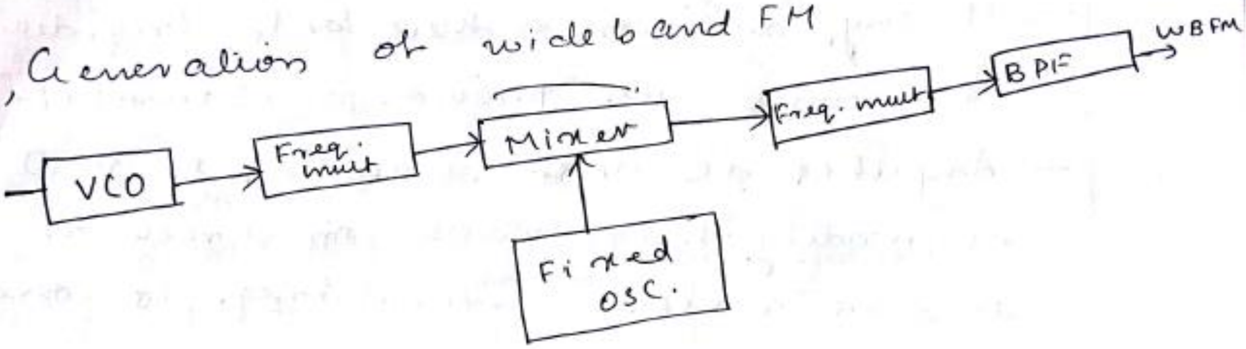
$$= f_0\left(1-\frac{\Delta C}{2C_0}\cos 2\pi f_m t\right)$$

Let $\frac{\Delta C}{2C_0} = \frac{-\Delta f}{f_0}$

$$\therefore \boxed{f_i(t) = f_0 + \Delta f \cos(2\pi f_m t)}$$

equ for inst. freq. of FM wave

Generation of wideband FM



2b) Discuss about the transmission bandwidth of FM signal

Sol:

→ FM contains infinite no of side freq so max BW required to transmit such a s/l is high(∞)

→ But FM s/l is effectively limited to a finite no. of significant side frequencies.

→ As $\beta > 1 \Rightarrow$ BW = greater than $2\Delta f$.

$\beta < 1 \Rightarrow BW = 2f_m$.

$\therefore \boxed{B_T \simeq 2\Delta f + 2f_m = 2\Delta f\left(1+\frac{1}{\beta}\right)}$ → Carson's rule

B_T = Transmission BW.

→ For a more accurate assessment of BW requirements, define

"Transmission BW of an FM wave as the separation b/n 2 freq. beyond which none of the side freq. is greater than 1% of the carrier amplitude obtained when modulation is removed".

ie, $\boxed{\text{Transmission BW} = 2n_{max}f_m}$

f_m = modulation freq

n_{max} = largest value of n that satisfies the requirement $|J_n(\beta)| > 0.01$

→ n_{max} varies with β (Table 4.1)

→ Transmission BW calculated using this method is plotted in the form of universal curve.
($B_T/\Delta f$ Vs β)

⇒ As β↑es, BW occupied by significant side freq's

onsider m(t) with max freq W.
BW in this case for FM transmission is obtained using worst-case tone modulation analysis.

$\boxed{D = \frac{\Delta f}{W}}$ → deviation ratio.

Replacing 'β' by 'D' & fm by W, we may use Carson's rule or universal curve to find out B_T

Universal curve yields ~~som~~ conservative result.

3a) Explain super heterodyne receiver for AM

Sol:

In a *broadcasting* system, irrespective of whether it is based on amplitude modulation or frequency modulation, the receiver not only has the task of demodulating the incoming modulated signal, but it is also required to perform some other system functions:

- *Carrier-frequency tuning*, the purpose of which is to select the desired signal (i.e., desired radio or TV station).
- *Filtering*, which is required to separate the desired signal from other modulated signals that may be picked up along the way.
- *Amplification*, which is intended to compensate for the loss of signal power incurred in the course of transmission.

Basically, the receiver consists of a radio-frequency (RF) section, a mixer and local oscillator, an intermediate-frequency (IF) section, demodulator, and power amplifier. Typical frequency parameters of commercial AM and FM radio receivers are listed in Table 2.3. Figure 2.32 shows the block diagram of a superheterodyne receiver for amplitude modulation using an envelope detector for demodulation.

The incoming amplitude-modulated wave is picked up by the receiving antenna and amplified in the RF section that is tuned to the carrier frequency of the incoming wave. The combination of mixer and local oscillator (of adjustable frequency) provides a *heterodyning* function, whereby the incoming signal is converted to a predetermined fixed *intermediate frequency*, usually lower than the incoming carrier frequency. This frequency translation is achieved without disturbing the relation of the sidebands to the carrier; see Section 2.4. The result of the heterodyning is to produce an intermediate-frequency carrier defined by

$$f_{IF} = f_{LO} - f_{RF} \tag{2.78}$$

where f_{LO} is the frequency of the local oscillator and f_{RF} is the carrier frequency of the incoming RF signal. We refer to f_{IF} as the intermediate frequency (IF), because the signal

is neither at the original input frequency nor at the final baseband frequency. The mixer–local oscillator combination is sometimes referred to as the *first detector*, in which case the demodulator is called the *second detector*.

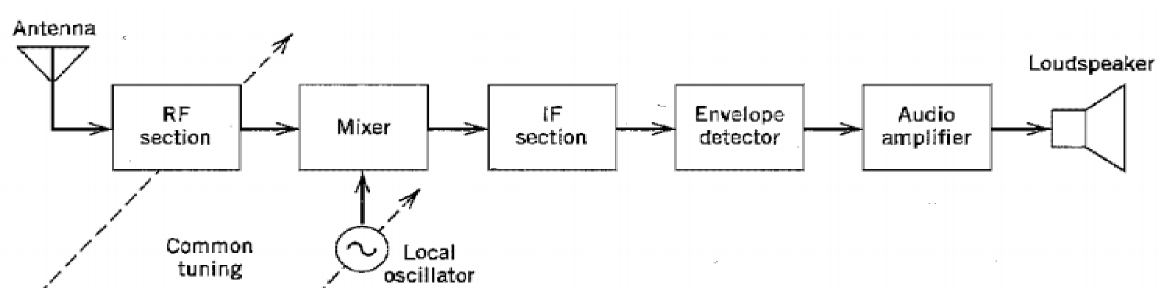


FIGURE 2.32 Basic elements of an AM radio receiver of the superheterodyne type.

The IF section consists of one or more stages of tuned amplification, with a bandwidth corresponding to that required for the particular type of modulation that the receiver is intended to handle. The IF section provides most of the amplification and selectivity in the receiver. The output of the IF section is applied to a demodulator, the purpose of which is to recover the baseband signal. If coherent detection is used, then a coherent signal source must be provided in the receiver. The final operation in the receiver is the power amplification of the recovered message signal.

3b) An angle modulated signal S(t) given by the equation; $S(t)=12\cos(12\pi 10^8 t+20\sin(2\pi 10^3 t)$.
Calculate its

(i) Un modulated carrier frequency (ii) Bandwidth (iii) Modulation index (iv) Frequency deviation

(v) Power dissipated in 100Ω resistor

Sol:

$$s(t) = 12\cos\left(12\pi 10^8 t + 0.5\sin(2\pi 10^3 t)\right)$$

$$s(t)_{FM} = A_c\cos\left(2\pi f_c t + \beta\sin(2\pi f_m t)\right)$$

$A_c = 12$ $\qquad\qquad$ $\beta = 0.5$

$f_c = 6\times10^8$ Hz $\qquad\qquad$ $f_m = 10^3$ Hz

Un modulated carrier frequency

$$f_c = 6\times10^8 \text{ Hz}.$$

$$\text{Bandwidth} = 2f_m(\beta+1)$$

$$= 2\times10^3(0.5+1)$$

$$BW = 3\text{KHz}$$

Modulation Index $\beta = 0.5$

$\beta = \frac{\Delta f}{f_m}$ $\quad \Delta f = \beta f_m$

Frequency deviation $\Delta f = 0.5 \times 10^3$

$= 500 \text{ Hz}$

Power $= \frac{A_c^2}{2R}$

4) Explain FM stereo multiplexing with neat block diagrams

Sol:

Stereo multiplexing is a form of frequency-division multiplexing (FDM) designed to transmit two separate signals via the same carrier. It is widely used in FM radio broadcasting to send two different elements of a program (e.g., two different sections of an orchestra, a vocalist and an accompanist) so as to give a spatial dimension to its perception by a listener at the receiving end.

The specification of standards for FM stereo transmission is influenced by two factors:

1. The transmission has to operate within the allocated FM broadcast channels.
2. It has to be compatible with monophonic radio receivers.

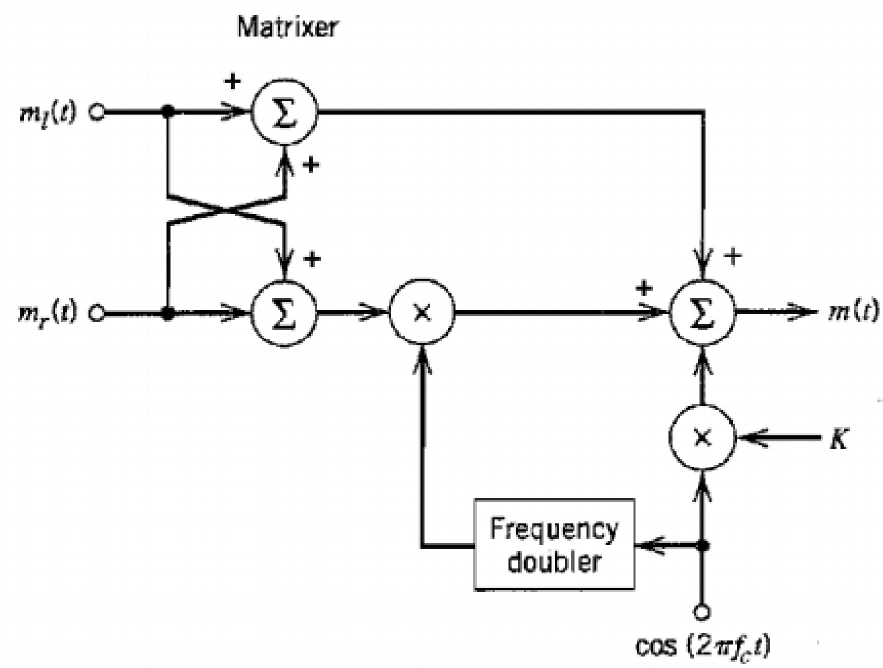
The first requirement sets the permissible frequency parameters, including frequency deviation. The second requirement constrains the way in which the transmitted signal is configured.

Figure 2.31*a* shows the block diagram of the multiplexing system used in an FM stereo transmitter. Let $m_l(t)$ and $m_r(t)$ denote the signals picked up by left-hand and right-hand microphones at the transmitting end of the system. They are applied to a simple *matrixer* that generates the *sum signal*, $m_l(t) + m_r(t)$, and the *difference signal*, $m_l(t) - m_r(t)$. The sum signal is left unprocessed in its baseband form; it is available for monophonic reception. The difference signal and a 38-kHz subcarrier (derived from a 19-kHz crystal oscillator by frequency doubling) are applied to a product modulator, thereby producing a DSB-SC modulated wave. In addition to the sum signal and this DSB-SC modulated wave, the multiplexed signal $m(t)$ also includes a 19-kHz pilot to provide a reference for the coherent detection of the difference signal at the stereo receiver. Thus the multiplexed signal is described by

$$m(t) = [m_l(t) + m_r(t)] + [m_l - m_r(t)] \cos(4\pi f_c t) + K \cos(2\pi f_c t) \qquad (2.72)$$

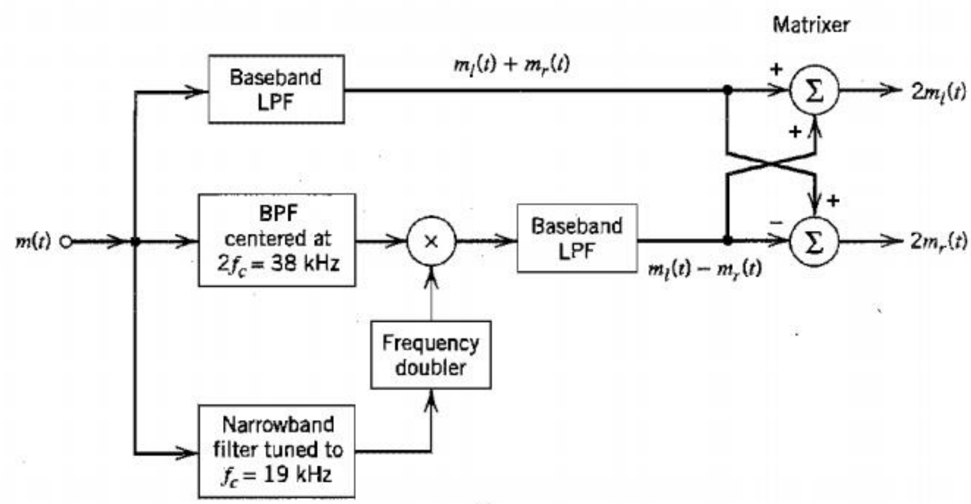
where $f_c = 19$ kHz, and K is the amplitude of the pilot tone. The multiplexed signal $m(t)$ then frequency-modulates the main carrier to produce the transmitted signal. The pilot is allotted between 8 and 10 percent of the peak frequency deviation; the amplitude K in Equation (2.72) is chosen to satisfy this requirement.

At a stereo receiver, the multiplexed signal $m(t)$ is recovered by frequency demodulating the incoming FM wave. Then $m(t)$ is applied to the *demultiplexing system* shown



(a)

(a)

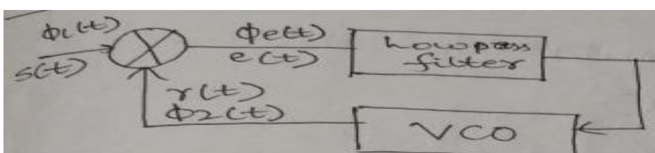


(b)

FIGURE 2.31 (*a*) Multiplexer in transmitter of FM stereo. (*b*) Demultiplexer in receiver of FM stereo.

in Figure 2.31*b*. The individual components of the multiplexed signal $m(t)$ are separated by the use of three appropriate filters. The recovered pilot (using a narrowband filter tuned to 19 kHz) is frequency doubled to produce the desired 38-kHz subcarrier. The availability of this subcarrier enables the coherent detection of the DSB-SC modulated wave, thereby recovering the difference signal, $m_l(t) - m_r(t)$. The baseband low-pass filter in the top path of Figure 2.31*b* is designed to pass the sum signal, $m_l(t) + m_r(t)$. Finally, the simple matrixer reconstructs the left-hand signal $m_l(t)$ and right-hand signal $m_r(t)$, except for scaling factors, and applies them to their respective speakers.

5a) Explain the non linear model of PLL with necessary equations



$v(t) \Rightarrow$

PLL

$\phi_e(t) = \phi_1(t) - \phi_2(t)$

$$s(t) = A_c \sin(2\pi f_c t + \phi_1(t))$$

$$r(t) = A_v \cos(2\pi f_c t + \phi_2(t))$$

$$\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau$$

$$\phi_2(t) = 2\pi k_v \int_0^t v(\tau)\,d\tau$$

$$e(t) = s(t)\, r(t)$$

$$= A_c \sin(2\pi f_c t + \phi_1(t))\, A_v \cos(2\pi f_c t + \phi_2(t))$$

$$= \frac{K_m' A_c A_v}{2} \left[\sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + \sin(\phi_1(t) - \phi_2(t)) \right]$$

K_m' = Multiplier gain.

Here we use : $\sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$

Since we pass it through the low pass filter, we obtain only the 2nd term

$$\boxed{e(t) = K_m A_c A_v [\sin(\phi_e(t))]}$$

From the circuit we obtain.

$$V(t) = e(t) * h(t)$$

$$V(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau$$

$$V(t) = \int_{-\infty}^{\infty} K_m A_c A_v \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau.$$

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$
$$= \phi_1(t) - 2\pi K_v \int_0^t v(\tau)\, d\tau$$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_v V(t)$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi K_v \int_{-\infty}^{\infty} K_m A_c A_v \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau$$

$$K_v K_m A_c A_v = K_o$$

$$\boxed{\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_o \int_{-\infty}^{\infty} \sin \phi_e(\tau)\, h(t-\tau)\, d\tau}$$

This is the non-linear integro-differential equation which is the descriptor of the dynamic nature of PLL.

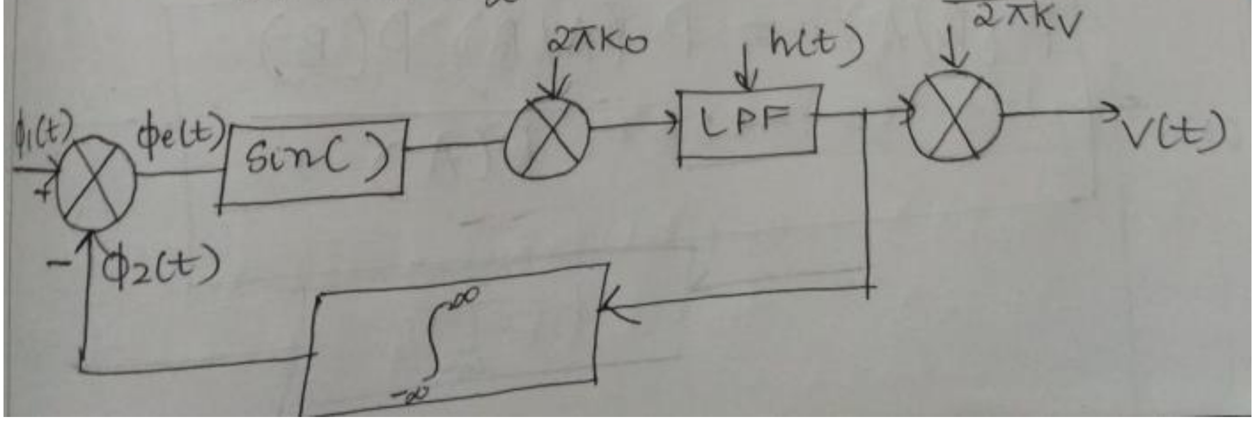
$$V(t) = \int_{-\infty}^{\infty} K_m A_c A_v \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau$$

$$K_m A_c A_v K_v = K_o$$

$$2\pi K_m A_c A_v K_v = 2\pi K_o$$

$$K_m A_c A_v = \frac{2\pi K_o}{2\pi K_{\cancel{v}}}$$

$$v(t) = \frac{1}{2\pi K_v} \int_{-\infty}^{\infty} 2\pi K_o \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau$$



5b) What is conditional probability? Show that P(B/A) = $\frac{P(A/B)P(B)}{P(A)}$

Sol:

Suppose we perform an experiment that involves a pair of events A and B. Let $P(B|A)$ denote the probability of event B, given that event A has occurred. The probability $P(B|A)$ is called the *conditional probability of B given A*. Assuming that A has nonzero probability, the conditional probability $P(B|A)$ is defined by

$$P(B|A) = \frac{P(AB)}{P(A)} \tag{A1.11}$$

where $P(AB)$ is the joint probability of A and B.

We may rewrite Equation (A1.11) as

$$P(AB) = P(B|A)P(A) \tag{A1.12}$$

It is apparent that we may also write

$$P(AB) = P(A|B)P(B) \tag{A1.13}$$

Accordingly, we may state that *the joint probability of two events may be expressed as the product of the conditional probability of one event given the other, and the elementary probability of the other.* Note that the conditional probabilities $P(B|A)$ and $P(A|B)$ have essentially the same properties as the various probabilities previously defined.

6) Discuss briefly about the posterior probabilities for a binary symmetric channel.

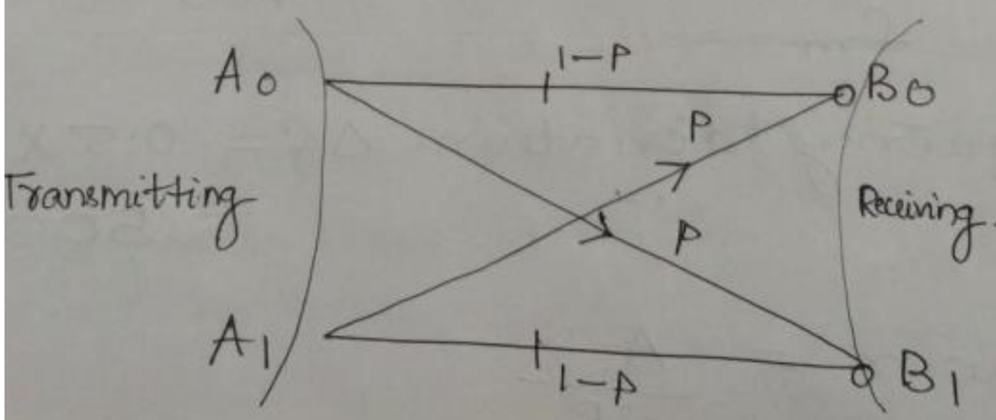
Sol:

Binary Symmetric channel

It is a discrete memory less binary channel used to transmit binary data (0 or 1)

Sent	Received
A_0	B_0
A_1	B_1

$$P(A_0) = P_0 \quad \text{—(1)}$$
$$P(A_1) = P_1 \quad \text{—(2)}$$



Transition diagram of binary symmetric channel

Error occurs when we send 0 but get 1 or when we send 1 and get a 0.

$$P(B_0/A_1).$$

~~The probability of~~

There are equal chances of getting an error $\Big\} \Rightarrow$ $P(B_0/A_1) = P$ — ③
$P(B_1/A_0) = P$ — ④

2 chances of getting B_0 $\Rightarrow P(B_0/A_0) + P(B_0/A_1) = 1$

$$\boxed{P(B_0/A_0) = 1 - P} \quad \text{— ⑤}$$

(using eq ③).

2 chances of getting B_1 $\Rightarrow P(B_1/A_0) + P(B_1/A_1) = 1$

$$\boxed{P(B_1/A_1) = 1 - P} \quad \text{— ⑥}$$

(using eq ④).

$$P(B_0) = P(B_0/A_0)\,P(A_0) + P(B_0/A_1)\,P(A_1)$$

$$\boxed{P(B_0) = (1-P)\,P_0 + P P_1} \quad \text{— ⑦}$$

$$P(B_1) = P(B_1/A_0)\,P(A_0) + P(B_1/A_1)\,P(A_1)$$

$$\boxed{P(B_1) = (1-P)\,P_1 + P P_0} \quad \text{— ⑧}$$

The requirement is to find the posterior probabilities which are $P(A_0/B_0)$ and $P(A_1/B_1)$

$$P(A_0/B_0) = \frac{P(B_0/A_0)\,P(A_0)}{P(B_0)}$$

$$\boxed{P(A_0/B_0) = \frac{(1-P)P_0}{(1-P)P_0 + PP_1}}$$

$$P(A_1/B_1) = \frac{P(B_1/A_1)\,P(A_1)}{P(B_1)}$$

$$\boxed{P(A_1/B_1) = \frac{(1-P)P_1}{(1-P)P_1 + PP_0}}$$