Internal Assessment Test III Solutions – MaY 2018

Sub: Principles of Communication Systems   Subject code: 15EC45

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1(a) Explain the following: random variable, mean, function of a random variable ,correlation and covariance

Sol:

Random Variables

→ It is convenient to assign a number or a range of values to the outcomes of a random experiment.

eg:- Head could correspond to '1' and tail to '0'.

→ The expression Random variable is used to describe this process of assigning a number to the outcome of a random experiment

→ The expected value or mean of a random variable 'X' is defined by

$$\mu_X = E[X] = \int_{-\infty}^{\infty} x \, f_X(x) \, dx \quad \text{—(1)}$$

where E - Statistical expectation operator.

- Function of a Random Variable
  - Let X denote a random variable, and let $g(X)$ denote a real-valued function defined on the real line. We denote as

$$Y = g(X) \qquad (5.37)$$

  - To find the expected value of the random variable Y.

$$\mathbf{E}[Y] = \int_{-\infty}^{\infty} y f_Y(y)\,dy \longrightarrow \mathbf{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f_x(x)\,dx \qquad (5.38)$$

→ Correlation is defined by $E[XY]$, which corresp... to Joint moment with $i=k=1$ in eqn(5)

ie, $E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{X,Y}(x,y)\,dx\,dy$.

→ The correlation of centered random variable $X - E[X]$ and $Y - E[Y]$, that is the joint mom...

$E\{(X - E[X])(Y - E[Y])\} = \text{COV}[XY]$ is calle... covariance of X and Y.

1 (b) Define autocorrelation function of the process X(t). Explain the properties of autocorrelation function.

Sol:

We define the *autocorrelation function* of the process $X(t)$ as the expectation of the product of two random variables $X(t_1)$ and $X(t_2)$.

$$R_X(t_1, t_2) = \mathbf{E}\left[X(t_1)X(t_2)\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_{X(t_1),X(t_2)}(x_1, x_2)\,dx_1 dx_2 \qquad (5.60)$$

We say a random process $X(t)$ is *stationary to second order* if the joint distribution $f_{X(t_1),X(t_2)}(x_1, x_2)$ depends on the difference between the observation time t_1 and t_2.

$$R_X(t_1, t_2) = R_X(t_2 - t_1) \quad \text{for all } t_1 \text{ and } t_2 \qquad (5.61)$$

For convenience of notation, we redefine the autocorrelation function of a stationary process $X(t)$ as

$$R_X(\tau) = \mathbf{E}\left[X(t+\tau)X(t)\right] \quad \text{for all } t \qquad (5.63)$$

This autocorrelation function has several important properties:

1. $R_X(0) = \mathbf{E}\left[X^2(t)\right]$ (5.64)

2. $R_X(\tau) = R_X(-\tau)$ (5.65)

3. $\left|R_X(\tau)\right| \le R_X(0)$ (5.67)

Proof of (5.64) can be obtained from (5.63) by putting $\tau = 0$.

Proof of (5.65):

$$R_X(\tau) = \mathbf{E}\left[X(t+\tau)X(t)\right] = \mathbf{E}\left[X(t)X(t+\tau)\right] = R_X(-\tau)$$

Proof of (5.67):

$$\mathbf{E}\left[\left(X(t+\tau)\pm X(t)\right)^2\right]\geq 0$$

$$\to \mathbf{E}\left[X^2(t+\tau)\right]\pm 2\mathbf{E}\left[X(t+\tau)X(t)\right]+\mathbf{E}\left[X^2(t)\right]\geq 0$$

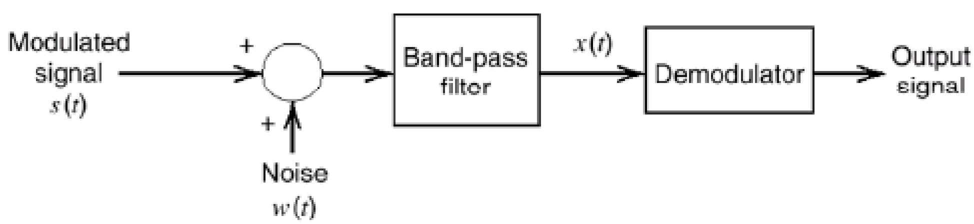
$$\to 2R_X(0)\pm 2R_X(\tau)\geq 0$$

$$\to -R_X(0)\leq R_X(\tau)\leq R_X(0)$$

$$\to \left|R_X(\tau)\right|\leq R_X(0)$$

2(a) With relevant equations, show how noises are produced in the receiver model.

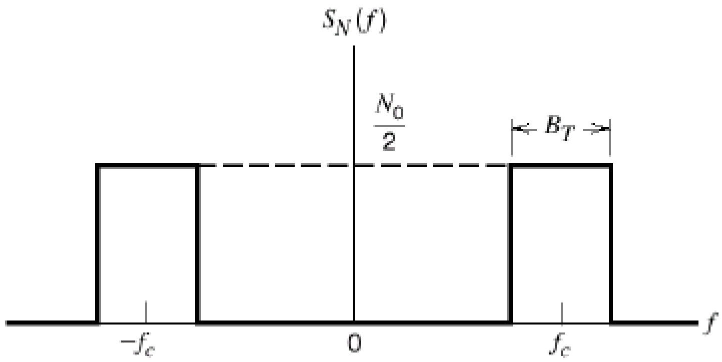
Sol:



- $s(t)$ denotes the incoming modulated signal.
- $w(t)$ denotes front-end receiver noise. The power spectral density of the noise $w(t)$ is denoted by $N_0/2$, defined for both positive and negative frequencies. N_0 is the average noise power per unit bandwidth measured at the front end of the receiver.
- The bandwidth of this band-pass filter is just wide enough to pass the modulated signal without distortion.
- Assume the band-pass filter is ideal, having a bandwidth equal to the transmission bandwidth B_T of the modulated signal $s(t)$, and a mid-band frequency equal to the carrier frequency f_c, $f_c >> B_T$.
- The filtered noise $n(t)$ may be treated as a narrow band noise represented in the canonical form:

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \qquad (6.1)$$

where $n_I(t)$ is the in-phase noise component and $n_Q(t)$ is the quadrature noise component, both measured with respect to the carrier wave $A_c\cos(2\pi f_c t)$.



Idealized characteristic of band-pass filtered noise.

- The filtered signal $x(t)$ available for demodulation is defined by

$$x(t) = s(t) + n(t) \qquad (6.2)$$

- The average noise power at the demodulator input is equal to the total area under the curve of the power spectral density $S_N(f)$:

$$P_{\text{avg-noise}} = 2 \times B_T \times \frac{N_0}{2} = B_T N_0$$

- *Input signal-to-noise ratio* $(SNR)_I$ is defined as:

$$(SNR)_I = \frac{\text{average power of the modulated signal } s(t)}{\text{average power of the filtered noise } n(t)}$$

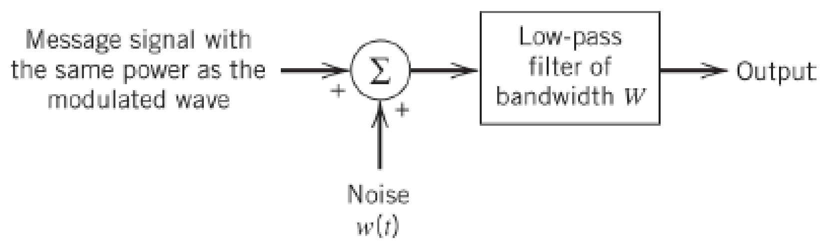
- *Output signal-to-noise ratio* $(SNR)_O$ is defined as:

$$(SNR)_O = \left.\frac{\text{average power of the demodulated message signal}}{\text{average power of the noise}}\right|_{\text{measured at the receiver output}}$$

- *Channel signal-to-noise ratio*

$$(SNR)_C = \left.\frac{\text{average power of the modulated signal}}{\text{average power of noise in the message BW}}\right|_{\text{measured at the receiver input}}$$

- This ratio may be viewed as the signal-to-noise ratio that results from baseband (direct) transmission of the message signal $m(t)$ without modulation, as demonstrated in the following figure:



  - The message power at the low-pass filter input is adjusted to be the same as the average power of the modulated signal
  - The low-pass filter passes the message signal and rejects out-of-band noise.

- *Figure of merit*
  - For the purpose of comparing different continuous-wave (CW) modulation systems, we normalize the receiver performance by dividing the output signal-to-noise ratio by the channel signal-to-noise ratio.
  - The higher the value of the figure of merit, the better will the noise performance of the receiver be.
  - The figure of merit may equal one, be less than one, or be greater than one, depending on the type of modulation used.

$$\text{Figure of merit} = \frac{(SNR)_O}{(SNR)_C}$$

2(b) Discuss the following: (i) capture effect (ii)threshold effect

Sol:

(i)Capture effect: In the frequency modulation, the signal can be affected by another frequency modulated signal whose frequency content is close to the carrier frequency of the desired FM wave. The receiver may lock such an interference signal and suppress the desired FM wave when interference signal is stronger than the desired signal. When the strength of the desired signal and interference signal are nearly equal, the receiver fluctuates back and forth between them, i.e., receiver locks interference signal for some times and desired signal for some time and this goes on randomly. This phenomenon is known as the capture effect.

(ii) Threshold effect: When the carrier-to-noise ratio at the receiver input of a standard AM is small compared to unity, the noise term dominates and the performance of the envelope detector changes completely.

In this case it is convenient to represent the narrow band noise n(t) in terms of its envelope r(t) and phase Ψ(t), as given by

$n(t) = r(t) \cos [2\pi f_c t + \Psi(t)]$ ---8.29

The phasor diagram of the detector input x(t) = s(t) + n(t) is shown in fig below,

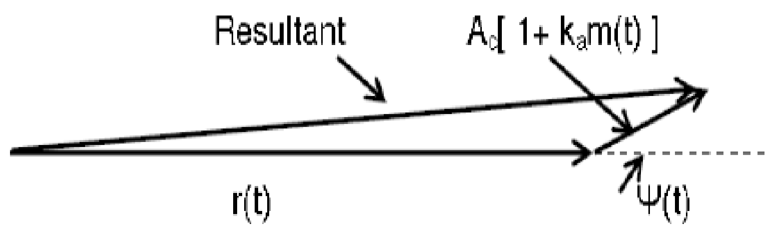


Fig 8.10 Phasor diagram for AM wave plus narrow-band noise for the case of low carrier-to-noise ratio

In the fig, we have used the noise as a reference, because it is a dominant term.

To the noise phasor, we added a phasor representing the signal term $A_c[1+ k_a m(t)]$, with the angle between them is Ψ(t).

From the fig it is observed that carrier amplitude is small compared to the noise envelope r(t). Then, we may approximate the envelope detector output as

$$y(t) = r(t) + A_c\cos[\Psi(t)] + A_c k_a m(t)\cos[\Psi(t)] \text{ ------------------------ } 8.30$$

The above relation shows that, the detector output is not proportional to the message signal m(t). The last term contains the message signal m(t) multiplied by

noise in the form of cos [Ψ(t)]. The phase Ψ(t) of a noise uniformly distributed over 2π radians, and it can have values between 0 to 2π with equal probability.

Therefore, we have a complete loss of information. The loss of a message in an envelope detector that operates at a low carrier to noise ratio is referred to as threshold effect.

It means a value of the carrier to noise ratio below which the noise performance of a detector deteriorates much more rapidly than high carrier to noise rario as in eqn8.24($(SNR)_0$).

3 Find the expression for figure of merit for DSB-SC receiver

Sol:

- The model of a DSB-SC receiver using a coherent detector

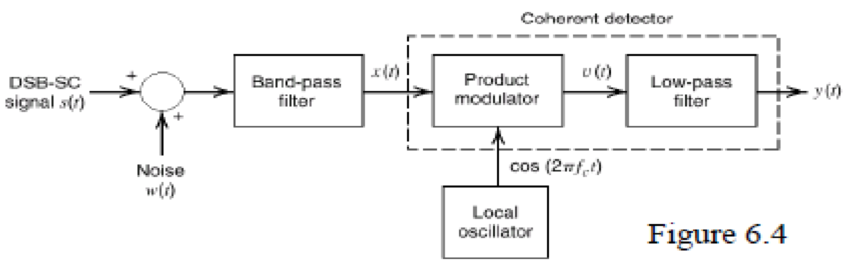


Figure 6.4

The DSB-SC component of the modulated signal $s(t)$ is expressed as

$$s(t) = CA_c\cos(2\pi f_c t)m(t) \qquad (6.4)$$

where C is the system dependent scaling factor. The purpose of which is to ensure that the signal component $s(t)$ is measured in the same units as the additive noise component $n(t)$.

$m(t)$ is the sample function of a stationary process of zero mean, whose power spectral density $S_M(f)$ is limited to a maximum frequency W, i.e. W is the message bandwidth.

The average power P of the message signal is the total area under the curve of power spectral density

$$P = \int_{-W}^{W} S_M(f)\,df$$

Therefore, the average power of the DSB-SC modulated signal component $s(t)$ is given by: $\dfrac{C^2 A_c^2 P}{2}$

The average power of the noise in the message BW is WN_0

The channel signal-to-noise ratio of the DSB-SC modulation system is:

$$(\text{SNR})_{C,\text{DSB}} = \frac{C^2 A_c^2 P}{2WN_0}$$

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= CA_c \cos(2\pi f_c t)\, m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \qquad (6.7) \end{aligned}$$

- The output of the product-modulator component of the coherent detector is:

$$\begin{aligned} \upsilon(t) &= x(t)\cos(2\pi f_c t) \\ &= \frac{1}{2} CA_c m(t) + \frac{1}{2} n_I(t) \\ &+ \frac{1}{2}\left[CA_c m(t) + n_I(t)\right]\cos(4\pi f_c t) - \frac{1}{2} A_c n_Q(t)\sin(4\pi f_c t) \end{aligned}$$

cos(α+β) + cos(α−β) = 2cos(α)cos (β)
sin(α+β) + sin(α−β) = 2sin(α)cos(β)

low-pass filter, BW=W

$$y(t) = \frac{1}{2} CA_c m(t) + \frac{1}{2} n_I(t) \qquad (6.8)$$

The receiver output signal : $y(t) = \frac{1}{2} CA_c m(t) + \frac{1}{2} n_I(t)$ (6.8)

The average power of message component may be expressed as

$$P_{avg} = \frac{C^2 A_c^2 P}{4}$$

The average power of the noise at the receiver output is

$$\left(\frac{1}{2}\right)^2 2WN_0 = \frac{1}{2}WN_0$$

- The output signal-noise ratio for DSB-SC

$$(\text{SNR})_{O,\text{DSB-SC}} = \frac{C^2 A_c^2 P/4}{WN_0/2}$$

$$= \frac{C^2 A_c^2 P}{2WN_0} \qquad (6.9)$$

- We obtain the figure of merit

$$\left.\frac{(\text{SNR})_O}{(\text{SNR})_C}\right|_{\text{DSB-SC}} = 1 \qquad (6.10)$$

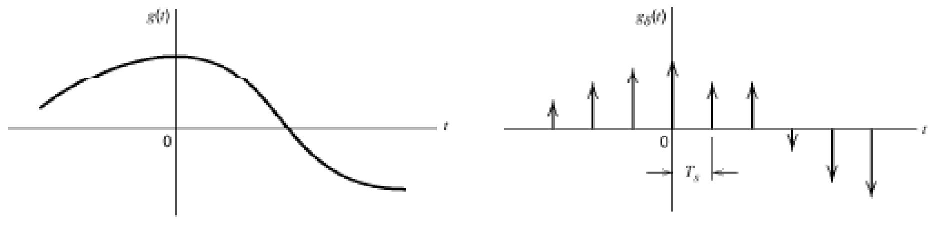
4 Explain sampling theorem. Derive an interpolation formula for reconstructing a message signal from its Samples. With neat frequency spectrum of signals, explain aliasing.

Sol:

- Through use of the sampling process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.
- It is necessary that we choose the sampling rate properly, so that the sequence of samples uniquely defines the original analog signal.
- Let $g_\delta(t)$ denote the *ideal sampled signal*

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s) \qquad (7.1)$$

- We refer to T_s as the *sampling period*, $f_s = 1/T_s$ as the *sampling rate*.



(a) (b)

- Applying Eq. (2.88), we get the result $\sum_{m=-\infty}^{\infty} g(t - mT_0) \rightleftharpoons f_0 \sum_{n=-\infty}^{\infty} G(nf_0)\,\delta(f - nf_0)$ (2.88)

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) \qquad (7.2)$$

where $G(f)$ is the Fourier transform of the original signal $g(t)$ and f_s is the sampling rate.

- Eq. (7.2) state that the process of uniformly sampling a continuous-times signal of finite energy results in a periodic spectrum with a period equal to the sampling rate.

- Taking the *discrete-time Fourier transform* of both sides of Eq. (7.1), we get

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\exp(-j2\pi n f T_s) \qquad (7.3)$$

- Hence, under the following two conditions

  1. $G(f) = 0$ for $|f| \geq W$ (Band-Limited Signal)

  2. $f_s = 2W \left(\text{or } T_s = \dfrac{1}{2W} \right)$

  we can get (from Eq. (7.3))

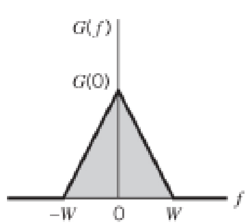
$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(-\frac{j\pi n f}{W}\right) \qquad (7.4)$$

- From Eq. (7.2), we readily see that the Fourier transform of $g_\delta(t)$ may also be expressed as

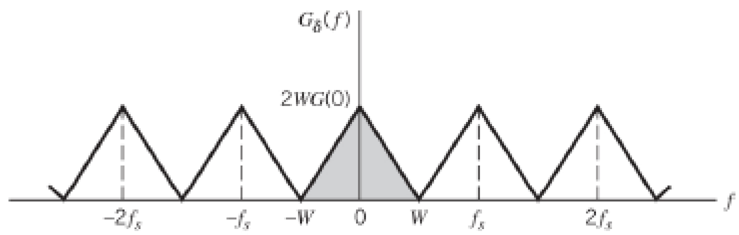
$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f - m f_s) \qquad (7.5)$$

- We find from Eq. (7.5) that

$$G(f) = \frac{1}{2W} G_\delta(f), \quad -W < f < W \qquad (7.6)$$



(a)



(b)

- Substituting Eq. (7.4) in Eq. (7.6), we may also write

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi nf}{W}\right), \quad -W < f < W \qquad (7.7)$$

- **Reconstructing the signal of $g(t)$**
  - Substituting Eq. (7.7) in the formula for the inverse Fourier transform $g(t)$ in terms of $G(f)$, we get

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi ft)\, df$$

$$= \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi nf}{W}\right) \exp(j2\pi ft)\, df$$

  - Interchanging the order of summation and integration

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \int_{-W}^{W} \exp\left[j2\pi f\left(t - \frac{n}{2W}\right)\right] df \qquad (7.8)$$

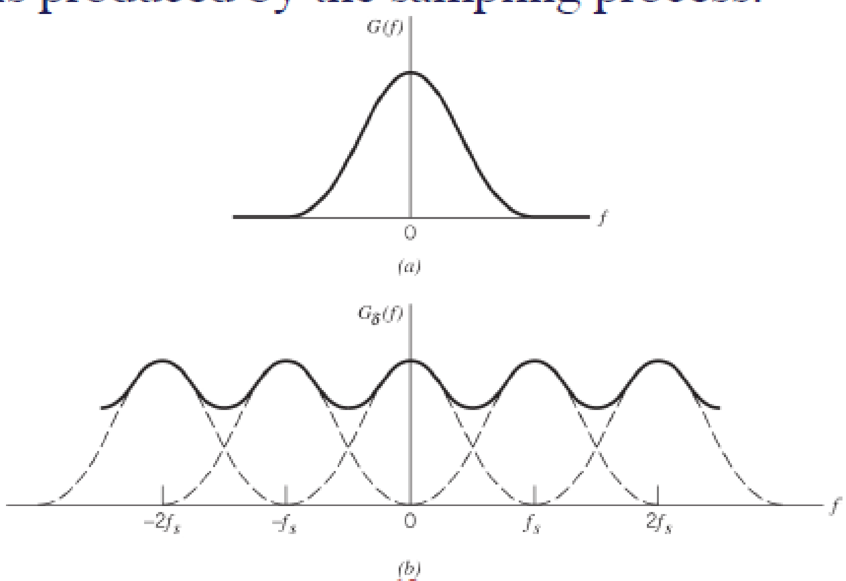
$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin(2\pi Wt - n\pi)}{(2\pi Wt - n\pi)} = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt - n) \quad -\infty < t < \infty$$

- **Reconstructing the signal of $g(t)$**

  - Eq. (7.9) provides an *interpolation formula* for reconstructing the original signal from the sequence of sample values $\{g(n/2W)\}$, with the $\operatorname{sinc}(2Wt)$ playing the role of an *interpolation function*.

- The *sampling theorem for strictly band-limited signals of finite energy* may be stated in two equivalent parts
  - A band-limited signal of finite energy, which only has frequency components less than W Hz, is completely described by specifying the values of the signal at instants of time separated by $1/2W$ seconds.
  - A band-limited signal of finite energy, which only has frequency components less than W Hz, may be completely recovered from a knowledge of its samples taken at the rate of $2W$ samples per second.
- The sampling rate of $2W$ samples per second, for a signal bandwidth of W Hz, is called the *Nyquist rate*; its reciprocal $1/2W$ (measured in seconds) is called *Nyquist interval*.

- In practice, however, an information-bearing signal is not strictly band-limited, with the result that some degree of undersampling is encountered. Consequently, some *aliasing* is produced by the sampling process.



5 Write a note on PAM. With a neat block diagram, explain how the message signal can be recovered from a PAM modulated signal?

Sol:

- In pulse-amplitude modulation (PAM), the amplitudes of regularly spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal; the pulses can be of a rectangular form or some other shape.

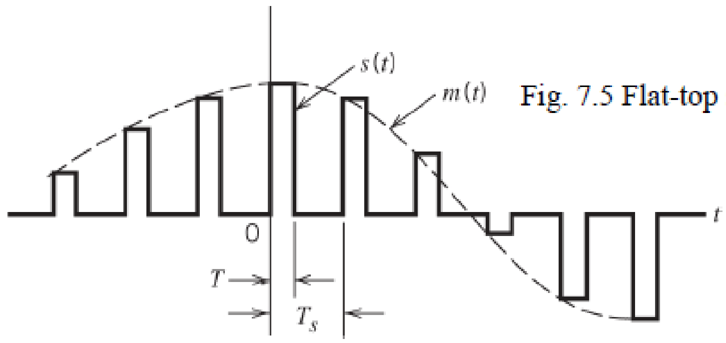


Fig. 7.5 Flat-top samples.

- Two operations are involved in the generation of the PAM signal
  - Instantaneous sampling of the message signal $m(t)$ every T_s seconds, where the sampling rate $f_s = 1/T_s$ is chosen in accordance with the sampling theorem.
  - Lengthening the duration of each sample so obtained to some constant value T.

- In digital circuit technology, these two operations are jointly referred to as "*sample and hold*."

Let $s(t)$ denote the sequence of flat-top pulses generated in the manner described in Fig. 5.5. Hence, we may express the PAM signal as

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s) \qquad (5.8)$$

where T_s is the sampling period and $m(nT_s)$ is the sample value of $m(t)$ obtained at time $t = nT_s$. The $h(t)$ is a standard rectangular pulse of unit amplitude and duration T, defined as follows (see Fig. 5.6(a)):

$$h(t) = \text{rect}\left(\frac{t - \frac{T}{2}}{T}\right) = \begin{cases} 1, & 0 < t < T \\ \frac{1}{2}, & t = 0, t = T \\ 0, & \text{otherwise} \end{cases} \tag{5.9}$$

By definition, the instantaneously sampled version of $m(t)$ is given by [see Eq. (5.1)]

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s) \tag{5.10}$$

where $\delta(t - nT_s)$ is a time-shifted delta function. To modify $m_\delta(t)$ so as to assume the same form as the PAM signal $s(t)$, we convolve $m_\delta(t)$ with the pulse $h(t)$, obtaining

$$\begin{aligned} m_\delta(t) \star h(t) &= \int_{-\infty}^{\infty} m_\delta(\tau)h(t - \tau)\, d\tau \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(nT_s)\delta(\tau - nT_s)h(t - \tau)\, d\tau \\ &= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t - \tau)\, d\tau \end{aligned} \tag{5.11}$$

where, in the last line, we have interchanged the order of summation and integration, both of which are linear operations. Using the sifting property of the delta function—namely,

$$\int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t - \tau)\, d\tau = h(t - nT_s)$$

we find that Eq. (5.11) reduces to

$$m_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s) \tag{5.12}$$

$$s(t) = m_\delta(t) \star h(t) \tag{5.13}$$

$$S(f) = M_\delta(f)H(f) \tag{5.14}$$

$$M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \tag{5.15}$$

where $f_s = 1/T_s$ is the sampling rate. Therefore, substitution of Eq. (5.15) into (5.14) yields

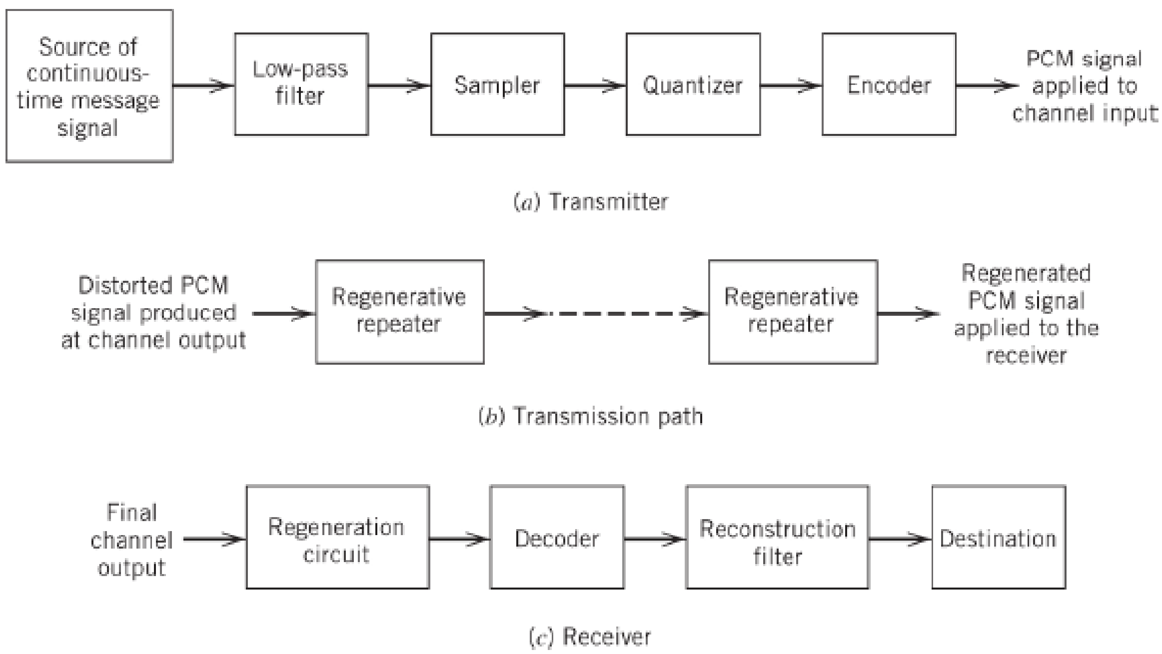
$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f) \tag{5.16}$$

$$H(f) = T\,\text{sinc}(fT)\exp(-j\pi fT) \tag{5.17}$$

6 With a neat diagram, explain PCM.

Sol:

- In *pulse-code modulation* (PCM) a message signal is represented by a sequence of coded pulses, which is accomplished by representing the signal in discrete form in both time and amplitude.



(a) Transmitter

(b) Transmission path

(c) Receiver

- Sampling
  - The incoming message signal is sampled with a train of narrow rectangular pulses so as to close approximate the instantaneous sampling process. Sampling rate must be greater than $2W$.
  - A pre-alias filter is used at the front end of the sampler in order to exclude frequencies greater than W before sampling.

- Quantization
  - The quantization process may follow a uniform law as described in the previous section.
    - Unacceptable signal-to-noise ratio for small signals.
    - Solution: Increasing quantization levels – price is too high.

- The use of a nonuniform quantizer is equivalent to pass the baseband signal through a *compressor* and applying the compressed signal to a uniform quantizer.
- In order to restore the signal samples to their correct relative level, we must use a device in the receiver with a characteristic complementary to the compressor. Such a device is called an *expander*.
- The compression and expansion laws are exactly inverse so that, except for the effect of quantization, the expander output is equal to the compressor input.
- The combination of a compressor and an expander is called a *compander*.

- μ - law (usually μ = 255; used in US, Canada, Japan)

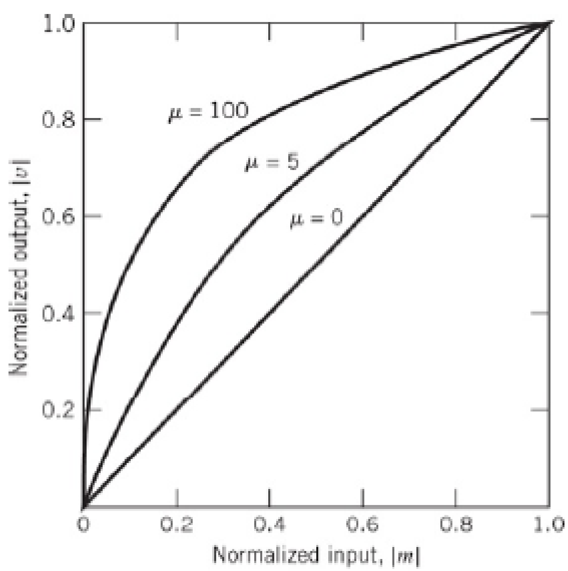
$$|v| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$

- A - law (usually A = 87.6; used in Europe)

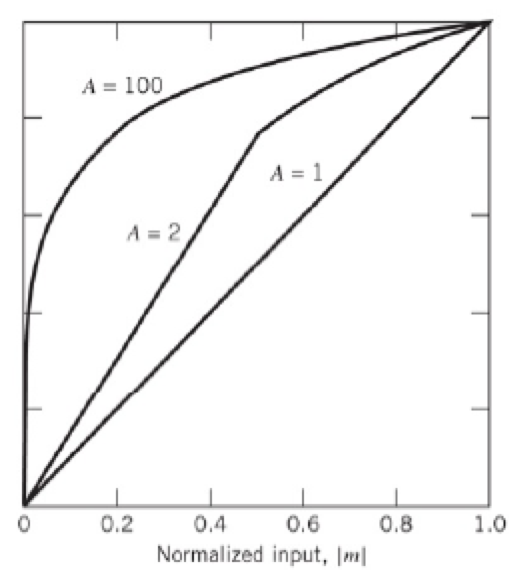
$$|v| = \begin{cases} \dfrac{A|m|}{1+\log A}, & 0 \le |m| \le \dfrac{1}{A} \\ \dfrac{1+\log(A|m|)}{1+\log A}, & \dfrac{1}{A} \le |m| \le 1 \end{cases} \qquad (7.48)$$

- m and v are the normalized input and output voltages, respectively.
- The case of uniform quantization corresponds to μ=0 and A=1.

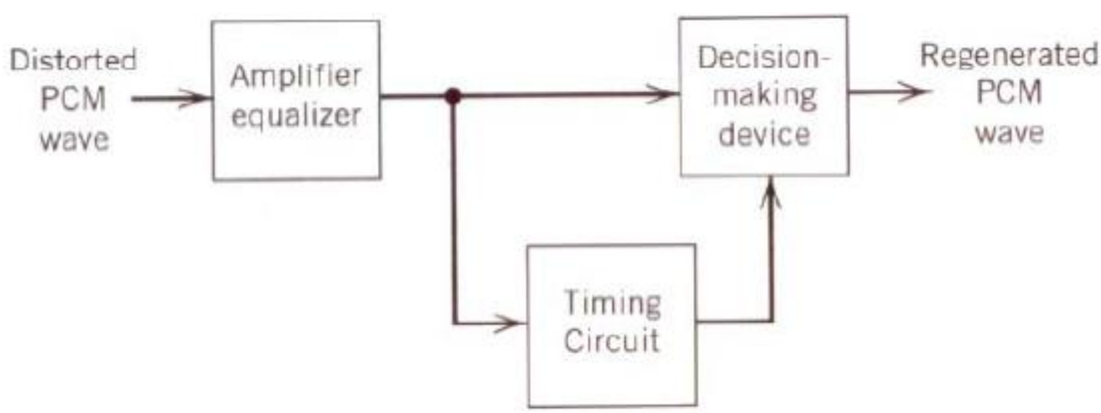
μ - law



A - law



- Encoding
  - In combining the processes of sampling and quantizing, the specification of a continuous message (baseband) signal becomes limited to a discrete set of values, but not in the form best suited to transmission over a line or radio path.
  - To exploit the advantages of sampling and quantizing for the purpose of making the transmitted signal more robust to noise, interference, and other channel degradations, we require the use of an *encoding process* to translate the discrete set of sample values to a more appropriate form of signal.
  - Any plan for representing each of this discrete events is called a *code*.
- Regeneration
  - The most important feature of any digital system lies in the ability to control the effects of distortion and noise produced by transmitting a digital signal through a channel.
  - This capability is accomplished by reconstructing the signal by means of a chain of *regenerative repeaters*, which perform three basic functions: *equalization*, *timing*, and *decision making*.



- *Equalizer*: shapes the received pulses so as to compensate for the effects of amplitude and phase distortions produced by the channel.
- *Timing* Circuitry: provides a periodic pulse train, derived from the received pulses, for sampling the equalized pulses at the instants of time where the signal-to-noise ratio is a maximum.
- *Decision-making* Device: the sample extracted is compared to a predetermined threshold.

- Decoding
  - The first operation in the receiver is to regenerate the received pulse one last time.
  - These clean pulses are then regrouped into code words and decoded in to a quantized PAM signal.
- Filtering
  - The final operation in the receiver is to recover the message signal wave by passing the decoder output through a low-pass reconstruction filter whose cutoff frequency is equal to the message bandwidth W.

7 Discuss generation and detection methods of PPM.

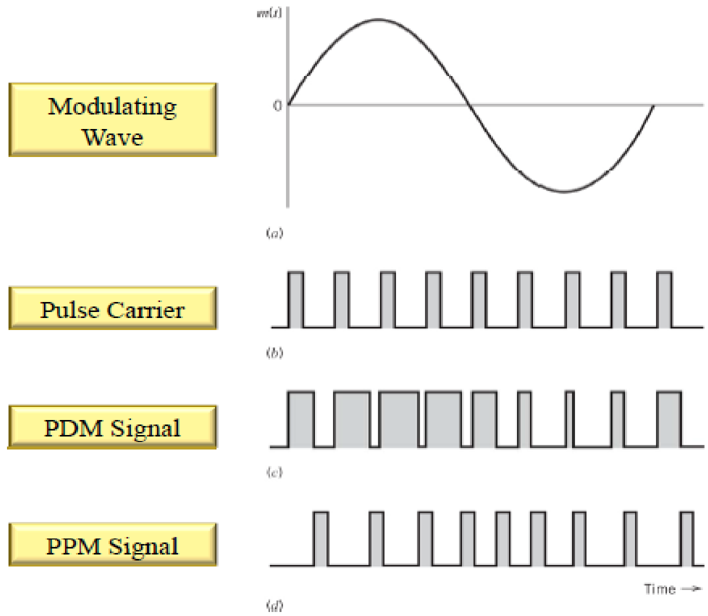
Sol:

- *Pulse-Position Modulation* (PPM): the position of a pulse relative to its unmodulated time of occurrence is varied in accordance with the message signal.

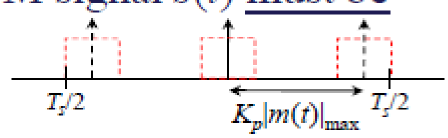
- Mathematical Representation of PPM Signal
  - Using the sample $m(nT_s)$ of a message signal $m(t)$ to modulate the position of nth pulse, we obtain the PPM signal

$$s(t) = \sum_{n=-\infty}^{\infty} g\left(t - nT_s - k_p m(nT_s)\right) \qquad (7.20)$$

where k_p is the *sensitivity* of the pulse-position modulator and $g(t)$ denotes a standard pulse of interest.



- The different pulses constituting the PPM signal $s(t)$ must be strictly non-overlapping.

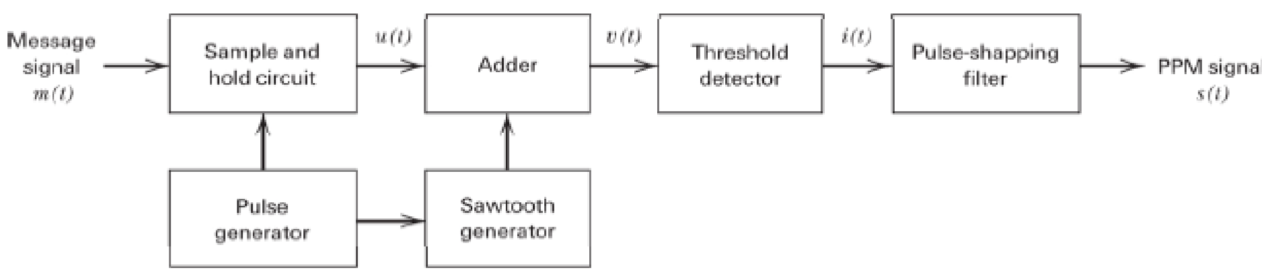


- A sufficient condition is given by:

$$g(t) = 0, \quad |t| > \frac{T_s}{2} - k_p |m(t)|_{\max} \qquad (7.21)$$

which in turn requires $\quad k_p |m(t)|_{\max} < \dfrac{T_s}{2} \qquad (7.22)$

- Generation of PPM waves



- The message signal $m(t)$ is first converted in to a PAM signal by means of a sample-and-hold circuit, generating a staircase waveform $u(t)$.

- Next, the signal $u(t)$ is added to a sawtooth wave, yielding the combined signal $v(t)$.

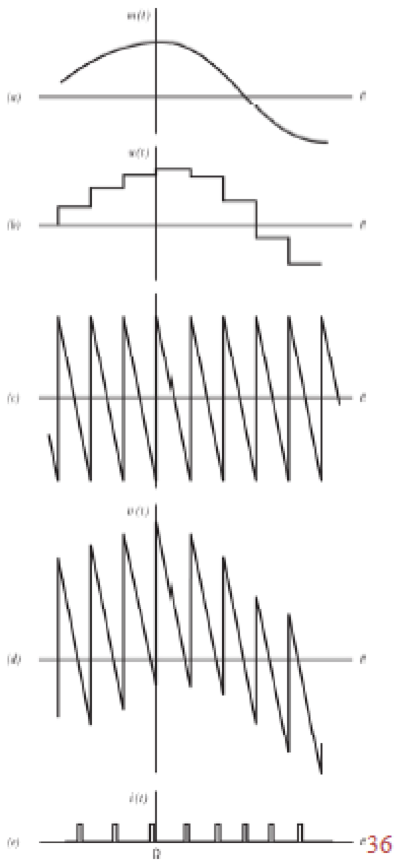
(a) Message signal

(b) Staircase approximation of the message signal

(c) Sawtooth wave

(d) Composite wave

(e) Sequence of Impulses used to generate the PPM signal



- The $v(t)$ is applied to a threshold detector that produces a very narrow pulse (approximating an impulse) each time $v(t)$ crosses zero in the negative-going direction.

- Finally, the PPM signal $s(t)$ is generated by using this sequence of impulses to excite a filter whose impulse response is defined by the standard pulse g(t).

Detection process:

- Convert PPM into PDM
- Integrate PDM
- Sample it at regular intervals to obtain PAM
- Use PAM demodulation method to recover message signal m(t)