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Internal Assesment Test - I

Sub:	RADAR Engineering							Code:	15EC833
Date:	07/ 03 / 2019	Duration:	90 mins	Max Marks:	50	Sem:	8th	Branch:	ECE

Answer Any FIVE FULL Questions

	Marks	OBE CO	OBE RBT
1. A 10 GHZ radar has the following characteristics P_t = 250 kW, p.r.f = 1500 pps, pulse width = 0.8 μs, power gain of antenna = 2500, $S_{min}=10^{-14}$ W, A_e = 10 m^2 , σ = 2 m^2 . Find (i) R_{unamb} (ii) Maximum possible range (iii) Duty cycle (iv) Average power.	[10]	CO1	L3

Soln:

Given: $f_p = 1500$ pps.
$\tau = 0.8\,\mu s$

Soln: (i) $R_{unamb} = \frac{c}{2f_p} = \frac{3\times10^8}{3000} = 100\,km$ 2.5 M

(ii) $R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 \cdot S_{min}}\right]^{1/4} = \left[\frac{1.25\times10^{10}}{1.58\times10^{-12}}\right]^{1/4} = \underline{\underline{298\,km}}$ 3 M

(iii) Duty cycle $= \frac{\tau}{T_p} = \tau f_p = \underline{\underline{0.12\%}}$ 2 M

(iv) Avg. Power ⇒ $P_{av} = \frac{\tau}{T_p} \cdot P_t = \tau \cdot f_p P_t = \underline{\underline{300\,W}}$ 2.5 M

	Marks	CO	RBT
2. Derive an expression for simple form of the RADAR range equation in three different forms starting from the power density of isotropic antenna.	[10]	CO1	L2

Soln

* If the transmitter power 'P_t' is radiated by an isotropic antenna, power density at a distance 'R' from radar is ⇒ the radiated power / surface area $4\pi R^2$ of an imaginary sphere of radius R.

i.e. $\boxed{\text{Power density at range R from an isotropic antenna} = \frac{P_t}{4\pi R^2}}$ —③

Power density → W/m^2.

[Isotropic antenna → one that radiates uniformly in all directions]

Radars employ directive antennas (with narrow beamwidths) to concentrate radiated power P_t in a particular direction.

← Radiation pattern of a directive antenna

* Gain of an antenna is a measure of the increased power density in some direction as compared to the power density that would appear in that direction from an isotropic antenna.

Maximum gain G of an antenna:

$$G = \frac{\text{maximum power density radiated by a directive antenna}}{\text{power density radiated by lossless isotropic antenna with the same power input}}$$ (4)

2 M

Power density at the target from a directive antenna with a transmitting gain G is:

$$\text{Power density at range R from directive antenna} = \frac{P_t G}{4\pi R^2}$$ (5)

The target intercepts a portion of the incident energy & re-radiates it in various directions.

- The echo signal of interest is only that which is reradiated in the direction of radar.
- The radar cross section of the target determines the power density returned to the radar for

particular power density incident on the target. It is denoted by 'σ' & is often called target cross section/radar cross section or simply cross section.

The radar cross section is defined by the equation: 1 M

$$\boxed{\text{Re-radiated power density back at the radar} = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2}} \quad -(6)$$

σ → units of area

Note:
Power intercepted by ~~radar~~ target of cross section σ = $\frac{P_t G \sigma}{4\pi R^2}$ watts.

→ this power is reradiated by target in all directions acting as an isotropic source

Hence,

$$\text{power density of echo signal at Radar} = \frac{P_t G \sigma}{4\pi R^2} \cdot \frac{1}{4\pi R^2} \ W/m^2$$

2 M

- Radar cross section is more dependent on the target's shape than on its physical size.
- The power received by the radar is given as the product of incident power density times the effective Area (A_e) of the receiving antenna.
- The effective area is related to the physical

area (A) by the relation: $\boxed{A_e = \rho_a A}$ 1 M

where ρ_a = antenna aperture efficiency.

(effective aperture → effective area or capture area.)

∴ Power received by radar is given by:

$$\boxed{P_r = \frac{P_t G}{4\pi R^2} \cdot \frac{\sigma}{4\pi R^2} \cdot A_e = \frac{P_t G A_e \sigma}{(4\pi)^2 R^4}} \quad \text{——} (7)$$

1 M

Maximum range of radar R_{max} is the distance beyond which the target cannot be detected. It occurs when the received signal 'P_r' just equals the minimum detectable signal 'S_{min}'.

Substituting $S_{min} = P_r$ in (7):

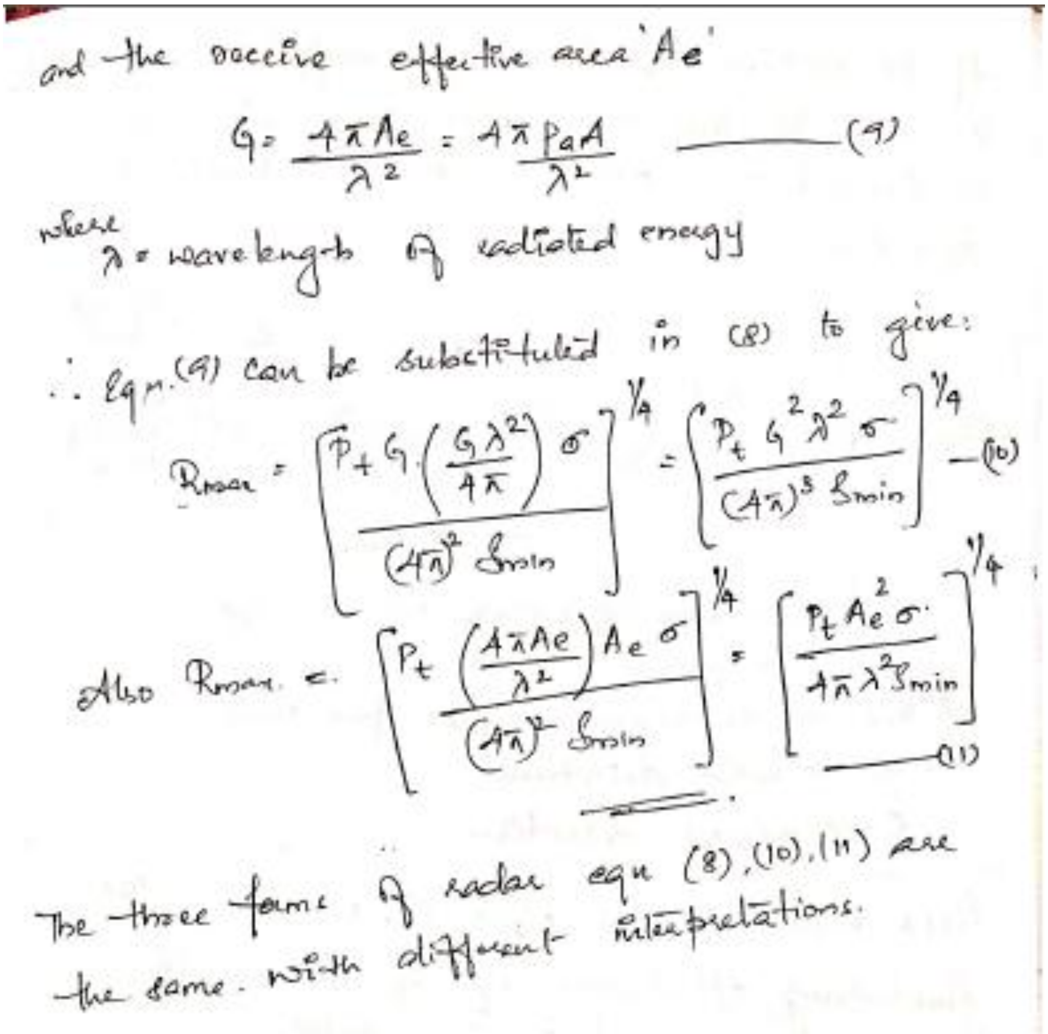
$$\boxed{R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}}\right]^{1/4}} \longleftarrow (8)$$

1M

This is the fundamental form of radar range equation (also called radar eqn or range eqn).

- Important antenna parameters are the transmitting gain & the receiving effective area.

- If the same antenna is used for both transmitting & receiving (usually in radar) antenna theory gives relation btwn transmit gain G



and the receive effective area 'A_e'

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \rho_a A}{\lambda^2} \quad \text{———} (9)$$

where
λ = wavelength of radiated energy

∴ eqn.(9) can be substituted in (8) to give:

$$R_{max} = \left[\frac{P_t G \left(\frac{G\lambda^2}{4\pi}\right)\sigma}{(4\pi)^2 S_{min}}\right]^{1/4} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4} \quad \text{—} (10)$$

$$\text{Also } R_{max} = \left[\frac{P_t \left(\frac{4\pi A_e}{\lambda^2}\right) A_e \sigma}{(4\pi)^2 S_{min}}\right]^{1/4} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}}\right]^{1/4} \quad \text{———} (11)$$

The three forms of radar eqn (8),(10),(11) are the same with different interpretations.

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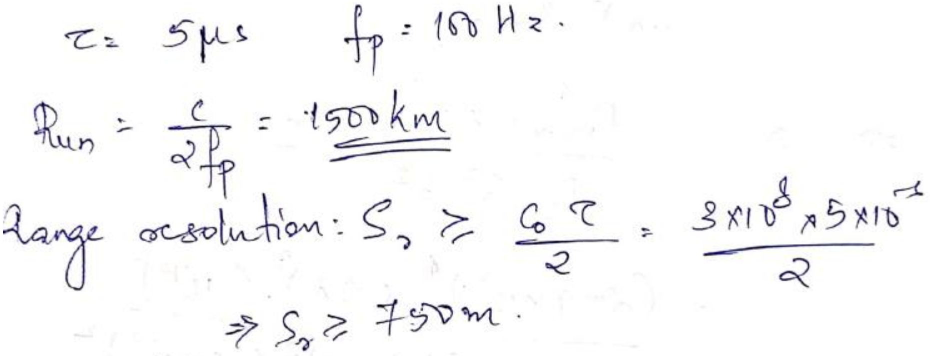
3. (a) A pulse radar having pulse width of 5μs and at p.r.f of 100 Hz. Find R_{unamb} and range resolution. [04] CO1 L3

Soln:



$\tau = 5\mu s \qquad f_p = 100\ Hz.$

$R_{un} = \frac{c}{2f_p} = \underline{\underline{1500\ km}}$

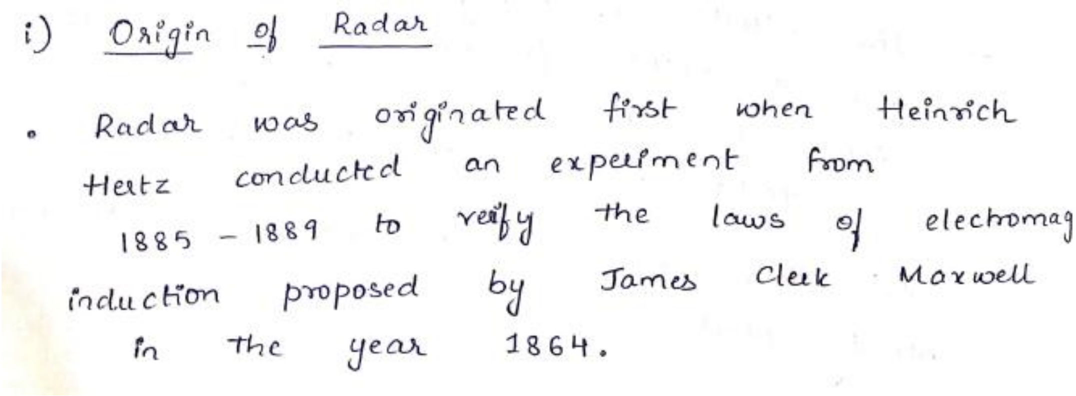
2 M

Range resolution: $S_r \geq \frac{c\tau}{2} = \frac{3\times10^8 \times 5\times10^{-6}}{2}$

2 M

$\Rightarrow S_r \geq 750m.$

(b) Write short notes on (i) Origins of RADAR (ii) Applications of RADAR [06] CO1 L2



i) <u>Origin of Radar</u>

- Radar was originated first when Heinrich Hertz conducted an experiment from 1885 – 1889 to verify the laws of electromag induction proposed by James Clerk Maxwell in the year 1864.

3 M

- The apparatus used by him was similar in function to a radar at a frequency in the vicinity of 875 Hz.
- It came to be known that the ~~[illegible]~~ radio waves were similar to the light waves except for the difference in the frequency.
- They also showed proofs that the waves were reflected from metallic objects & also ~~went~~ underwent refraction.
- In 1920, S.G Marconi, a well known pioneer of radio experiments, proved the detection of radio waves in his experiment.
- In was in the late 1920s or early 1930s, when radar found application in radar, due to a bomber aircraft used for military purpose.
- These radars used the old extension of the current - leading edge technologies.
- However, microwave radars are more efficient.

The various countries involved in the origination of radar are.

- United states
- United kingdom

- Germany
- USSR
- Italy
- France
- Japan

and after World War II

ii) Applications of Radar.

- **Military :**

  - Radar was widely used in air-defence systems for surveilance control and weapon control.

3 M

  - Surveilance control included target detection, target identification, target tracking and then assessment of engagement.
  - Weapon control involved target detection & fuzzing & guidance to weapon.

- **Remote sensing :**

  - It implies sensing the environment.
  - Used for weather observation which is necessary for regular TV information

- planetary observation like location of Venus below opaque clouds
- short - range below ground probing
- sea - ice detection for efficient shipping.

- Air traffic control :

- Use air surveilance control (ASR) for detection of clear sky.
- en-route from one airport to another by airport Router surveilance control (ARSC)

- Law enforcement and Highway

- Radar play a very important role for vehicle safety by keeping a track of speed
- Helps for air bag, warning about obstruction or people behind the vehicle.

- Aircraft safety and navigation :

- Military aircrafts travel along high terrain
- Prevent collision in air.

Ship safety :

- Radars are widely used in ships & boats to prevent collision.

- It is also used to track the presence of harbours when there is no visuality.

- **Space :**

- Radars are widely used for planetary observations
- It also gives accurate information about astronomical unit.
- When space vehicles were not free, radar was used to view space objects ~~near~~ from near.

- **Other :**

- Radars are used for non-contact measurement of speed & distance.
- Entologists & Ornithologists use radar for detection of birds & insects flying.

4. With a neat block diagram, explain the conventional pulse radar with a super heterodyne receiver. [10] CO1 L2

Soln: 4 M

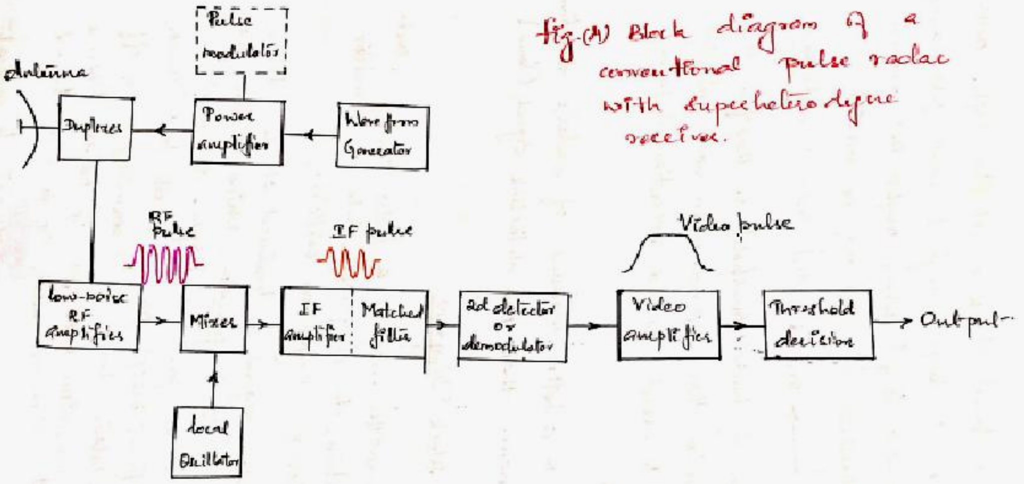


fig-(A) Block diagram of a conventional pulse radar with superheterodyne receiver.

RADAR Block Diagram

- The transmitter may be a power amplifier, such as klystron, traveling wave tube or transistor amplifier. or magnetron oscillator.

6 M

- The radar signal is produced at low power by a waveform generator, which is then the i/p to the power amplifier. In most power amplifiers, except for solid state power sources, a modulator turns the transmitter on & off in synchronism with the i/p pulses. When a power oscillator is used, it is also turned on & off by a pulse modulator to generate a pulse wave form

- The o/p of the transmitter is delivered to the antenna by a waveguide or other form of transmission line, where it is radiated into space. Antennas can be mechanically steered parabolic reflectors, planar arrays or phased arrays.
- On transmit, the parabolic reflector focuses the energy into a narrow beam. Using phase shifters at each radiating elements of a phased array, an electronically steered phased array can rapidly change the direction of the antenna beam in space without mechanically moving the antenna.

- Duplexer allows a single antenna to be used on a time-shared basis for both transmitting & receiving.
- The receiver is almost always a superheterodyne. The input or RF stage can be a low noise amplifier. The mixer & local oscillator (LO) convert the RF signal to an intermediate freq (IF) where it is amplified by an IF amplifier. The signal BW of a superheterodyne receiver is determined by the BW of its IF stage.
- The IF amplifier is designed as a matched filter (i.e. one which maximizes the o/p peak-signal-to-mean-noise ratio)

∴ matched filter maxi [illegible] detectability of weak echo signals & attenuates unwanted signals.

- A receiver with mixer as i/p stage (without low noise i/p stage) will be less sensitive because of the mixer's high NF
- IF amplifier is followed by a crystal diode which is traditionally called the second detector, or demodulator. Its purpose is to assist in extracting the signal modulation from the carrier.
- Combination of IF amplifier, video amplifier & second detector → act as an envelope detector to pass the pulse modulation (envelope) & reject the carrier freq.
- In radars which detect the doppler shift of the echo signal, the envelope detector is replaced by a phase detector.

- At the o/p of receiver, a decision is made whether or not a target is present. The decision is based on the magnitude of the receiver o/p. If the o/p is large enough to exceed a predetermined threshold, the decision is that a target is present, else only noise is assumed to be present.

5. Explain single delay line canceller with neat block diagram. Derive an expression for frequency response of single DLC. Also obtain the expression for blind speeds. [10] CO2 L2

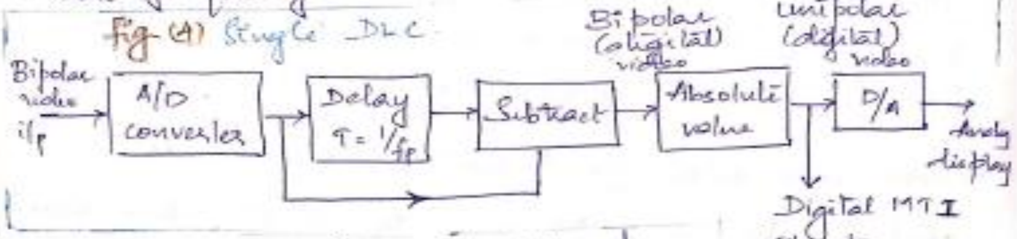
Soln:

Delay-line Cancelers

Simple MTI delay-line canceler (DLC) is a time-domain filter that rejects stationary clutter at zero frequency.

3 M

fig (4) Single DLC



If one sweep is subtracted from previous sweep, fixed clutter echoes will cancel & will not be detected or displayed.

Note: Sweep → ~~the~~ what occurs in time between two transmitted pulses.

On the other hand, moving targets change in amplitude from sweep to sweep because of their doppler frequency shift. If one sweep is subtracted from the other, result will be an uncancelled residue.

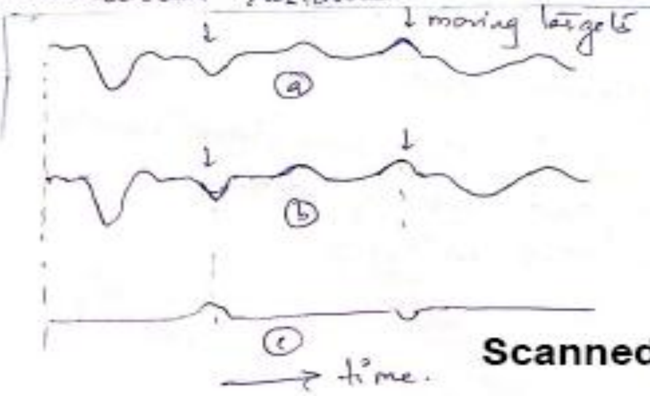


fig (5)
2 successive sweeps ⓐ & ⓑ
ⓒ → echoes from stationary targets are cancelled leaving only moving targets

3 M

DLC → has a freq response function H(f) that can be derived from time-domain representation of signals.

freq. response of single DLC

Signal from target at range R_0 at the output of the phase detector can be written as:

$$V_1 = k \sin(2\pi f_d t - \phi_0) \quad \text{———} \quad (9)$$

3 M

where.

f_d = doppler freq shift

ϕ_0 = constant phase = $\frac{4\pi R_0}{\lambda}$

R_0 = range at time = 0.

k = amplitude of the signal

The signal from the previous radar transmission is similar, except it is delayed by time T_p (T_p = pulse repetition interval)

$$V_2 = k \sin\left[2\pi f_d (t - T_p) - \phi_0\right] \quad \text{———} \quad (10)$$

(k = assumed to be same for both pulses)

* The DLC subtracts these two signals.

i.e. $$V = V_1 - V_2 = 2k \cdot \sin(\pi f_d T_p) \cos\left[2\pi f_d \left(t - \frac{T_p}{2}\right) - \phi_0\right] \quad \text{———} \quad (11)$$

$$\left\{ \because \sin A - \sin B = 2 \sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right) \right\}$$

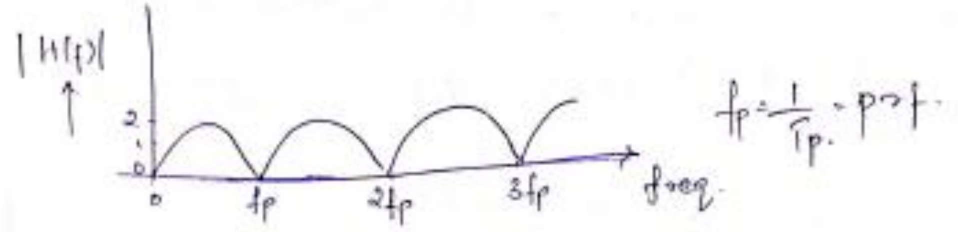
The o/p from DLC is seen to consist of a cosine wave with the same freq f_d as i/p but with amplitude $2k \sin(\pi f_d T_p)$

∴ the amplitude of the [illegible] depends
on the doppler freq shift & the pulse

repetition period (T_p).

∴ freq of response function of single delay-line canceler (o/p amplitude ÷ i/p amplitude k) is:

$$\boxed{H(f) = 2\sin(\pi f_d T_p).} \quad \text{———} (12)$$



Blind speeds

The response of the single delay-line canceler will be zero whenever the magnitude of $\sin(\pi f_d T_p)$ in eqn (12) is zero. → this occurs when $\pi f_d T_p = 0, \pm\pi, \pm 2\pi, \ldots$

$$\therefore \quad f_d = \frac{2v_r}{\lambda} \quad \text{(from eqn (5))}$$

$$\Rightarrow f_d = \frac{n}{T_p} = n f_p \quad ; \; n = 0, 1, 2, \ldots \quad \text{———} (13)$$ 1 M

↳ This states that: in addition to zero response at zero frequency, there will also be zero response of delay-line canceler whenever doppler freq. $f_d = \frac{2v_r}{\lambda}$ is a multiple of prf (doppler shift can be -ve or +ve)

The radial ~~speeds~~ velocities that produce blind speeds are found by equating eqn(13) and (5), and solving for radial velocity; which gives:

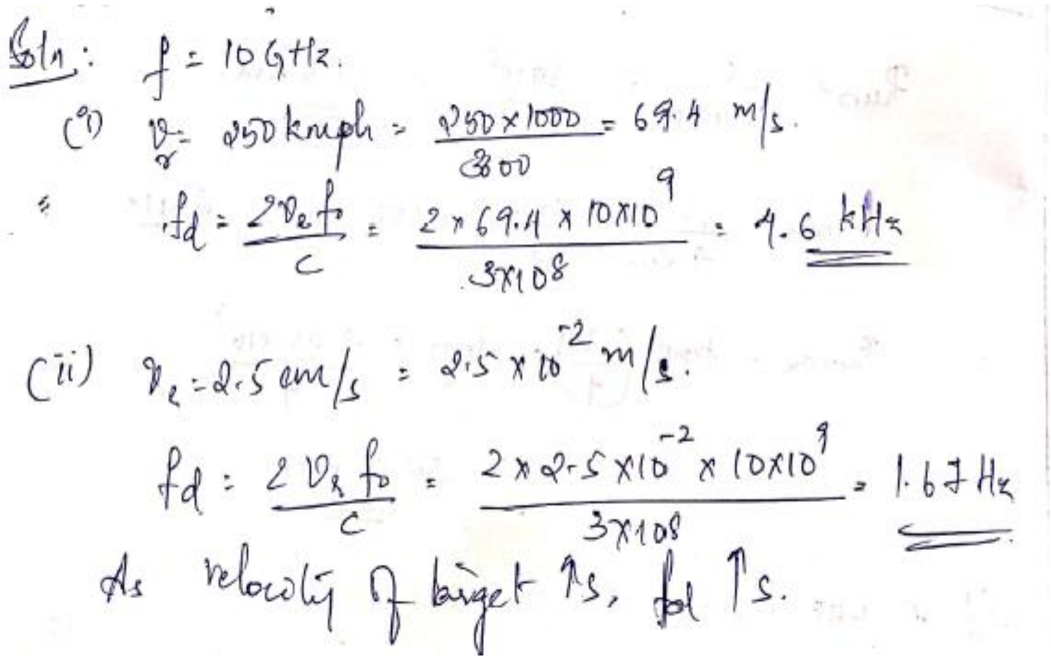
$$v_n = \frac{n\lambda}{2T_p} = \frac{n\lambda f_p}{2} \quad ; \; n = 1, 2, 3, \ldots \quad \text{———} (14)$$

v_r is replaced by v_n (n^{th} blind speed).

Those relative target velocities which result in zero MTI response are called "blind speeds" given by eqn (14).

6. (a) A CW RADAR operates at a frequency of 10GHz. What is the Doppler frequency produced by (i) an aero plane plying at a speed of 250 kmph (ii) a man crawling at 2.5 cm/sec. What do you understand? [06] CO2 L3

Soln:



Soln: $f = 10\,GHz$.

(i) $v_r = 250\,kmph = \frac{250 \times 1000}{3600} = 69.4\ m/s$.

$f_d = \frac{2v_r f_0}{c} = \frac{2 \times 69.4 \times 10 \times 10^9}{3 \times 10^8} = 4.6\ kHz$ 2.5 M

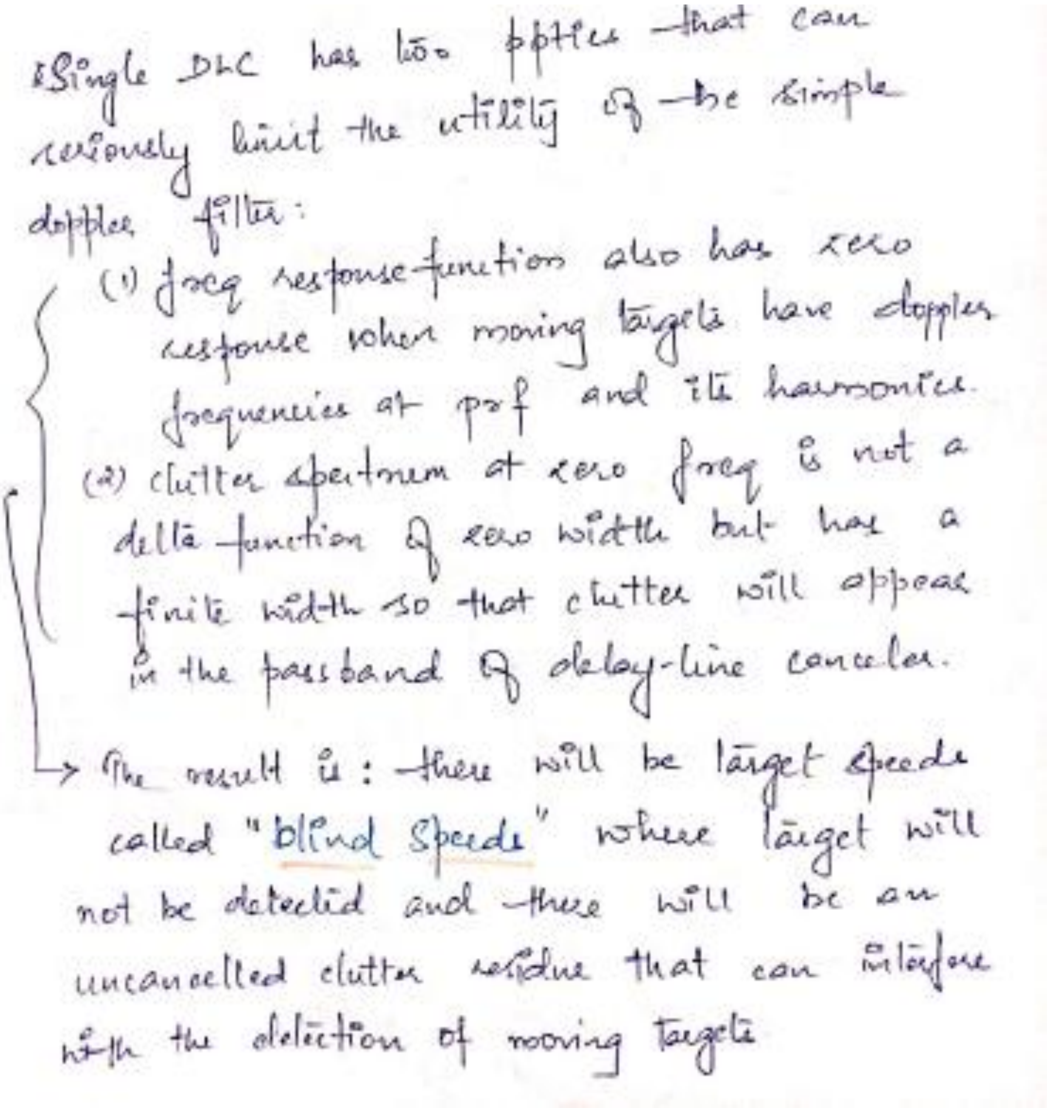
(ii) $v_r = 2.5\ cm/s = 2.5 \times 10^{-2}\ m/s$.

$f_d = \frac{2 v_r f_0}{c} = \frac{2 \times 2.5 \times 10^{-2} \times 10 \times 10^9}{3 \times 10^8} = 1.67\ Hz$ 2.5 M

As velocity of target ↑s, fd ↑s. 1M

(b) What is blind speed? How can we eradicate it? [04] CO2 L1

Soln:



Single DLC has two ppties that can seriously limit the utility of the simple doppler filter:

(1) freq response function also has zero response when moving targets have doppler frequencies at prf and its harmonics.

(2) clutter spectrum at zero freq is not a delta function of zero width but has a finite width so that clutter will appear in the passband of delay-line canceler.

→ The result is: there will be target speeds called "blind speeds" where target will not be detected and there will be an uncancelled clutter residue that can interfere with the detection of moving targets 2 M

Blind speeds can be a serious limitation in MTI radar since they cause some desired moving targets to

be cancelled along with the undesired clutter at zero frequency.

Based on eqn (4), there are four methods for reducing the detrimental effects of blind speeds.

ⓐ Operate radar at long wavelengths (low freqs)

ⓑ operate with high pulse repetition freq. (prf)

ⓒ operate with more than one prf.

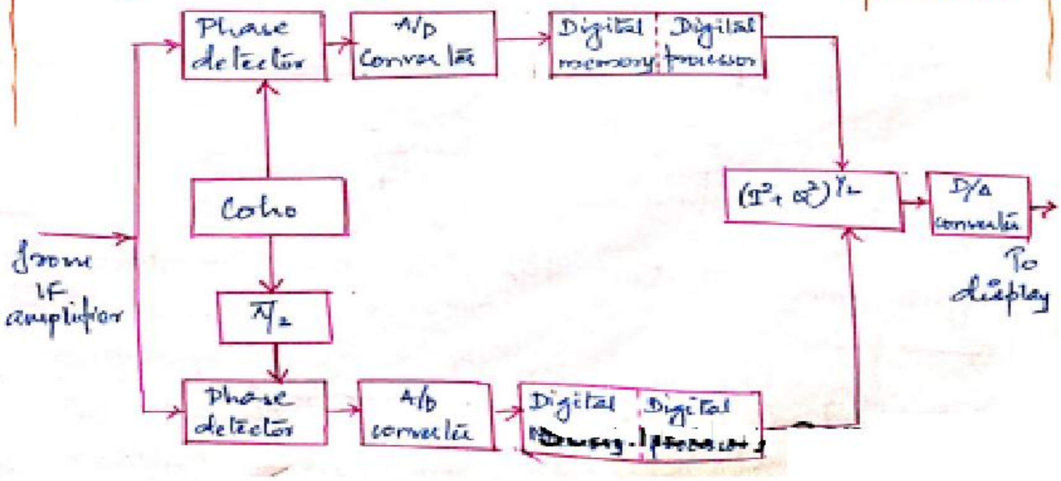
ⓓ operate with more than one RF freq. (wavelength)

2 M

7. Explain the operation of Digital MTI Doppler Signal Processor with neat block diagram. [10] CO2 L2

Soln:

fig (c) Block diagram of digital MTI doppler signal processor



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** Digital MTI processing

Sophisticated MTI doppler filters were difficult to implement with analog methods. Rapid development of digital technology allowed the delays to be obtained by storing digital words in memory for whatever length of time was required. Advantages of digital MTI processing are:

- * Compensation for 'blind phases' which cause a loss due to difference in phase btwn echo signal and MTI reference signal → this is achieved by use of I & Q processing.
- * Greater dynamic range
- * Digital MTI is more stable & reliable
- * More flexible, reprogrammable.

The block dgm of MTI radar shown in fig(3) had a single phase detector & filter channel. Hence, there is a loss when doppler-shifted signal is not sampled at peak positive & -ve values of sinewave.

⇒ When the phase btwn doppler signal & the sampling at prf results in a loss, it is called a blind phase. 6 M

(Blind speed occurs when sampling pulse appears at the same point in the doppler cycle at each sampling time)

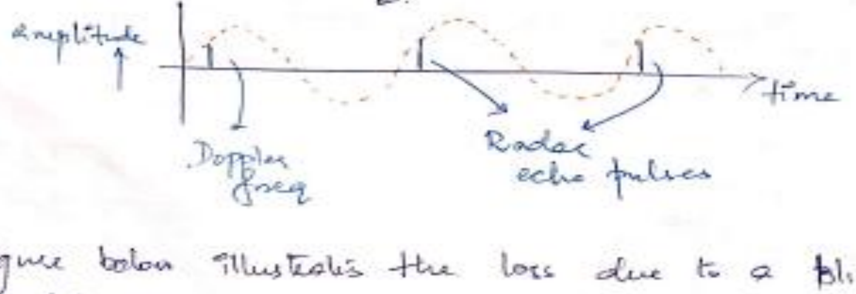
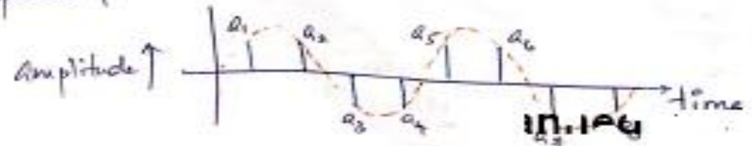


figure below illustrates the loss due to a blind ~~speed~~ phase.



The sampled signals are of same amplitude & with a spacing such that when pulse a_2 ~~&~~ is subtracted from a_1, result is zero. But a_3 is subtracted from a_2 → there is a finite o/p.

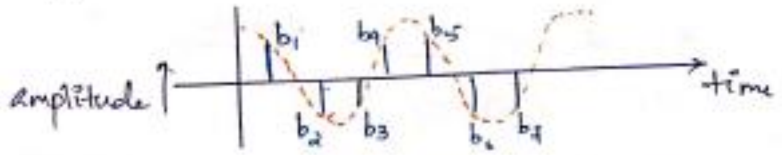
∴ half of signal energy is lost.

other half can be recovered if a second identical processing channel is used & there is a 90° phase change of (coho) reference signal which is applied to phase detector.

. This second channel is called Q (quadrature) channel. Original channel is called I (in-phase) channel.

. If coho signal in I-channel is $\sin(2\pi f_{if} t)$, the coho in Q-channel is $\cos(2\pi f_{if} t)$

. Result of 90° phase change in Q-channel is shown below:



The pulse pairs which had zero o/p in the I-channel now have a finite residue in Q-channel.

And, those pulse pairs which had a finite residue in I-channel now have zero o/p in Q-channel. ∴ what was lost in I-channel is recovered in Q-channel & vice versa.

Combination of I & Q channels results in a uniform o/p with no loss.

fig(6) shows the block diagram of digital MTI signal processor with I and Q channels. Signal from IF amplifier is split into two channels. The phase detectors in each channel extract the doppler shifted signal. In the I channel, the doppler signal is represented as $A_d \cos(2\pi f_d t + \phi_0)$ & in Q channel it is the same except that sine replaces cosine.

- The signals are then digitized by the analog-to digital A/D converter. A sample & hold circuit is often on the same chip as the A/D converter. It is usually needed ahead of A/D converter for more effective digitizing.
- The digital words are stored in a digital memory for the required delay time(s) & are processed with a suitable algorithm to provide the desired doppler filtering..
- The magnitude of doppler signal is obtained by taking the $\sqrt{I^2 + Q^2}$. Sometimes, for simplicity, sum of magnitudes of 2 channels $|I| + |Q|$ is taken or "greater of" two channels might be used instead.
- The I & Q processor of fig(6) has a square-law detector characteristic.

8.	Explain (i) Clutter attenuation (ii) MTI Improvement factor (iii) Doppler shift	[10]	CO2	L1

Soln:

(i) Clutter attenuation

→ One of the solution to reduce blind speeds is is used multiple prfs.

. When two or more prfs are used in a radar, the blind speeds at one prf generally are different from the blind speeds at the other prfs. Thus targets that are highly attenuated with one prf might be readily seen with another prf.

4 M

↳ This technique is widely used with air-surveillance radars.

↓

Disadv : multiple time-around echoes are not cancelled.

→ A radar that can operate at two or more RF freqs can also unmask blind speeds → but the reqd freq change is often bigger than might be possible within usual freq. bands allocated for radar use.

↳ limitⁿ : need for greater s/m B.W.

→ Another limitation of single DLC is insufficient attenuation of clutter that results from finite clutter spectrum. In real world, width of clutter spectrum has a finite width due to things like internal motions of the clutter; instabilities of stalo & coho oscillators, other imperfections of the radar & its signal processor, & the finite duration.

For present purposes, we assume that the clutter power spectral density is represented by a gaussian function & is written as:

$$W(f) = W_0 \exp\left(-\frac{f^2}{2\sigma_c^2}\right) = W_0 \exp\left(-\frac{f^2\lambda^2}{8\sigma_v^2}\right) \quad f \geq 0 \qquad ①$$

where, W_0 = peak value of clutter power spectral density @ $f = 0$

σ_c = std deviation of clutter spectrum (in hertz)

σ_v = std deviation of clutter spectrum (in km/s) $\quad \sigma_c = \frac{2\sigma_v}{\lambda}$

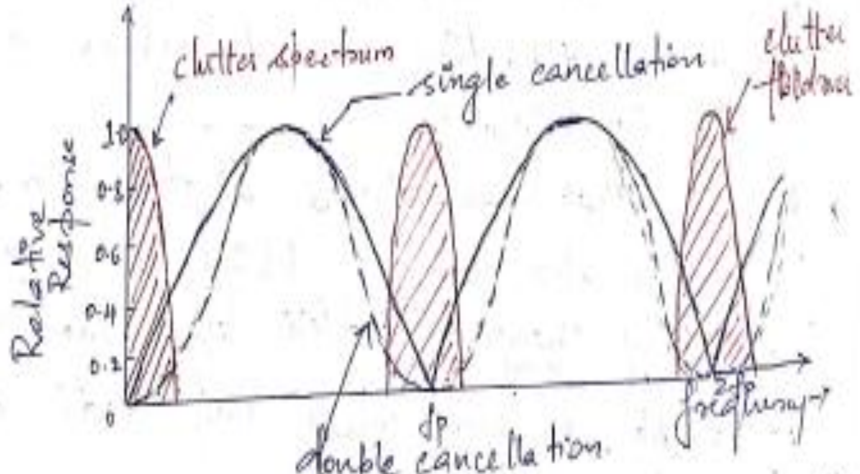


fig: Relative response of single DLC and the double DLC along the freq. ~~response~~ spectrum of the clutter

- As shown above, the freq response of single DLC (solid curve) encompasses a portion of the clutter spectrum → ∴ clutter will appear in the o/p.
- Greater the σ_c, the greater the amount of clutter that will be passed by the filter

∴ interfere with moving target detection.

- The clutter attenuation (CA) produced by a single DLC is :

$$\boxed{CA = \frac{\int_0^\infty W(f)\,df}{\int_0^\infty W(f)\,|H(f)|^2\,df}} \quad \text{②}$$

where $H(f)$ = freq response of DLC
$= 2\sin(\pi f_d T_p)$

final exp^n: $$\boxed{CA = \frac{f_p^2}{4\pi^2\sigma_c^2} = \frac{f_p^2\lambda^2}{16\pi^2\sigma_v^2}} \quad (3)$$

(ii) **MTI Improvement factor**

Clutter attenuation: useful measure of performance of an MTI radar in canceling clutter

To avoid the problem of using clutter attenuation at the expense of desired signals. IEEE defined a measure of performance → MTI Improvement factor.

Defined as: Signal to clutter ratio at the o/p of clutter filter divided by the signal-to-clutter ratio at the input of the clutter filter, averaged uniformly over all target velocities of interest.

3 M

It is expressed as:

$$\text{improvement factor} = I_f = \frac{(\text{signal}/\text{clutter})_{out}}{(\text{signal}/\text{clutter})_{in}}$$

$$= \frac{C_{in}}{C_{out}} \times \left.\frac{S_{out}}{S_{in}}\right|_{f_d}$$

$$\rightarrow \boxed{I_f = CA \times \text{avg. gain}} \qquad \text{—— (1)}$$

(iii) **Doppler Effect:**

Doppler effect is the change in freq of a wave for an observer moving relative to its source. When target is in motion relative to radar, then an apparent shift in freq will result. This is called doppler effect & is the basis of continuous wave (CW) RADAR.

3 M

- The received freq is higher during the approach & is lower during recession.
- Variation of freq also depends on the direction the wave source is moving w.r.t the observer, it is maximum when the source is moving directly towards or away from the observer & diminishes with increasing angle between the direction of motion & direction of waves.

If R is the distance from RADAR to the moving target at any instant of time,

then,

$$\text{total no. of wavelengths contained in two-way path} = \frac{2R}{\lambda} \quad \text{——— (1)}$$

where, λ = wavelength of total wave.

One wavelength corresponds to a phase shift of 2π radians

$$\therefore \text{total phase shift, } \phi_d = \left(\frac{2R}{\lambda}\right)(2\pi) = \frac{4\pi R}{\lambda}. \quad \text{——— (2)}$$

When target is in motion, R and ϕ_d are continuously changing.

$$\text{But } \omega_d = \frac{d}{dt}\phi_d$$

$$\therefore \omega_d = \frac{d}{dt}\phi_d = \frac{d}{dt}\left[\frac{4\pi R}{\lambda}\right] = \frac{4\pi}{\lambda}\frac{dR}{dt} = 2\pi f_d.$$

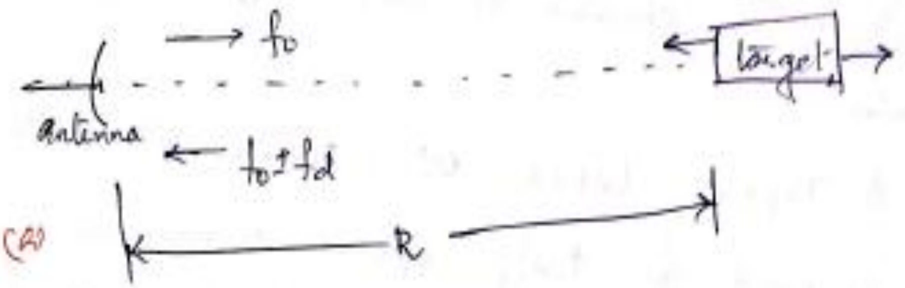
Now $\frac{dR}{dt} = v_r$ = velocity of target.

$\omega_d = 2\pi f_d = \frac{4\pi}{\lambda} v_r$

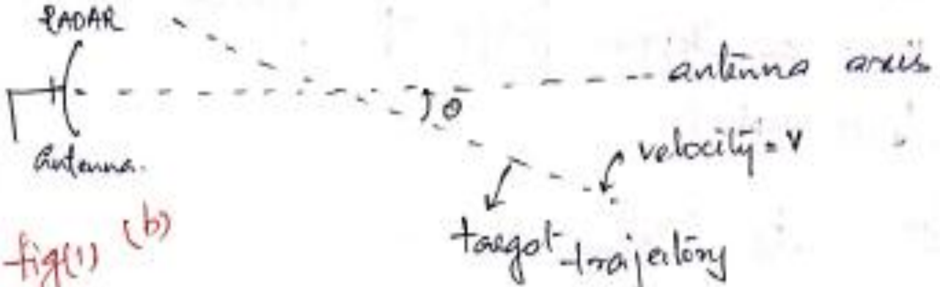
$\therefore$ doppler freq shift, $f_d = \frac{2v_r}{\lambda}$ ——— (4)

But $\lambda = \frac{c}{f_0}$; f_0 = txtd freq

$$\therefore \boxed{f_d = \frac{2 v_r f_0}{c}} \text{ ——— } (5)$$



(A)



fig(1) (b)

Eqns (4) & (5) are applicable when the target moves along the antenna axis.

If the trajectory of target makes an angle 'θ' with the antenna axis, then.

velocity component along the axis, $v_r = v\cos\theta$.

$$(5) \Rightarrow \therefore \boxed{f_d = \frac{2 f_0 v\cos\theta}{c}} \text{ ——— } (6)$$

- The direction of motion of target can be determined from the sign of fd.
- When target is approaching,
  received freq → $f_0 + f_d$
- When target is receding,
  received freq → $f_0 - f_d$.