(1) Distinguish between:

i. Periodic and Non-periodic signals

A continuous time signal $x(t)$ is said to be periodic if it satisfies $x(t) = x(t+T), \forall t$ —— (1)

The smallest value of T for which eq^n (1) gets satisfied is called periodicity or fundamental time period. (T_0).

Ex: 1. $x(t) = \sin t \Rightarrow \cancel{x t = \frac{2\pi}{\pi}}$

2. $x(t+\pi) = \sin(t+\pi)$.

Non periodic: A continuous time signal $x(t)$ is said to be not periodic if it doesnot satisfies $x(t) = x(t+T), \forall t$.

Ex: $\sin 2t + \cos \pi t$

ii. Even and Odd signals:

Even: A signal is said to be even if and only if it satisfies $x(t) = x(-t)$

Ex: $x(t) = \cos(\pi t)$.

Odd: A signal is said to be odd if it does not satisfies $x(t) \cancel{=} x(-t)$. [i.e $x(t) \neq x(-t)$]

Ex: $x(t) = \sin(\pi t)$.

iii Energy and power signals:

Energy signals: A signal is said to be energy signal if energy is finite and average power is zero.

$0 < E < \infty$

$P = 0$

A signal is said to be power signal if power is finite and energy is infinite.

$0 < P < \infty$

$E = \infty$.

iv. Causal and Non-causal signals.

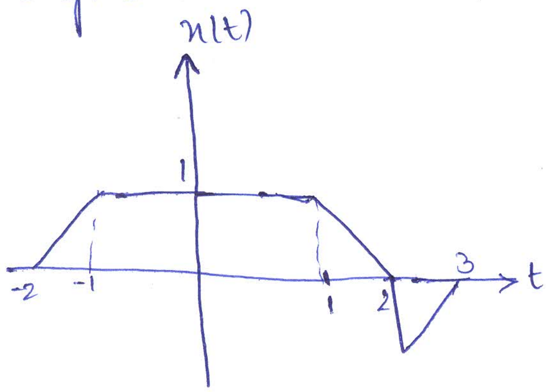
A signal which posses zero amplitude for all negative value of time, then the signal is known as a causal signal

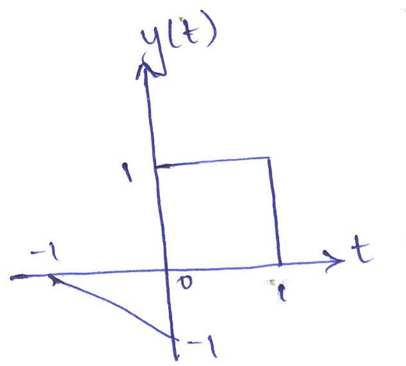
$x(t) > 0$, for $t \geq 0$ & $x(t) = 0$ for $t < 0$.

~~(A signal which posses zero value for all positive value of time, but has amplitude which is greater than zero for all negative value of time, then the signal is known as)~~

A signal that has positive value of amplitude for both positive and negative instances of time is a non causal signal.

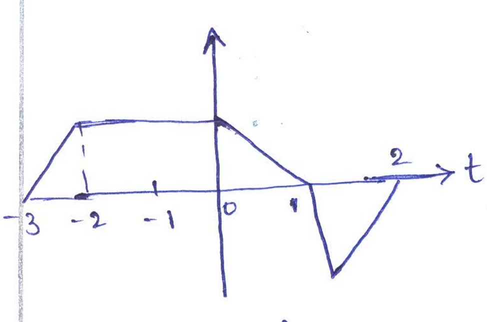
3.(a) For a signal $x(t)$ and $y(t)$ as shown sketch the following signals

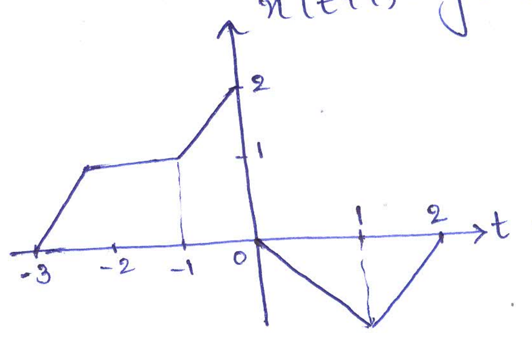




i. $x(t+1) - y(t)$.

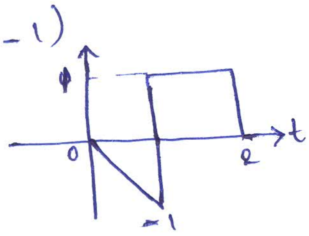
$x(t+1)$:

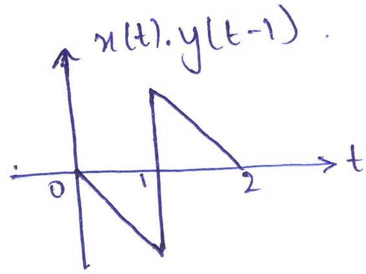




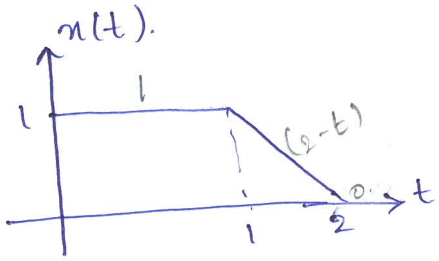
ii. $x(t) . y(t-1)$.

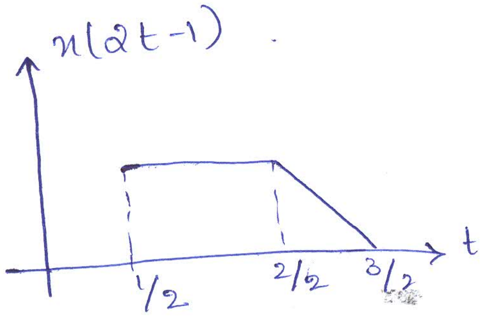
$y(t-1)$





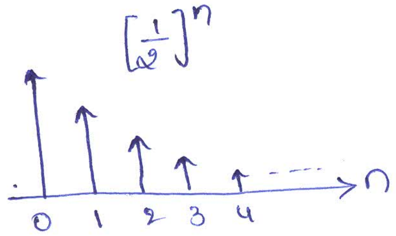
(b) Given $x(t) = \begin{cases} 1 & , 0 \le t \le 1 \\ 2-t & , 1 \le t \le 2 \\ 0 & , \text{otherwise} \end{cases}$ Sketch $y(t) = x(2t-1)$.

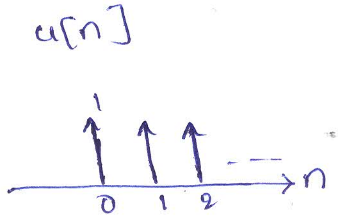




(4) Determine the following signals are energy signals or power signals.

i. $x[n] = 2^{-n} u[n] = \left[\frac{1}{2}\right]^n u[n]$





$E = \sum_{n=0}^{\infty} |x[n]^2| = \sum_{n=0}^{\infty} \left|\left(\frac{1}{2}\right)^n\right|^2$

$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}}$ $\left[\because \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}\right]$

$= \frac{4}{3}$ J

Signal is energy signal.

ii. $x(t) = e^{j(2t)}$.

$$E = \int_{-\infty}^{\infty} |\cancel{e^{j2t}}|^2 \, dt$$

$$= \int_{-\infty}^{\infty} 1 \, dt = t\Big]_{-\infty}^{\infty} = \infty$$

$P = e^{j2t}$

$P = 1$

It is a power signal

iii. $x(t) = e^{-t} u(t)$.

$$E = \int_{0}^{\infty} |e^{-t} \cdot u(t)|^2 dt = \int_{0}^{\infty} e^{-2t} \, dt$$

$$= \frac{e^{-2t}}{-2}\Big]_{0}^{\infty} = \frac{1}{-2}[0 - 1]$$

$$= \frac{1}{2} J$$

$P = \infty$

It is energy signal

iv. $x[n] = \left[\frac{1}{4}\right]^n u[n]$.

$$E = \sum_{n=0}^{\infty} \left| \left[\frac{1}{4}\right]^n \right|^2$$

$$= \sum_{n=0}^{\infty} \left[\frac{1}{16}\right]^n$$

$$= \frac{1}{1 - \frac{1}{16}} = \frac{1}{\frac{15}{16}} = \frac{16}{15} J$$

$P = 0$

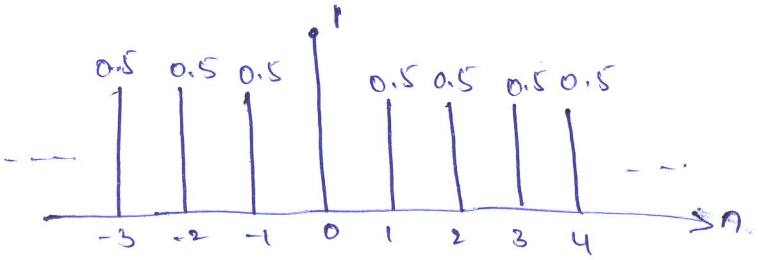
It is energy signal.

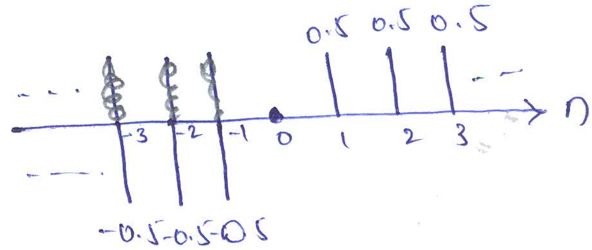
7. (a) Sketch the even and odd part of unit step signal (5)

$$u_e[n] = \frac{1}{2}\left[u[n] + u[-n]\right]$$

$$u_o[n] = \frac{1}{2}\, \mathrm{signam}[\delta(n)]$$

$$u_o[n] = \frac{1}{2}\left[u[n] - u[-n]\right]$$





$$u_e[n] = \begin{cases} 1 \; ; & n = 0 \\ 0.5 \; ; & n \neq 0 \end{cases}$$

$$u_o[n] = \begin{cases} 0 \; ; & n = 0 \\ 0.5 \; ; & n > 0 \\ -0.5 \; ; & n < 0 \end{cases}$$

$$u_e[n] = \frac{1}{2} + \frac{1}{2}\delta(n)$$

$$u_o[n] = \frac{1}{2}\, \mathrm{signam}[\delta(n)]$$

b. Define unit impulse function for continuous time signal. Write the properties of impulse function.

Impulse function is defined as

~~$f(x) =$~~

$$\int_{-\infty}^{\infty} \delta(t)\, dt = \begin{cases} 1, & t > 0 \\ 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$

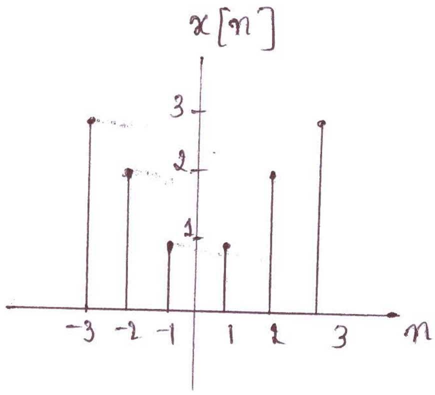
The impulse function is actually the area of a rectangular pulse at $x = 0$, whose width is considered to be tending to zero.

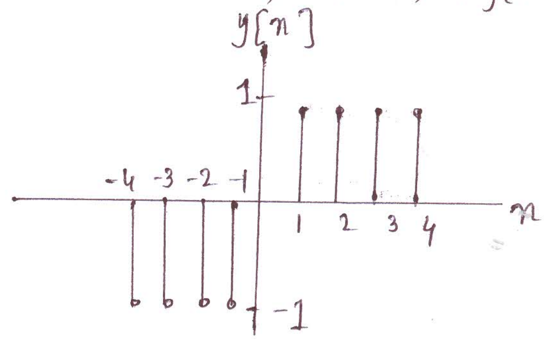
Properties:

1. $\delta(-t) = \delta(t)$
2. $\delta(at) = \frac{1}{|a|}\delta(t)$
3. $\int_{-\infty}^{\infty} \delta(t)\, dt = 1, \; t = 0$
4. $\int_{-\infty}^{\infty} \delta(t - t_0)\, dt = 1, \; t = t_0$
5. $x(t)\delta(t) = x(0)\delta(t)$.
6. $x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$
7. $\int_{-\infty}^{\infty} x(t)\delta(t)\, dt = x(0), \; t = 0$
8. $\int_{-\infty}^{\infty} x(t) \cdot \delta(t - t_0)\, dt = x(t_0), \; t = t_0$.

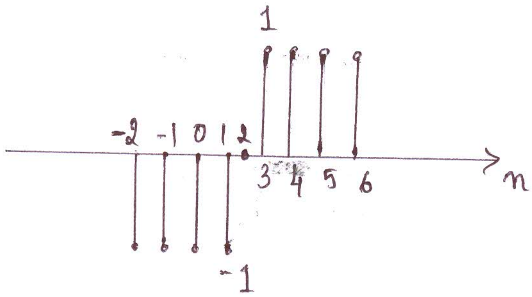
2. Let $x(n)$ and $y(n)$ be as given in the Fig. Sketch the following signals. [6]

i) $y(n-2)$ ii) $x(-n)y(-n)$ iii) $y(2-2n)$ iv) $x(n-2)+y(n+2)$

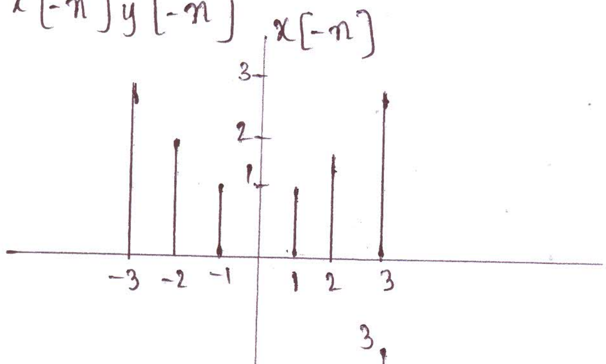


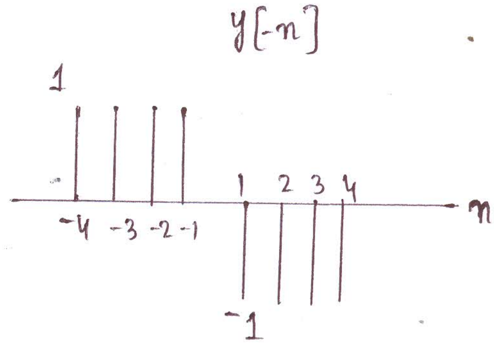


Sol. i) $y(n-2)$

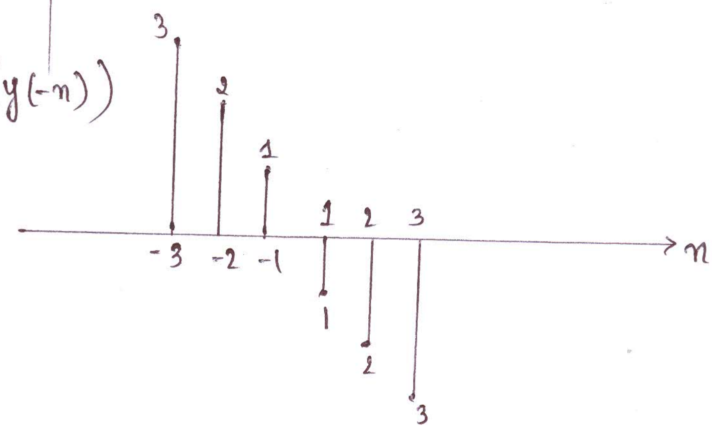


ii) $x[-n]y[-n]$





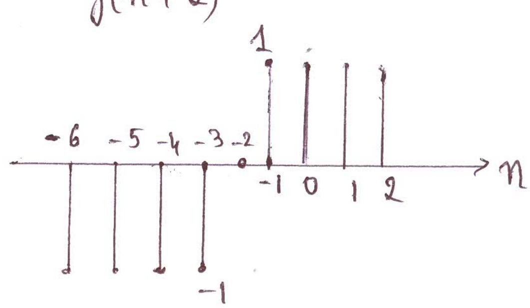
$x[-n](y(-n))$



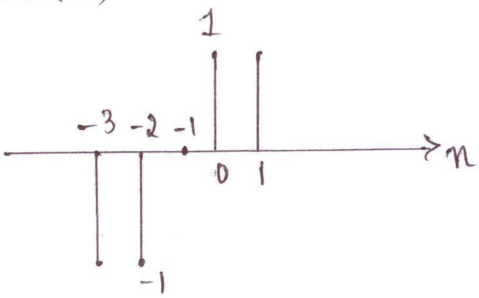
iii) $y(2-2n)$

$= y(-2n+2)$

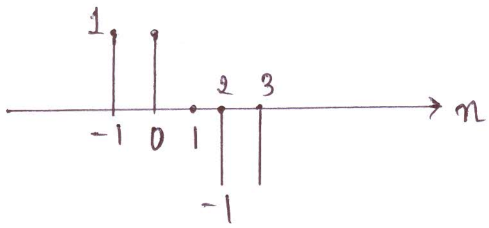
$y(n+2)$



$y(2n+2)$

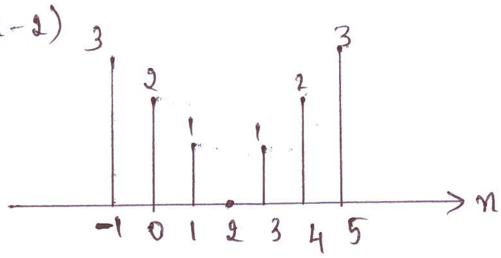


$y(-2n+2)$

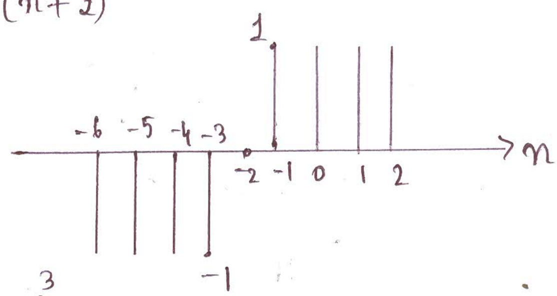


iv) $x(n-2) + y(n+2)$

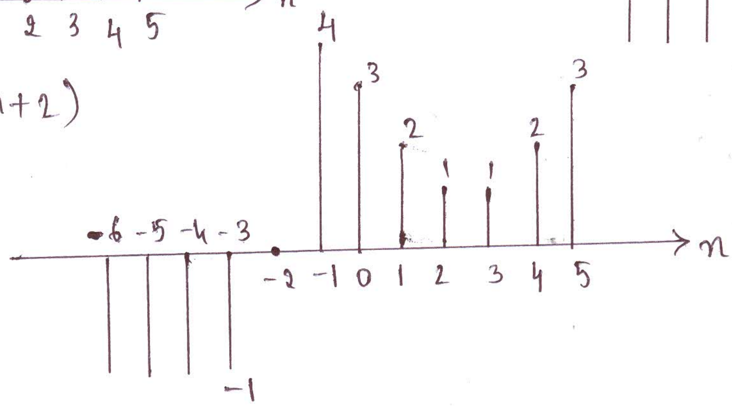
$x(n-2)$



$y(n+2)$



$x(n-2) + y(n+2)$



5. Determine whether or not each of the following signals is periodi[c]. If the signal is periodic, determine its period.

i) $x(t) = \sin(2\pi t) + \sin(3\pi t)$

$$T_1 = \frac{2\pi}{\omega_1} \qquad T_2 = \frac{2\pi}{\omega_2}$$

here, $\omega_1 = 2\pi$, $\omega_2 = 3\pi$

$$\Rightarrow T_1 = \frac{2\pi}{2\pi} = 1 \text{ sec} \qquad T_2 = \frac{2\pi}{3\pi} = \frac{2}{3} \text{ sec}$$

$$\frac{T_1}{T_2} = \frac{1}{(2/3)} = \frac{3}{2}$$

$$T = 2T_1 = 3T_2 = 2 \text{ sec}$$

$\therefore$ $x(t)$ is a periodic signal with periodicity, $T = 2$ sec.

ii) $x(t) = e^{j10t}$

$$\omega = 10, \quad T = \frac{2\pi}{10} = \frac{\pi}{5} \text{ sec}$$

$x(t)$ is periodic, $T = \frac{\pi}{5}$ sec

iii) $$x[n] = \cos\left(\frac{\pi n}{2}\right)\cos\left(\frac{\pi n}{4}\right)$$

$$x[n] = \frac{1}{2}\left[\cos\left(\frac{3\pi n}{4}\right) + \cos\left(\frac{\pi n}{4}\right)\right]$$

$$N_1 = \frac{2\pi}{\Omega_1}, \quad \Omega_1 = \frac{3\pi}{4}$$

$$\Rightarrow N_1 = \frac{2\pi}{(3\pi/4)} = \frac{2\pi}{3\pi} \times 4 = \frac{8}{3} \times m$$

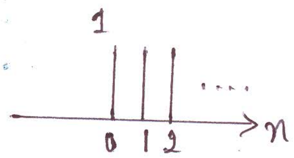
$m = 3$, $N_1 = 8$ samples

$$N_2 = \frac{2\pi}{\Omega_2} = \frac{2\pi}{(\pi/4)} = \frac{2\pi}{\pi} \times 4 = 8 \text{ samples}$$

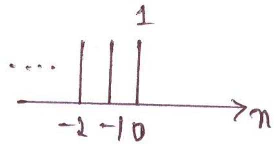
$$\frac{N_1}{N_2} = \frac{8}{8} = 1 \Rightarrow N = N_1 = N_2 = 8 \text{ samples.}$$

iv) $x[n] = u[n] + u[-n]$

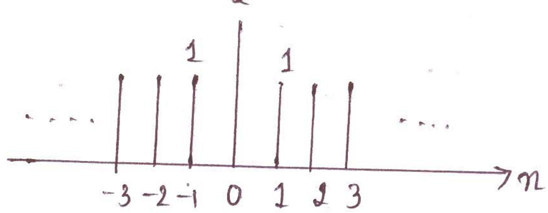
$u[n]$



$u[-n]$

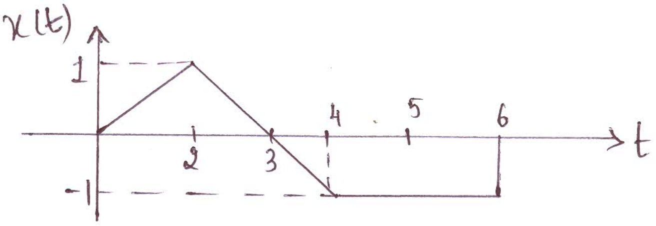


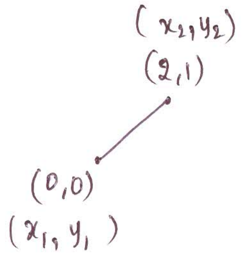
$u[n] + u[-n]$



$x[n]$ is not periodic signal.

6. Determine energy and power of the signal $x(t)$ shown in the fig.

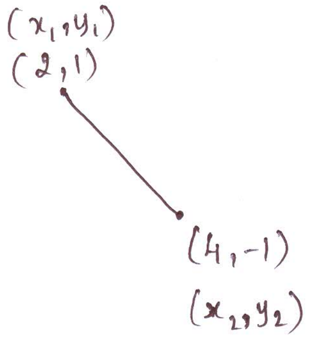




$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{1-0}{2-0}[x - 0]$$

$$y = \frac{1}{2}(x) \Rightarrow x(t) = \frac{t}{2}$$



$$y - 1 = \frac{-1-1}{4-2}[x - 2]$$

$$y - 1 = \frac{-2}{2}(x-2)$$

$$y - 1 = -x + 2$$

$$y = -x + 2 + 1 \Rightarrow x(t) = -t + 3$$

$$y - (-1) = \frac{-1-(-1)}{6-4}(x - 4)$$

$$y + 1 = 0 \Rightarrow y = -1$$

$$x(t) = -1$$

$$\therefore x(t) = \begin{cases} t/2, & 0 \le t \le 2 \\ -t+3, & 2 \le t \le 4 \\ -1, & 4 \le t \le 6 \end{cases}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E = \int_0^2 \left(\frac{t}{2}\right)^2 dt + \int_2^4 (-t+3)^2 \, dt + \int_4^6 -1 \, dt$$

$$= \frac{1}{4}\int_0^2 t^2 \, dt + \int_2^4 (9 + t^2 - 6t) \, dt - \int_4^6 dt$$

$$= \frac{1}{4}\left(\frac{t^3}{3}\right)_0^2 + 9(t)_2^4 + \left(\frac{t^3}{3}\right)_2^4 - 6\left(\frac{t^2}{2}\right)_2^4 - (t)_4^6$$

$$= \frac{1}{12}(8) + 9(2) + \frac{1}{3}(56) - 3(12) - 2 = -2/3 \text{ J}$$