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I INTERNAL ASSESSMENT TEST

Sub:	DIGITAL COMMUNICATION							Code:	15EC61
Date:	05/ 03 / 2019	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	TCE

Answer any 5 full questions

		Marks	CO	RBT
1	Define Hilbert transform. Plot the magnitude response and phase response of the ideal Hilbert transformer. Derive the impulse response of the ideal Hilbert transformer.	[10]	CO1	L3
2(a)	State and prove the properties of Hilbert transform.	[06]	CO1	L2
2(b)	Determine the Hilbert transform of $m(t)\sin(2\pi f_c t)$ assuming that $m(t)$ is a low pass signal bandlimited to W Hz and $f_c \gg W$.	[04]	CO1	L2
3	Discuss pre-envelope and complex envelope of bandpass signals with relevant equations. Plot the spectra of a bandpass signal, its pre-envelope and complex envelope.	[10]	CO1	L2
4(a)	Determine the Hilbert transform of the signal $x(t)$ given by $$x(t) = \begin{cases} 1 & for\ -\frac{T}{2} \leq t \leq \frac{T}{2} \\ 0 & otherwise \end{cases}$$	[04]	CO1	L2
4(b)	Determine the pre-envelope and complex envelope of the signal $x(t) = m(t)\cos(2\pi f_c t)$ assuming that $m(t)$ is a low pass signal bandlimited to W Hz and $f_c \gg W$.	[06]	CO1	L2
5	Derive an expression for the canonical representation of bandpass signals. Obtain a scheme for extracting in-phase and quadrature components of bandpass signals. Draw the corresponding block diagram.	[10]	CO1	L2
6	Derive an expression for the power spectral density of NRZ unipolar signal assuming equiprobable and statistically independent 0s and 1s. Plot the resulting power spectral density.	[10]	CO1	L3

CMR INSTITUTE OF TECHNOLOGY

DEPT OF ECE/TCE

DIGITAL COMMUNICATION

IAT-1

5/3/2019

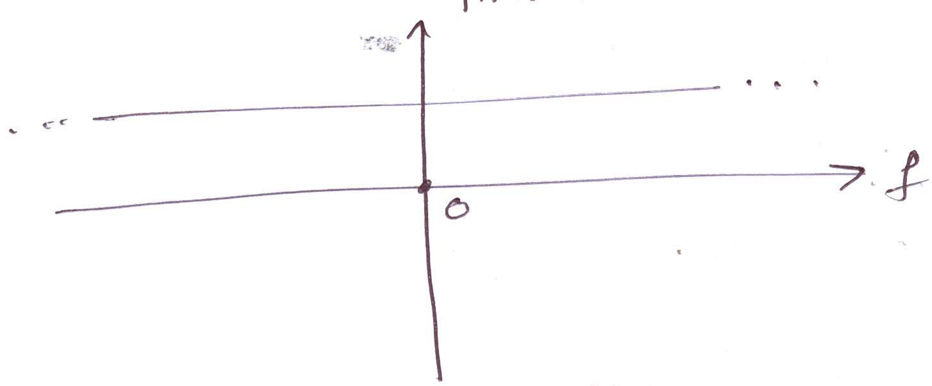
Solution and Scheme of Evaluation

1 Hilbert transform

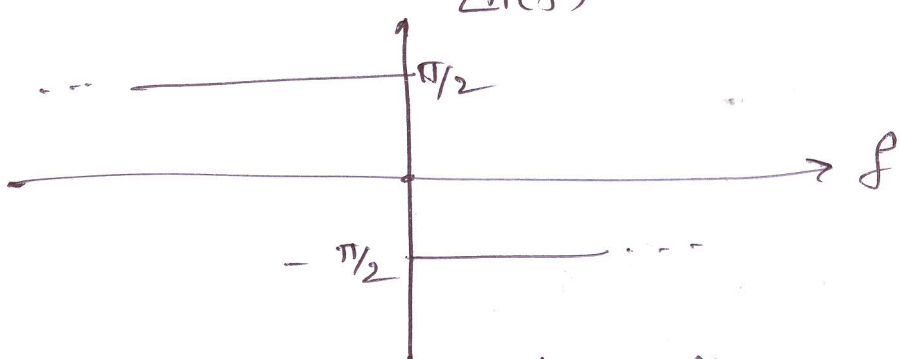
90° phase shift for $f < 0$

-90° phase shift for $f > 0$ (2Marks)

$|H(f)|$



$\angle H(f)$



(2Marks)

$$h(t) = \int_{-\infty}^{0} j\left(\lim_{a \to 0} e^{+af}\right) e^{j2\pi ft}\, df + \int_{0}^{\infty} (-j)\left(\lim_{a \to 0} e^{-af}\right) e^{j2\pi ft}\, df$$

$$= j \int_{-\infty}^{0} \lim_{a \to 0} e^{(j2\pi t+a)f}\, df - j \int_{0}^{\infty} \lim_{a \to 0} e^{(j2\pi t-a)f}\, df$$

$$= \frac{j}{(j2\pi t+a)} \lim_{a \to 0} \left[e^{(j2\pi t+a)f} \right]_{-\infty}^{0} - j \lim_{a \to 0} \frac{1}{j2\pi t-a} \left[e^{j2\pi t-a} \right]_{0}^{\infty}$$

$$= \frac{1}{2\pi t} + \frac{1}{2\pi t}$$

$$= \frac{1}{\pi t}$$

(6 Marks)

2a HT of HT of $x(t)$ is $-x(t)$

Proof: $\hat{X}(f) = X(f)\,[-j\,\text{sgn}(f)]$

$$\hat{\hat{X}}(f) = -X(f)$$

$$\hat{\hat{x}}(t) = -x(t)$$

(2Marks)

$x(t)$ and $\hat{x}(t)$ have same magnitude spectrum.

Proof: $\hat{X}(f) = X(f)\,[-j\,\text{sgn}(f)]$

$$|\hat{X}(f)| = |X(f)|\,|-j\,\text{sgn}(f)|$$

$$= |X(f)|$$

(2Marks)

$x(t)$ and $\hat{x}(t)$ are orthogonal over $(-\infty, \infty)$

Proof: $\int_{-\infty}^{\infty} X(f)\,\hat{X}^{*}(f)\,df$

$$= \int_{-\infty}^{\infty} X(f)\,X^{*}(f)\,j\,\text{sgn}(f)\,df$$

$$= j\int_{-\infty}^{\infty} |X(f)|^{2}\,\text{sgn}(f)\,df$$

(2Marks)

$$= 0.$$

2a

$$x(t) = m(t)\sin(2\pi f_c t)$$

$$X(f) = \frac{1}{2j} M(f-f_c) - \frac{1}{2j} M(f+f_c)$$

$$\hat{X}(f) = \frac{1}{2j}(-j) M(f-f_c) - \frac{1}{2j} j M(f+f_c)$$ (2 Marks)

$$\hat{x}(t) = -\frac{m(t)e^{j2\pi f_c t}}{2} - \frac{m(t)e^{-j2\pi f_c t}}{2}$$

$$= -\frac{m(t)}{2} \cdot 2\cos(2\pi f_c t)$$

$$= -m(t)\, \cancel{\sin}\, \cos(2\pi f_c t)$$ (2 Marks)

3

$$x_+(t) = x(t) + j\hat{x}(t)$$ (2 Marks)

$$X_+(f) = X(f) + j\left[-j\,\mathrm{sgn}(f)\, X(f)\right]$$

$$= \begin{cases} 2X(f) & \text{for } f>0 \\ 0 & \text{for } f<0 \\ X(f) & @\, f=0 \end{cases}$$ (2 Marks)

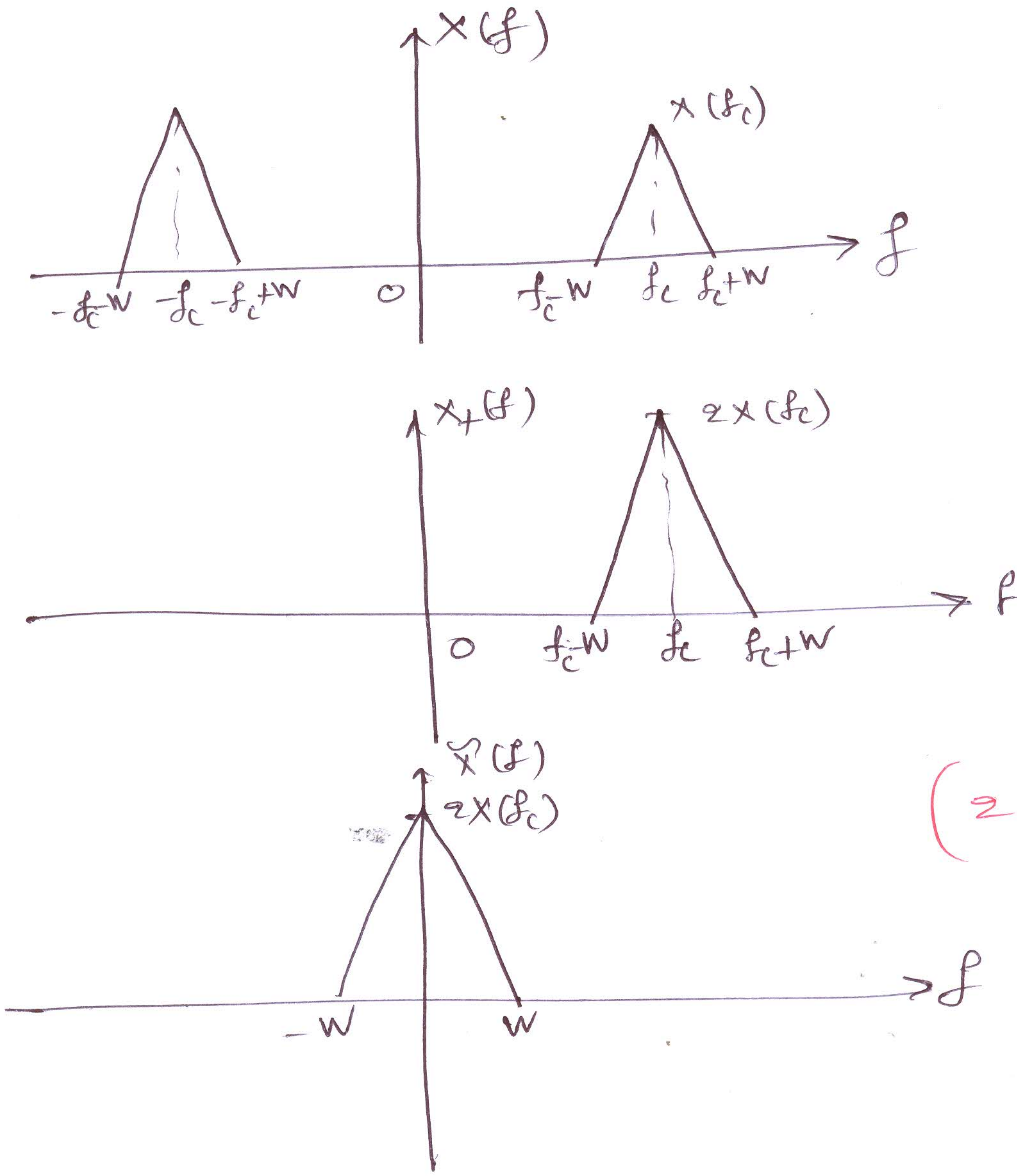
$$x_-(t) = x(t) - j\hat{x}(t)$$ (2 Marks)

$$X_-(f) = X(f) - j\hat{X}(f)$$

$$= \begin{cases} 0 & \text{for } f>0 \\ X(f) & @\, f=0 \\ 2X(f) & \text{for } f<0 \end{cases}$$ (2 Marks)

$$\tilde{x}(t) = x_+(t)\, e^{-j2\pi f_c t}$$

$$\tilde{X}(f) = X_+(f+f_c)$$



(2 Marks)

(2 Marks)

4a $\hat{x}(t) = x(t) * h(t)$

$$= \int_{-T/2}^{T/2} \frac{1}{\pi(t-\tau)}\, d\tau$$

$$= -\frac{1}{\pi} \ln(t-\tau) \Big|_{-T/2}^{T/2}$$

$$= \frac{-1}{\pi} \ln\left[\frac{t - T/2}{t + T/2}\right]$$

(2 Marks)

4b $x(t) = m(t)\cos(2\pi f_c t)$

$\hat{x}(t) = m(t)\sin(2\pi f_c t)$ (2 Marks)

$$x_+(t) = x(t) + j\hat{x}(t)$$

$$= m(t)\cos(2\pi f_c t) + j\, m(t)\sin(2\pi f_c t)$$

$$= m(t)\, e^{j2\pi f_c t}$$ (2 Marks)

$$\tilde{x}(t) = x_+(t)\, e^{-j2\pi f_c t}$$ (2 Marks)

$$= m(t)$$

5 $$\tilde{x}(t) = x_I(t) + j\, x_Q(t)$$ (2 Marks)

$$x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}$$

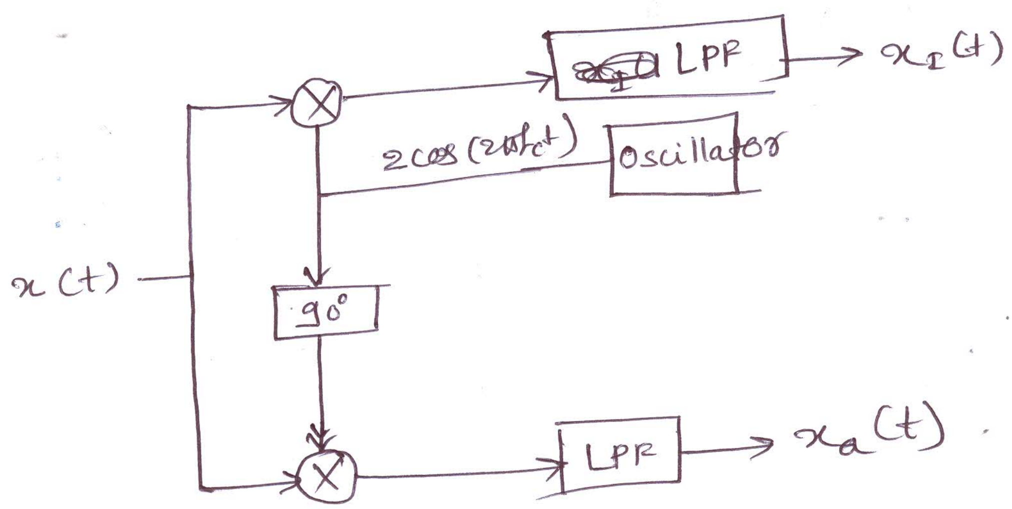
$$x_+(t) = x(t) + j\hat{x}(t)$$ (2 Marks)

$$x(t) = \mathrm{Re}[x_+(t)]$$

$$= x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

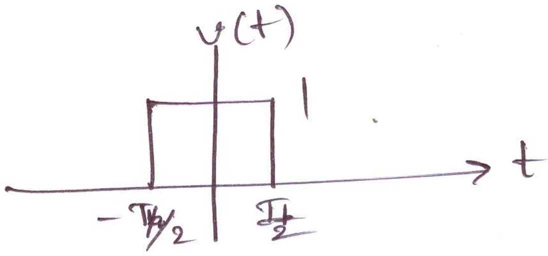
$$x(t)\cos(2\pi f_c t) = x_I(t)\left[\frac{1+\cos(4\pi f_c t)}{2}\right] - \frac{x_Q(t)}{2}\sin(4\pi f_c t)$$ (2 Marks)

$$x(t)\sin(2\pi f_c t) = \frac{x_I(t)}{2}\sin(4\pi f_c t) - \frac{x_Q(t)}{2}\left[1-\cos(4\pi f_c t)\right]$$ (2 Marks)



(2 Marks)

6



(2 Marks)

$$V(f) = T_b \operatorname{sinc}(f T_b)$$

$$R_A(n) = \begin{cases} \frac{a^2}{2} & \text{for } n = 0 \\ \frac{a^2}{4} & \text{for } n \neq 0 \end{cases}$$

(2 Marks)

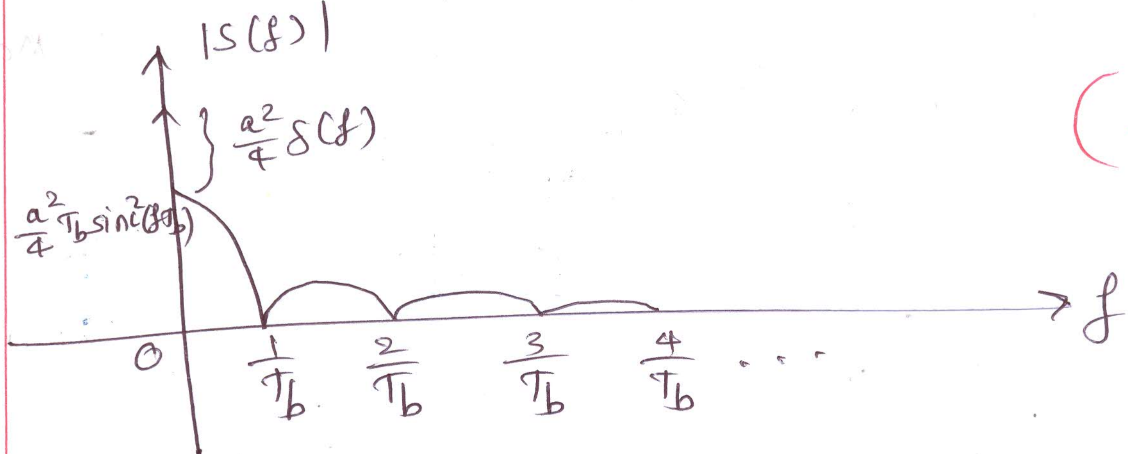
$$S(f) = \frac{1}{T_b} |V(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}$$

$$= \frac{1}{T_b} T_b^2 \operatorname{sinc}^2(f T_b) \left[\frac{a^2}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{a^2}{4} e^{-j2\pi f n T_b} \right]$$

$$= T_b \operatorname{sinc}^2(f T_b) \left[\frac{a^2}{4} + \frac{a^2}{4} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_b}\right) \right]$$

(4 Marks)

$$= \frac{a^2}{4} T_b \operatorname{sinc}^2(f T_b) + \frac{a^2}{4} \delta(f)$$



(2 Marks)