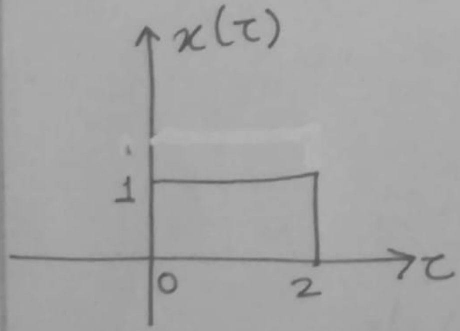
Q1 i)

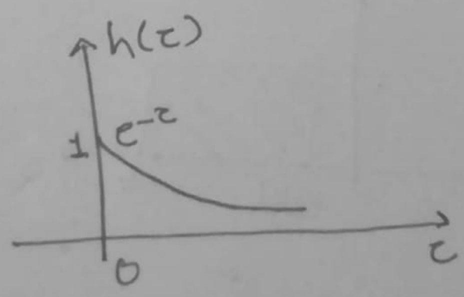
$$x(t) = u(t) - u(t-2)$$

$$h(t) = e^{-t} u(t)$$

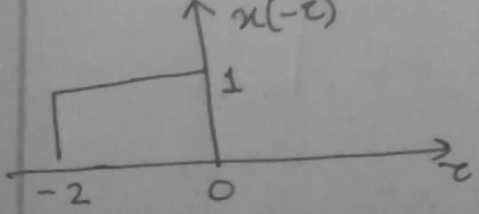
$$y(t) = h(t) * x(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$



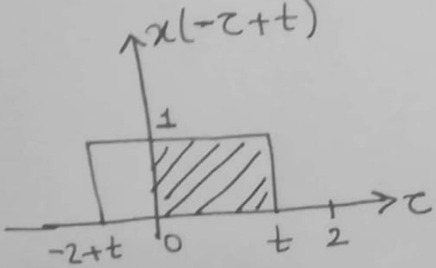


If $t<0$,



$$y(t) = 0$$

If $0 \le t < 2$

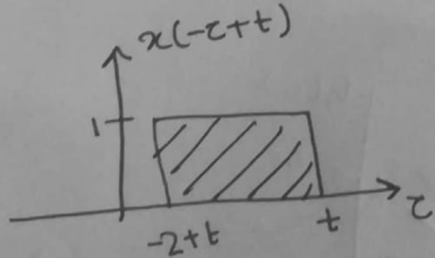


$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{0}^{t} e^{-\tau}\, d\tau = -e^{-\tau}\Big|_{0}^{t}$$

$$= 1 - e^{-t}$$

If $t \geq 2$

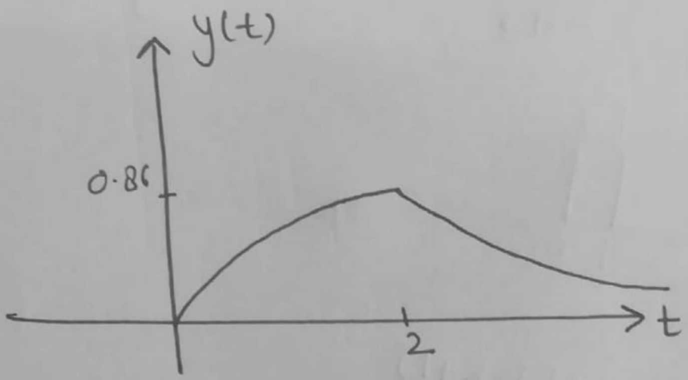


$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-2+t}^{t} e^{-\tau}\, d\tau = -e^{-\tau}\Big|_{-2+t}^{t}$$

$$= e^{2-t} - e^{-t}$$

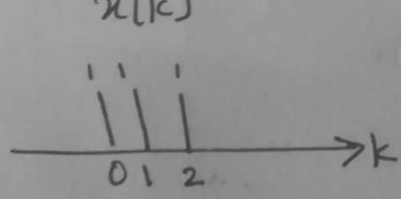
$$\therefore\ y(t) = \begin{cases} 0 & t<0 \\ 1-e^{-t} & 0\le t<2 \\ (e^{2}-1)\,e^{-t} & t\geq 2 \end{cases}$$

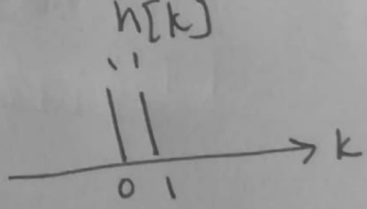


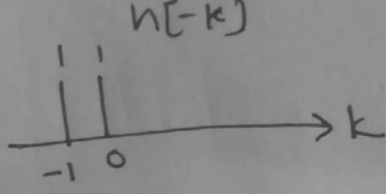
Q1 11) $x[n] = u[n] - u[n-3]$

$h[n] = u[n] - u[n-2]$

$$y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

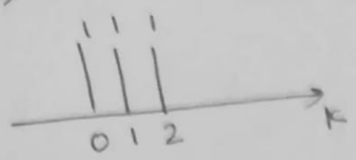




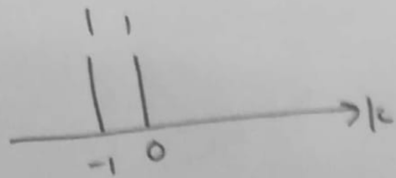


If $n<0$, $y[n]=0$

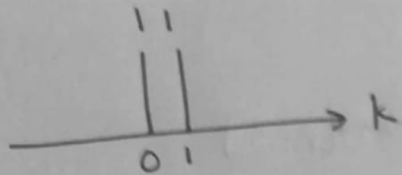
at $n=0$, $x[k]$



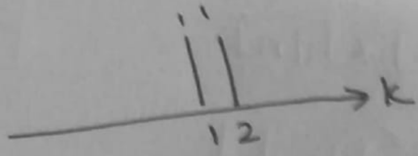
$h[-k]$ $y[0]=1$



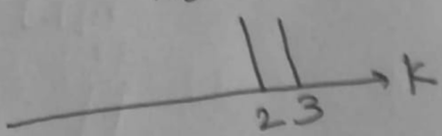
at $n=1$, $h[-k+1]$ $y[1]=2$



at $n=2$ $h[-k+2]$ $y[2]=2$



at $n=3$ $h[-k+3]$ $y[3]=1$



for $n>3$

$y[n]=0$

$\therefore \; y[n] = \{\underset{\uparrow}{1}, 2, 2, 1\}$

Q2 i) $x(t) * h(t) = h(t) * x(t)$

Proof:

LHS : $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Let $t-\tau = p$

$d\tau = -dp$

$\tau = t-p$

$\Rightarrow x(t) * h(t) = \int_{-\infty}^{\infty} x(t-p)\, h(p)\, (-dp)$

$= \int_{\infty}^{-\infty} x(t-p)\, h(p)\, dp$

$= \int_{-\infty}^{\infty} h(p)\, x(t-p)\, dp$

$= h(t) * x(t)$

Hence proved

Q2 II) $x[n] * \delta[n-n_0] = x[n-n_0]$

Proof:

LHS : $x[n] * \delta[n-n_0] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-n_0-k]$

$$= x(k)\Big|_{k=n-n_0}$$

$$= x(n-n_0)$$

// QED

Q2 III) $x[n] * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

Proof

LHS: $x[n] * [h_1(n) + h_2(n)]$

$$= \sum_{k=-\infty}^{\infty} x(k)\left(h_1(n-k) + h_2(n-k)\right)$$

$$= \sum_{k=-\infty}^{\infty} x(k)h_1(n-k) + \sum_{k=-\infty}^{\infty} x(k)h_2(n-k)$$

$$= x(n) * h_1(n) + x(n) * h_2(n)$$

QED

Q3

1) $h(n) = \left(\frac{1}{2}\right)^n u[n]$

step response, $s(n) = h(n) * u(n)$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \quad , n \geq 0$$

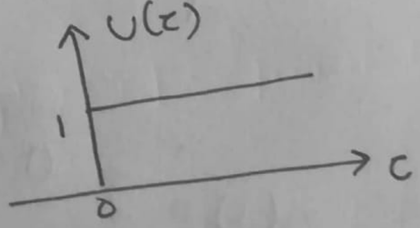
$$= \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}$$

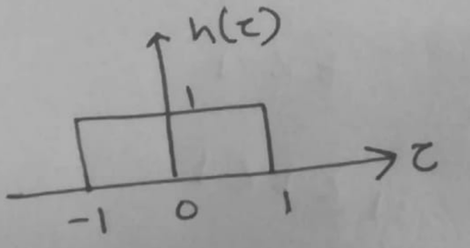
$$= 2\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$$

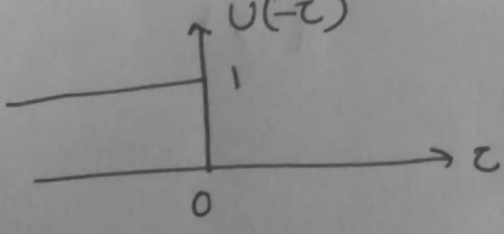
$$\therefore \; s(n) = \left(2-\left(\frac{1}{2}\right)^n\right) u[n]$$

11) $h(t) = u(t+1) - u(t-1)$

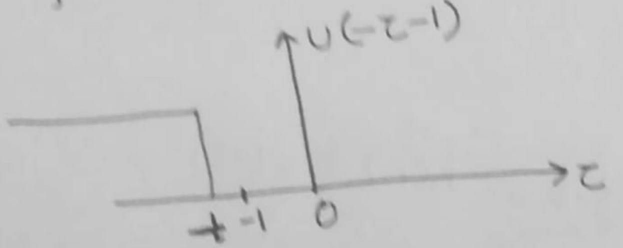
$s(t) = h(t) * u(t)$





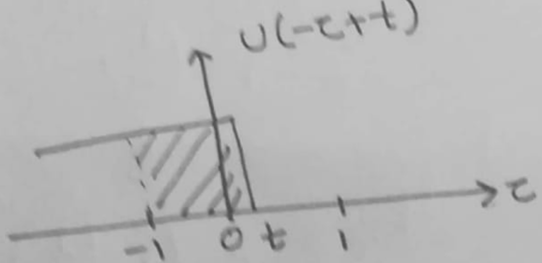


If $t < -1$,



$s(t) = 0$

If $-1 \leq t < 1$

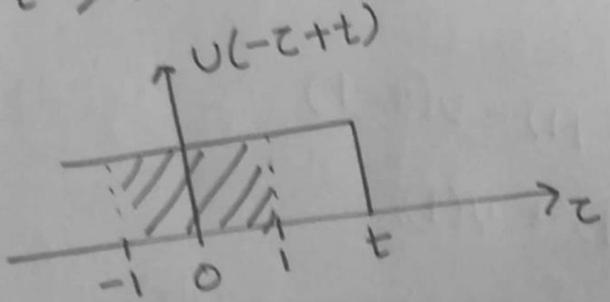


$$s(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$

$$= \int_{-1}^{t} 1\, d\tau$$

$$= t+1$$

If $t \geq 1$



$$s(t) = \int_{-1}^{1} 1\, d\tau$$

$$= 2$$

$$\therefore\ s(t) = \begin{cases} 0 & t < -1 \\ t+1 & -1 \leq t < 1 \\ 2 & t \geq 1 \end{cases}$$

Q4 a) $X(-1) = \frac{3}{2} e^{-j\pi/4}$

$X(1) = \frac{3}{2} e^{j\pi/4}$

$X(k) = 0 \quad , \quad k \neq -1, 1$

$T_0 = 2$

$\omega_0 = \frac{2\pi}{T_0} = \pi$

$$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t}$$

$$= X(-1) e^{-j\omega_0 t} + X(1) e^{j\omega_0 t}$$

$$= \frac{3}{2} e^{-j\pi/4} \cdot e^{-j\omega_0 t} + \frac{3}{2} e^{j\pi/4} e^{j\omega_0 t}$$

$$= 3 \left(\frac{e^{j(\omega_0 t + \pi/4)} + e^{-j(\omega_0 t + \pi/4)}}{2} \right)$$

$$= 3 \cos(\omega_0 t + \pi/4)$$

$$= 3 \cos(\pi t + \pi/4)$$

Q4 b) $x(t) = 1 + 2 \sin(2\pi t - 3) + \sin(6\pi t)$

$$= 1 + \frac{e^{j(2\pi t - 3)} - e^{-j(2\pi t - 3)}}{j} + \frac{e^{j6\pi t} - e^{-j6\pi t}}{2j}$$

$$= 1 + \frac{e^{-3j}}{j} e^{j2\pi t} - \frac{e^{j3}}{j} e^{j2\pi t} + \frac{1}{2j} e^{j6\pi t} - \frac{1}{2j} e^{j6\pi t}$$

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T_0 : Time period of $x(t)$ is 1

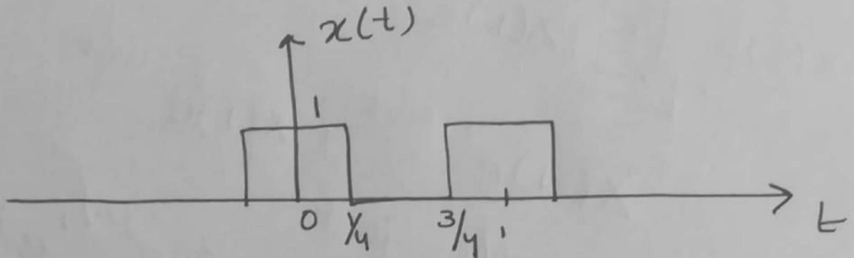
$\therefore \omega_0 = 2\pi$

$\Rightarrow \quad C_0 = 1$

$C_1 = \frac{e^{-3j}}{j} \qquad C_{-1} \cdot \frac{-e^{3j}}{j}$

$C_3 = \frac{1}{2j} \qquad C_{-3} = -\frac{1}{2j}$

Q5



$T_0 = 1 \;, \Rightarrow \omega_0 = 2\pi$

$$C_k = \frac{1}{T_0}\left(\int_{<T_0} x(t)\, e^{-jk\omega_0 t}\, dt\right)$$

$$= \frac{1}{T_0}\left(\int_0^{1/4} e^{-jk\omega_0 t}\, dt + \int_{3/4}^{1} e^{-jk\omega_0 t}\, dt\right)$$

$$= \frac{1}{T_0}\left(\left.\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right|_0^{1/4} + \left.\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right|_{3/4}^{1}\right)$$

$$= \frac{e^{-jk\frac{2\pi}{4}} - 1 + e^{-jk2\pi} - e^{-jk\frac{3\pi}{2}}}{-jk2\pi} = \frac{e^{-jk\frac{3\pi}{2}} - e^{-jk\frac{2\pi}{4}}}{j2\pi k}$$

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$T_0 = \pi, \quad \omega_0 = 2$

Q6

a) $y(t) = x(t) + x(-t)$

$x(t) \xleftrightarrow{CTFS} X(k)$

$x(-t) \longleftrightarrow X(-k)$ Time reversal property

$x(t) + x(-t) \longleftrightarrow X(k) + X(-k)$: Linearity

$\therefore y(t) \longleftrightarrow -k2^{|k|} + (k2^{|k|})$

$= 0$

b) $y(t) = x(2t)$

$x(t) \xleftrightarrow{CTFS} X(k)$

Time scaling property

$x(at) \longleftrightarrow X(k)$

$y(t) \longleftrightarrow -k2^{|k|}$

c) $y(t) = \frac{d}{dt} x(t)$

Using time differentiation

$\frac{d}{dt} x(t) \xleftrightarrow{CTFS} jk\omega_0 X(k)$

$\therefore y(t) \longleftrightarrow jk\omega_0 (-k2^{|k|})$

$= -j\omega_0 k^2 2^{|k|}$

$= -2jk^2 2^{|k|}$

d) $y(t) = x\left(t-\frac{1}{4}\right)$

Time shift prop.

$x(t) \longleftrightarrow X(k)$

$x(t-1/4) \longleftrightarrow e^{-jk\omega_0 \cdot 1/4} \cdot X(k)$

$\Rightarrow \quad y(t) \longleftrightarrow -e^{-jk/2} \cdot k2^{|k|}$

e) $y(t) = x(t) * x(t-1/2)$

Using time shift prop.

$x(t-1/2) \xleftrightarrow{CTFS} e^{-jk\cancel{\pi}/\cancel{2}} \cdot (-k2^{|k|})$

Using convolution prop.

$y(t) \xleftrightarrow{CTFS} Y(k)$

$x(t) \xleftrightarrow{CTFS} X(k)$

$x(t-\frac{1}{2}) \xleftrightarrow{CTFS} X_1(k)$

$y(t) = x(t) * x\left(t-\frac{1}{2}\right)$

$\Rightarrow Y(k) = \cancel{T_0}\, X(k)\, X_1(k)$

$= \cancel{T_0} \cdot (-k2^{|k|})(-e^{-jk}k2^{|k|})$

$= \pi.$

Using convolution property of CTFS

If $y(t) = x(t) * x(t-\frac{1}{2})$

then $Y(k) = T_0 . X(k) \cdot X(k) e^{-jk}$

$= \pi k^2 2^{2|k|} \cdot e^{-jk}$

Q7

$$x(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & 5 \le n \le 6 \end{cases}$$

$N_0 = 7, \quad \Omega_0 = 2\pi/7$

$$X(k) = \frac{1}{N_0} \sum_{<N_0>} x(n) e^{-jk\Omega_0 n}$$

$$= \frac{1}{7} \sum_{n=0}^{4} e^{-jk\frac{2\pi}{7}n}$$

$$= \frac{1}{7} \frac{1 - \left(e^{-jk\frac{2\pi}{7}}\right)^5}{1 - e^{-jk2\pi/7}}$$

$$= \frac{1}{7} \frac{e^{-jk\frac{10\pi}{14}} \left(e^{jk\frac{10\pi}{14}} - e^{-jk\frac{10\pi}{14}}\right)}{e^{-jk\frac{2\pi}{14}} \left(e^{jk2\pi/14} - e^{-jk2\pi/14}\right)}$$

$$= \frac{1}{7} e^{-jk\frac{8\pi}{14}} \cdot \frac{\sin\left(\frac{10\pi}{14}k\right)}{\sin\left(\frac{2\pi}{14}k\right)}$$

$k = 0, 1, \cdots 6$

Q8 a) $x(n) = \sin\left(\frac{4\pi n}{21}\right) + \cos\left(\frac{10\pi n}{21}\right) + 1$

$N_0 = 21$

$\Omega_0 = \frac{2\pi}{21}$

Using Euler's eq: $x(n) = \frac{e^{j\frac{4\pi}{21}n} - e^{-j\frac{4\pi}{21}n}}{2j} + \frac{e^{j\frac{10\pi}{21}n} + e^{-j\frac{10\pi n}{21}}}{2} + 1$

$X(0) = 1$

$X(-2) = \frac{-1}{2j}$

$X(2) = \frac{1}{2j}$

$X(-5) = 1/2$

$X(5) = \frac{1}{2}$

Q8 b) $X(k) = \left(\frac{1}{2}\right)^k \quad k = 0, 1 \ldots 9$

$N_0 = 10$

$x(n) = \sum_{\langle N_0 \rangle} X(k) e^{jk\Omega_0 n}$

$= \sum_{k=0}^{9} \left(\frac{1}{2}\right)^k e^{jk\Omega_0 n}$

$= \frac{1 - \left(\frac{1}{2} e^{j\frac{2\pi}{10}\cdot n}\right)^{10}}{1 - \frac{1}{2} e^{j\frac{2\pi}{10}n}}$

$$= \frac{1-\frac{1}{2}e^{j2\pi n}}{1-\frac{1}{2}e^{j\frac{2\pi}{10}n}}$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{5}n}}\right)$$

$x(0) =$