6. $G(s) \cdot H(s) = \dfrac{K(s+1)(s+2)}{\cancel{s}(s+0.1)(s-1)}$

Step-1:-

The ~~root~~ locus is symmetrical about real axis.

→ NO of poles are two [2]
there are -0.1 and 1

→ NO of ~~zero's~~ are two [2]

there are -1, -2

Step-2:-

NO of pole's are equal to NO of zero's

P=N=Z

8*
→ NO root locus branches starts and terminates to infinite.

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Starting points:-

All '2' Root locus points will starts at open loop pole location

$\longrightarrow -0.1 \; ; \; 1$

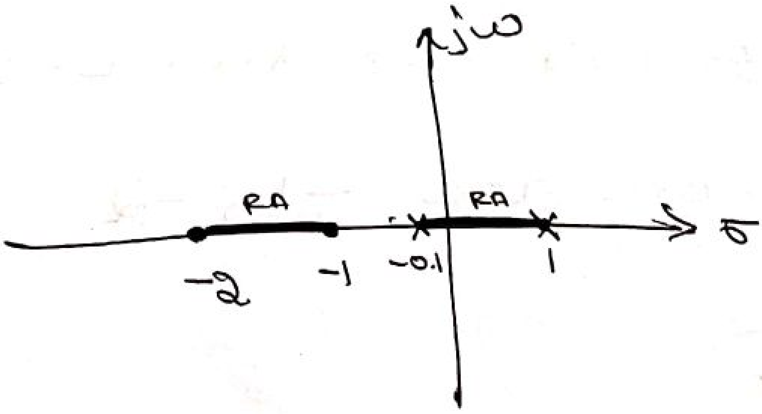
terminating points:-

All the '3' root locus branches will terminate to open loop zero location

$\longrightarrow -1 \; ; \; -2$

Step-3:-

The real axis of root locus section is marked in the graph sheet.



Step-4:-

Asymptotic lines

i) Centroid [σ]:-

$$\sigma = \frac{\Sigma_i RPP - \Sigma_i RPZ}{P - Z}$$

$$\sigma = \frac{(-0.1+1) - (-1-2)}{2-2}$$

$$\boxed{\sigma = \infty}$$

IN this Question there is not Asymptotic line's because of $P = Z$

step-5:-

Break away point

Both the condition are occuring in this question

Determination of Break away points

1. characterstics eq^n

$$1 + G(s) \cdot H(s) = 0$$

$$1 + \frac{K(s+1)(s+2)}{(s+0.1)(s-1)} = 0$$

$$(s+0.1)(s-1) + K(s+1)(s+2) = 0$$

$$s^2 - s + 0.1s - 0.1 + K[s^2 + 2s + 1s + 2] = 0$$

$$s^2 - 0.9s - 0.1 + Ks^2 + 3Ks + 2K = 0$$

$$(1+K)s^2 + (3K-0.9)s + (2K-0.1) = 0$$

ii) $K = f(s)$.

$$K = \frac{-s^2+0.9s+0.1}{(s^2+3s+2)}$$

iii) $\frac{dK}{ds} = 0$

$$\frac{dK}{ds} = \frac{(-2s+0.9)(s^2+3s+2) - (-s^2+0.9s+ \quad (2s+3}{(s^2+3s+2)^2}$$

$$\frac{dK}{ds} = (-2s+0.9)(s^2+3s+2) - (-s^2+0.9s+0.1)(2s+3) =$$

$$\frac{dK}{ds} = -2s^3 - 6s^2 - 4s + 0.9s^2 + 2.7s + 1.8 - (-2s^3 + 1.8s^2 + 0.2s - 3s^2 + 2.7s + $$
$$= 0$$

$$\frac{dK}{ds} = -2s^3 - 5.1s^2 - 1.3s + 1.8 - (-2s^3 - 1.2 \quad 2.9s + 0.3) = 0$$

$$\frac{dK}{ds} = -\cancel{2s^3} - 5.1s^2 - 1.3s + 1.8 + \cancel{2s^3} + 1.2s^2 - 2.9s - 0.3$$

$$\frac{dK}{ds} = -3.9s^2 - 4.2s + 1.5 = 0$$

roots of $\frac{dK}{ds}$ are

$$s = 0.2828, -1.359$$

at

$$s = 0.2828$$

$$K = \frac{-(0.2828)^2 + 0.9(0.2828) + 0.1}{(0.2828)^2 + 3(0.2828) + 2}$$

$$\boxed{K = 0.0937}$$

BAP at $s = 0.2828$ is valid.

at

$$s = -1.359$$

$$K = \frac{-s^2 + 0.9s + 0.1}{s^2 + 3s + 2}$$

$\boxed{K = 12.9062}$ is valid

Step-6:-

intersection of root locus and imagi-nary axis

is characteristics eqn

$$1 + G(s)H(s) = 0$$

$$(1+k)s^2 + (3k-0.9)s + (2k-0.1) = 0$$

s^2	$1+k$	$2k-0.1$	0
s^1	$3k-0.9$	0	0
s^0	$2k-0.1$		

row of zeros

Range of 'k';

$$k > 0$$

then; at s^0 row;

$$2k - 0.1 > 0$$

$2K > 0.1$

$$\boxed{K > 0.05}$$

at s^1 row;

$$3K - 0.9 > 0$$

$$3K > 0.9$$

$$\boxed{K > 0.3}$$

lower limit is defined and upper limit is not defined

$$\boxed{0.05 < K < \infty}$$

→ Marginal of 'K'

$$3K_{mar} - 0.9 = 0$$

$$\boxed{K_{mar} = 0.3}$$

→ Auxillary eqn $A_1(s)$ is

$$A_1(s) = (1 + K_{mar})s^2 + 2K_{mar} - 0.1$$

$$A_1(s) = (1 + 0.3)s^2 + 2(0.3) - 0.1$$

$$A_1(s) = 1.3s^2 + 0.5$$

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roots of $A_1(s)$ are;

$$\boxed{s = \pm 0.62j}$$

4. Characterstics eqn

$$s^6 + 4s^5 + 3s^4 - 16s^2 - 64s - 48 = 0$$

$+\ s^6$	1	3	-64	0
$+\ s^5$	4	-16	-48	0
$+\ s^4$	7	-52	0	0
$+\ s^3$	13.71	-48	0	0
$-\ s^2$	-27.49	0	0	0
$-\ s^1$	-48	0	0	0
$-\ s^0$	0 -48	0 0	0 0	0 0

row of zer

① ② ③

Auxillary eqn $A_1(s) = -48s^1$.

$$\frac{dA_1(s)}{ds} = -48,$$

roots of $A_1(s)$ are

$$\boxed{s=0}$$

→ $\underline{No}$ of roots lies on RHS poles are three [3]

→ $\underline{No}$ of roots lies on LHS Poles are three [3]

→ $\underline{No}$ of roots lies of imaginary axis is zero

⟹ Total roots are ~~six~~ six (6)

so; there are three sign chang.
which indicates 3 poles lies on RHS of s-plane

~~so~~; therefore
⟹ the system is unstable

2. Step response of a second order under-damped system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

∴ Under-damped system.

$$0 < \xi < 1$$

then;
roots of. $s^2 + 2\xi\omega_n s + \omega_n^2$ a

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$\therefore \omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\alpha = \xi\omega_n$$

$$\boxed{s_{1,2} = -\alpha \pm j\omega_d}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

∴ unit step response $R(s) = \frac{1}{s}$

$$c(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

but $c(t) = L^{-1}\{C(s)\}$

$$c(t) = L^{-1}\left\{\frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}\right\}$$

$$c(t) = L^{-1}\left\{\frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n s + \omega_n^2}\right\}$$

$$\therefore \frac{\omega_n^2}{s(\cancel{s^2 + 2s\xi\omega_n + \omega_n^2})} = \frac{A(s^2 + 2\xi\omega_n s + \omega_n^2) + Bs^2 + Cs}{s(\cancel{s^2 + \omega_n \xi 2s + \omega_n^2})}$$

at $s = 0$

$A = 1$

at $s = -\omega_n$

$$\omega_n^2 = A(\omega_n^2 - 2\xi\omega_n^2 + \omega_n^2) + B\omega_n^2 - C\omega_n \quad \text{(1)}$$

at $s = \omega_n$

$$\omega_n^2 = A(\omega_n^2 + 2\xi\omega_n^2 + \omega_n^2) + B\omega_n^2 + C\omega_n \quad \text{(2)}$$

By solving eq^n ① and ② we get

$$B = -1$$
$$C = -2\xi\omega_n$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{1s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}\right\}$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{1s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}\right\}$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \left[\frac{1s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} + \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}\right]\right\}$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \left[\frac{1s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} + \frac{\xi\omega_n\omega_d}{\omega_d((s + \omega_n\xi)^2 + \omega_d^2)}\right]\right\}$$

$$\because \omega_d = \omega_n\sqrt{1 - \xi^2}$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{(s + \xi\omega_n)}{(s + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\cancel{\omega_n}\omega_d}{\cancel{\omega_n}\sqrt{1 - \xi^2}((s + \xi\omega_n)^2 + \omega_d^2)}\right.$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{\xi\omega_d}{\sqrt{1 - \xi^2}((s + \xi\omega_n)^2 + \omega_d^2)} - \frac{(s + \xi\omega_n)}{(s + \xi\omega_n)^2 + \ldots}\right.$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\cos\omega_d t . \sqrt{1-\xi^2} + \sin\omega_d t \; \xi\right.$$

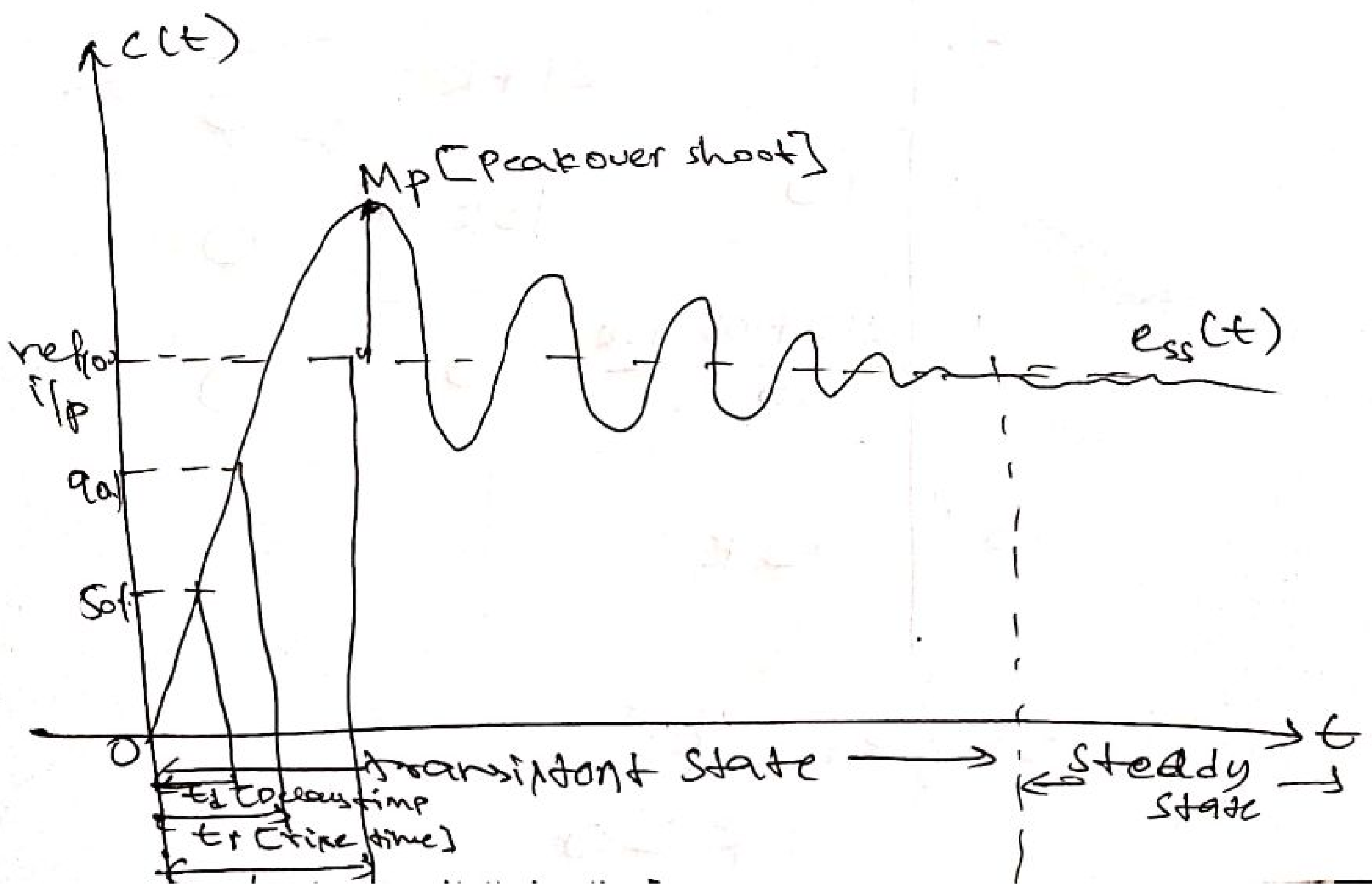
$\therefore$ wkt,

$$\sin\theta = \xi$$

$$\cos\theta = \sqrt{1-\xi^2}$$

$$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sin(\omega_d t + \theta)\right)}$$



3.

$$T.F = G(s)H(s) = \frac{K(s+13)}{s(s+3)(s+7)}$$

i)

→ Characterstics eqn

$$1 + G(s) \cdot H(s) = 0$$

$$s(s+3)(s+7) + K(s+13) = 0$$

$$(s^2+3s)(s+7) + K(s+13) = 0$$

$$s^3 + 7s^2 + 3s^2 + 21s + Ks + 13K = 0$$

$$s^3 + 10s^2 + (21+K)s + 13K = 0 \rightarrow (1)$$

s^3	1	$21+K$	0
$A_1(s) \rightarrow s^2$	10	$13K$	0
s^1	$\frac{210+10K-13K}{10}$	0	0
s^0	$13K$		

row of ze

Range of 'K'

$$K > 0$$

then;

$$\frac{210-3K}{10} > 0$$

$$210 > 3K$$

$$\boxed{K < 70}$$

Then range of 'K' is

$$\boxed{0 < K < 70}$$

→ K marginal value is

$$\frac{210 - 3K_{mar}}{10} = 0$$

$$\boxed{K_{mar} = 70}$$

Auxillary eqn $A_1(s)$ is

$$A_1(s) = 10s^2 + 13K_{mar} = 0$$

$$A_1(s) = 10s^2 + 910 = 0$$

roots of $A_1(s)$ are

$$\boxed{s = \pm 9.539j}$$

→ Marginal Stable

ii.)

Subpstitute $s = s' - 1$

then characterstics eq^n

$$1 + G(s) \cdot H(s) = 0$$

$$(s'-1)^3 + 10(s'-1)^2 + (21+K)(s'-1) + 13K = 0$$

$$(s')^3 - 1 - 3s'(s'-1) + 10((s')^2 + 1 - 2s') + 21s' - 21$$
$$Ks' - K + 13K = 0$$

$$(s')^3 + 10(s')^2 - 3(s')^2 + 3s' - 1 + 10 - 20s' + 21s'$$
$$-21 + Ks' + 12K$$

$$(s')^3 + 7(s')^2 + 4s' - 12 + Ks' + 12K$$

$$(s')^3 + 7(s')^2 + (4+K)s' - 12(K+1) = 0$$

s'^3	1	$4+K$	0
s'^2	7	$12K-12$	0
$(s')^1$	$\frac{28+7K-12K+12}{7} = 0$	0	row of zero's
$(s')^0$	$12K-12$	0	

AC

→ range of 'K'

$K > 0$

→ s^0 row;

$12K - 12 > 0$

$\boxed{K > 1}$

→ s^1 row;

$$\frac{40 - 5K}{7} \geq 0$$

$40 > 5K$

$K \leq 8$

So;

range of 'K' is

$\boxed{1 < K < 8}$

→ $K_{marginal}$ Value is

$$\frac{40 - 5K_{max}}{7} = 0$$

$\boxed{K_{max} = 8}$

Auxillary eq^n $A_1(s')$

$$A_1(s') = 7s'^2 + 12K_{mar}^{-12}$$

$$A_1(s') = 7s'^2 + 84$$

roots of $A_1(s)$ are

$$\boxed{s' = \pm 2\sqrt{3}j}$$

→ system in marginal stable

$$G(s).H(s) = \frac{K(s+0.5)}{s(s^2+2s+2)}$$

No. of poles are '3'

→ 0, $-1+j$, $-1-j$

No. of zero's are '1'

→ -0.5

Step-1:-

root locus is symmetrical ab
real axis

Step-2:-

Starting points :-

→ open loop Poles location

→ 0, −1+j, −1−j

terminating points :-

− open loop zeros locatio

→ −0.5, ∞, ∞

Step-3:-

Real axis Root locus is about section is marked in sheet graph

Step-3:-

Asymptotic line

i) centroid

$$\sigma = \frac{\Sigma RPP - \Sigma RPZ}{P - Z}$$

$$\frac{(-1-1) - (-0.5)}{3-1} = \frac{-2+0.5}{2}$$

$$\boxed{\sigma = -0.75}$$

axis

3-1-1

iii) Angle of Asymptotics :-

$$\theta = \frac{(2q+1)180^\circ}{p-q} \quad \therefore q = 0,1$$

at $q=0$

$$\theta_1 = \frac{180^\circ}{2} = 90^\circ$$

at $q=1$

$$\theta_2 = \frac{3(180^\circ)}{2} = 270^\circ$$

Step-4 :- Break away point

Determination of BAP :-

→ characterstic eqn

$1+G(s)H(s) = 0$

$$s^3 + 2s^2 + 2s + Ks + 0.5K = 0$$

s^3	1	$(2+K)$
s^2	2	$0.5K$

$$\begin{array}{c|c} s^1 & \frac{4+1.5K}{2} \quad 0 \\ s^0 & 0.5K \end{array}$$

ii) $K = f(s)$

$$K = \frac{-s^3 - 2s^2 - 2s}{(s+0.5)}$$

iii) $\frac{dK}{ds} = 0$

$$\frac{dK}{ds} = \frac{(-3s^2 - 4s - 2)(s+0.5) - (-s^3 - 2s^2 - 2s)(1)}{(s+0.5)^2} =$$

$$\frac{dK}{ds} = \cancel{-3s^3} - 4s^2 - \cancel{2s} - 1.5s^2 - 2s - \cancel{+s^3} + 2s^2 + \cancel{2s}$$

$$\frac{dK}{ds} = -2s^3 \underbrace{-2s^2 - 1.5s^2}_{-3.5s^2} - 2s - 1 =$$

$$\frac{dK}{ds} = -2s^3 - 3.5s^2 - 2s - 1 =$$

roots are $\frac{dK}{ds}$; $s = -1.2729; -0$

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Step-6:-

intersection of root locus and imaginary axis

i) Characteristic eqn:

$s^3 + 2s^2 + 2s + ks + 0.5k$

s^3	1	2+k	0
s^2	2	0.5	
s^1	$\frac{4+1.5k}{2}$	0	0
s^0	0.5	0	0

range of 'k' and $\boxed{k_{mar} = -2.}$

k > 0

$\therefore \frac{4+1.5k}{2} > 0$

$\boxed{k > 2.66}$ → if k_{mar} is -ve Not c with in

range:-
0 < K < ∞

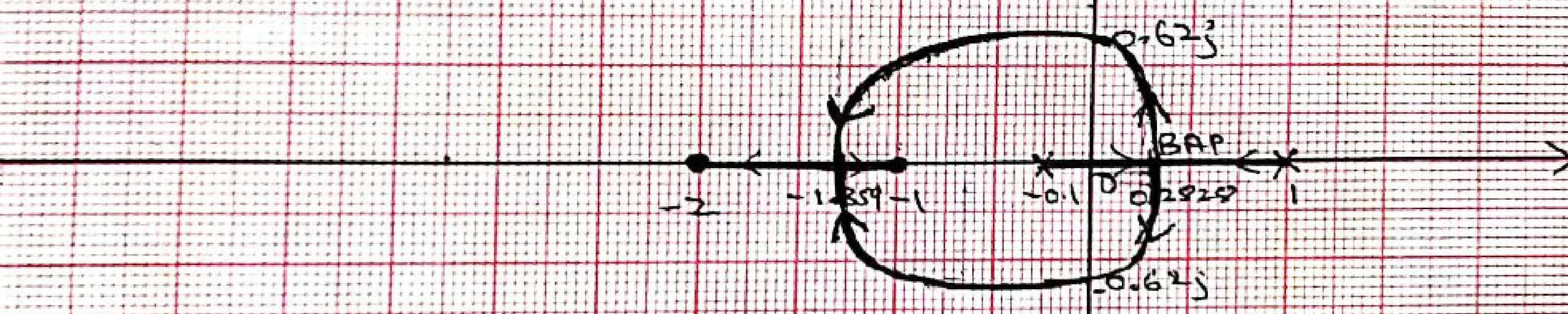
$K_{mar} = 0.3$ [marginal stable]

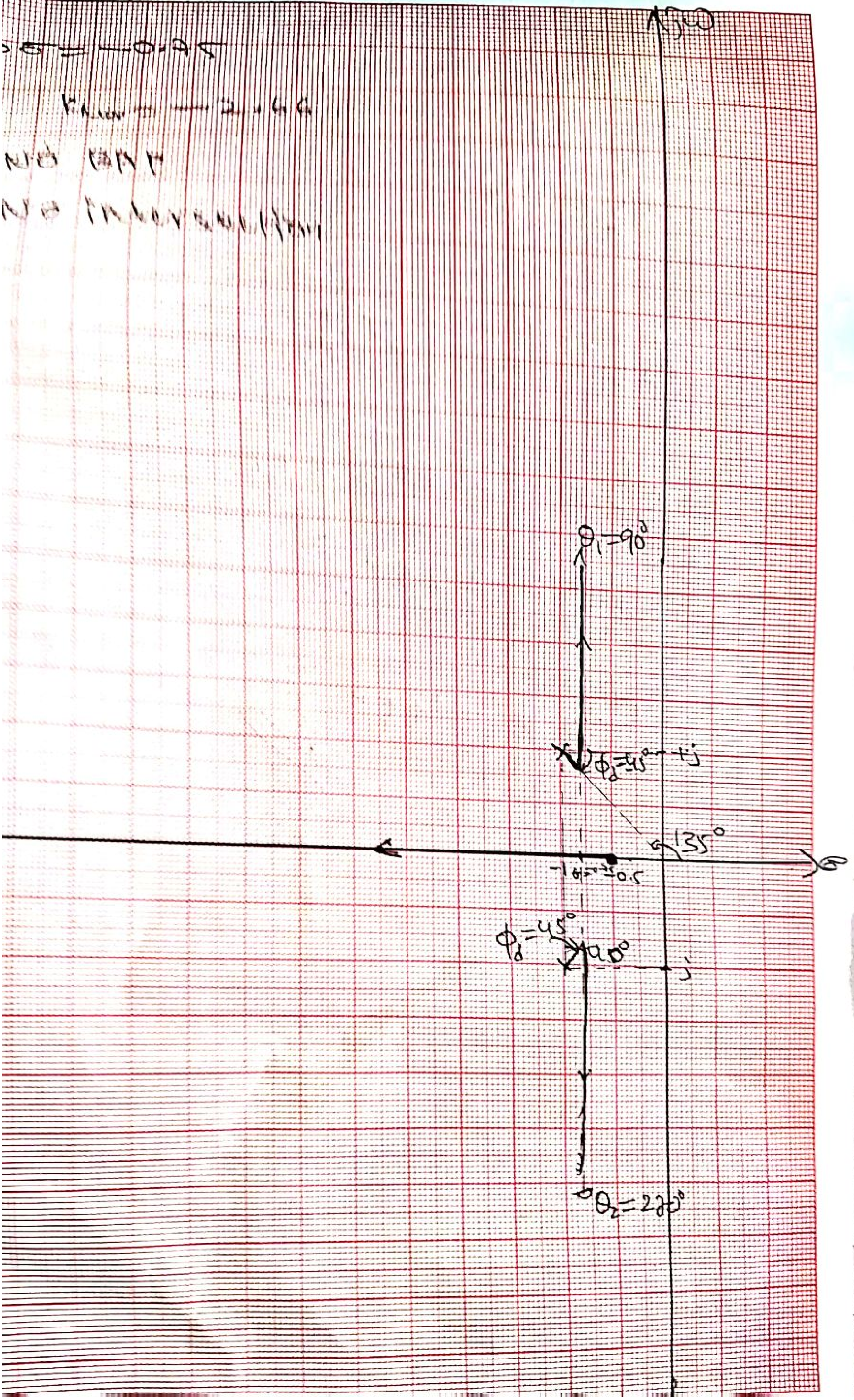
→ intersection;

$s = \pm 0.62j$

→ Breakaway point;

$s = -1.359, 0.2828$




jω
θ1=90°
φd=45°
+j
135°
σ
-1
0.5
φd=45°
90°
-j
θ2=270°

we have to find angle of departure

$$\phi_d = 180^\circ - \phi$$

$$\phi = \Sigma \phi_p - \Sigma \phi_z$$

$$\phi = 135 + 90 - 0$$

$$\boxed{\phi = 225^\circ}$$

$$\boxed{\phi_d = -45^\circ}$$

1)
a) t_r (Rise time) :- It is the time required by the system to reach 10% to 100% of its output during its first attempt.

$$t_r = \frac{n\pi - \theta}{\omega_d}.$$

w.k.t

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} (\sin \omega_d t + \theta).$$

$c(t) = 1$ since input is unit step $R(s) = 1$.

$$\cancel{1} = \cancel{1} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} (\sin \omega_d t + \theta)$$

$$\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) = 0.$$

$\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}$ cannot be zero.

$\therefore \sin(\omega_d t_r + \theta) = 0$ $\quad\quad \sin(0^\circ) = n\pi$

$$\omega_d t_r + \theta = n\pi$$

$$\omega_d t_r = n\pi - \theta$$

$$\boxed{t_r = \frac{n\pi - \theta}{\omega_d}} \text{ // } s$$

where ω_d = damping frequency.

ω_n = natural. frequency

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi} \right)$$

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