Scheme Of Evaluation

Internal Assessment Test II – April .2019

Sub:	Principles Of Communication Systems							**Code:**	17EC44
Date:	15/04/2019	Duration:	90mins	Max Marks:	50	**Sem:**	IV	**Branch:**	ECE

Note: Answer Any Five Questions

Question #	Description	Marks Distribution	Max Marks
1	Derive the expression of narrowband FM both in time domain and frequency domain, plot the NBFM frequency spectrum also compare with an AM signal using phasor diagrams • Derivation • Phasor diagrams	- 6 4	10
2a)	Explain generation of FM wave using direct method. • Circuit diagram • Equations • Theory	- 2 2 1	5
2b)	With a neat diagram, explain the generation of wideband FM signals. How the frequency stability is achieved. • Block Diagrams • Theory	- 3 2	5
3	With the help of linear model of PLL, obtain output expression of demodulation of FM signals. • Theory • Derivation • Block Diagram	- 2 5 3	10
4	With the help of amplitude response of VSB filter and necessary block diagrams explain VSB modulation and demodulation • Block Diagram • Theory • Equations	- 4 3 3	10
5a	Derive an expression of SSB for which USB should be retained • Spectrums • Theory	- 2 3	5
5b	Explain the working of superheterodyne receiver • Block Diagram • Theory	- 2 3	5
6	A FM wave is represented by the following equation: $s(t) = 10\sin(5 \times 10^8 t + 4\sin(1520t)$. Determine (i) Carrier wave (ii) modulation index (iii) Frequency deviation iv) Power dissipated by FM wave across5Ω (i) Carrier wave (ii) modulation index (iii) Frequency deviation iv) Power dissipated by FM wave across5Ω	- 2 2 3 3	10

7	State and prove sampling theorem. • Statement • Spectrum • Derivation	- 2 4 4	10

Solution

1.

NBFM - Narrow Band FM.

Considering an FM equation, $S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \longrightarrow (1)$

On expanding (1) we get, Hint $\{\cos[A+B] = \cos A \cdot \cos B - \sin A \cdot \sin B\}$

$$S(t) = A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)] \longrightarrow (2)$$

Assuming modulation index β is small compared to one radian, then

$$\left.\begin{aligned} &\cos[\beta \sin(2\pi f_m t)] \simeq 1 \\ \& \quad &\sin[\beta \sin(2\pi f_m t)] \simeq \beta \sin(2\pi f_m t) \end{aligned}\right\}$$

For less θ: $\cos\theta \simeq 1$, $\sin\theta \simeq \theta$

$$\therefore S(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \underline{\sin(2\pi f_m t) \cdot \sin(2\pi f_c t)}$$

using $\sin A \cdot \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$$S(t) \simeq A_c \cos(2\pi f_c t) + \frac{1}{2}\beta A_c \{\cos[2\pi(f_c+f_m)t] - \cos[2\pi(f_c-f_m)t]\} \longrightarrow (3)$$

This equation is similar to AM signal,

$$S_{AM}(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \{\cos[2\pi(f_c+f_m)t] + \cos[2\pi(f_c-f_m)t]\} \longrightarrow (4)$$

Comparing eq^n (3) & (4), the basic difference between an AM signal and a narrow band FM signal is the algebraic sign of lower side frequency in the narrow band FM is reversed. Thus a narrow band FM signal requires essentially the same transmission bandwidth $2f_m$ as AM signal.

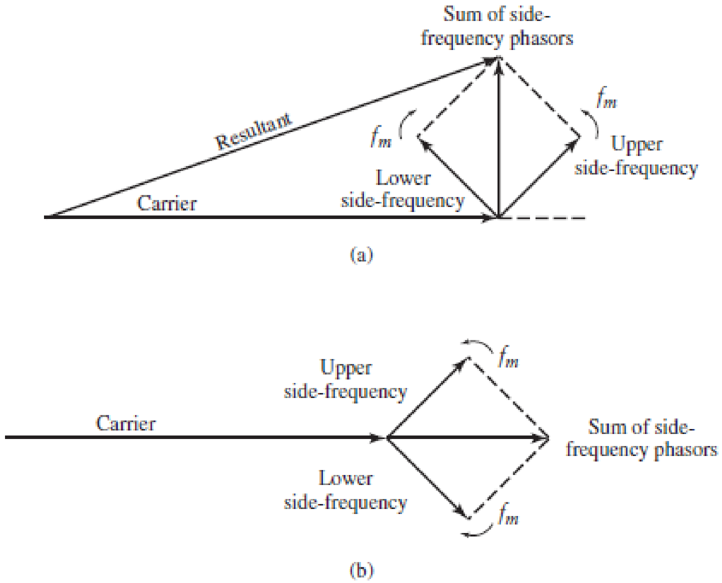


Figure 4.5 Phasor comparison of narrow-band FM and AM waves for sinusoidal modulation. (*a*) Narrow-band FM wave. (*b*) AM wave.

2

Generation of FM signals.

In a direct FM system, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal by means of a device known as a Voltage Controlled oscillator.

One way of implementing is to use a sinusoidal oscillator having a highly selective frequency-determining resonant network & to control the oscillator by incremental variations of the reactive components.

The Block diagram for generating FM signal using Hartley oscillator is shown in fig.

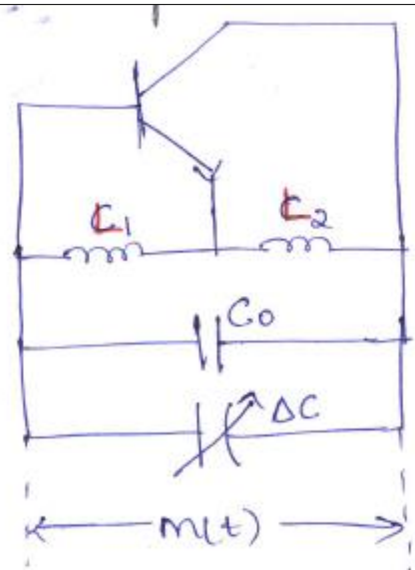


Fig : Hartley oscillator

The capacitive component of the ~~frequency~~ network in the oscillator consists of fixed capacitor shunted by a voltage variable capacitor.

The resultant capacitance is represented by $c(t)$.

A voltage variable capacitor commonly called a Varactor or varicap, is one whose capacitance depends on the voltage applied across its electrodes.

The frequency of oscillation of the Hartley oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L1+L2)\,c(t)}} \qquad (1)$$

where $c(t)$ is the total capacitance of the fixed capacitor & the variable-vtg capacitor. L_1 & L_2 are the two inductances.

Let f_m is the Sinusoidal modulating wave frequency. The Capacitance $C(t)$ is expressed as

$$C(t) = C_0 + \Delta C \cos(2\pi f_m t) \quad \text{--- (2)}$$

where C_0 is the total capacitance in the absence of modulation.

ΔC is the maximum change in Capacitance.

(2) in (1)

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0+\Delta C\cos(2\pi f_m t))}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{C_0(L_1+L_2)}} \cdot \frac{1}{\sqrt{1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)}}$$

$$f_i(t) = f_0\left[1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right]^{-1/2} \quad \text{--- (3)}$$

where f_0 is unmodulated frequency of oscillation.

$$f_0 = \frac{1}{2\pi\sqrt{C_0(L_1+L_2)}} \quad \text{--- (4)}$$

Note: Binomial theorem.

$$(1+x)^{-1/2} \simeq 1 - \frac{1}{2}x \quad \text{if } |x| << 1$$

If $\left|\frac{\Delta C}{C_0}\right| << 1$ then (3) is

$$f_i(t) = f_0\left[1 - \frac{\Delta C}{2C_0}\cos 2\pi f_m t\right]$$

$$f_i(t) = f_0 - f_0 \cdot \frac{\Delta C}{2C_0} \cos 2\pi f_m t \quad \text{— (5)}$$

WKT,

$$f_i(t) = f_c + K_f m(t)$$

$$f_i(t) = f_c + K_f A_m \cos(2\pi f_m t)$$

$$f_i(t) = f_0 + \Delta f \cos 2\pi f_m t \quad \text{— (6)}$$

compare (5) & (6)

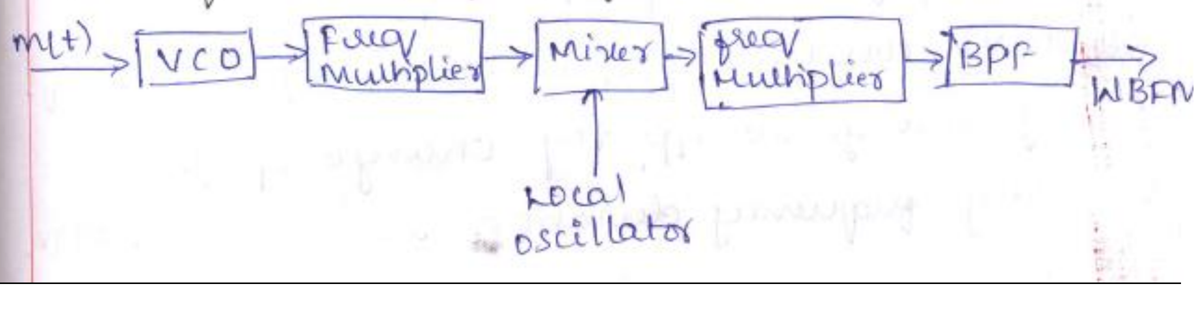
$$-\frac{\Delta C}{2C_0} f_0 = \Delta f \quad \text{— (7)}$$

use (7) in (5)

$$\boxed{f_i(t) \simeq f_0 + \Delta f \cos 2\pi f_m t} \quad \text{— (8)}$$

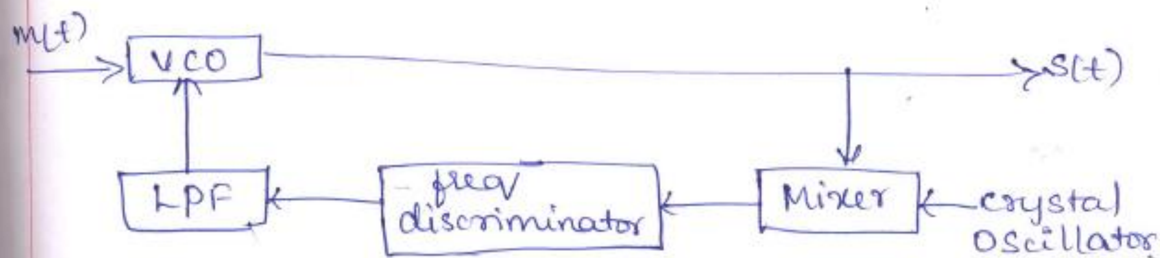
(8) is the desired relation for the instantaneous frequency of an FM wave assuming sinusoidal modulation.

The configuration below ~~shows~~ is used to generate a wide band FM wave with the required frequency deviation.



This configuration provides the constant proportionality b/w o/p frequency change & i/p vtg change, & the necessary frequency deviation to achieve wide band FM.

The disadvantage is that the carrier frequency is not obtained from a highly stable oscillator. One method of effecting this control is shown below.



The o/p of a FM generator is applied to a mixer together with the output of a crystal controlled oscillator.

The mixer o/p is next applied to a frequency discriminator & then low pass filtered.

This configuration provides good oscillator stability, constant proportionality b/w o/p frequency change to i/p vtg change & the necessary frequency deviation to acheive WBFM

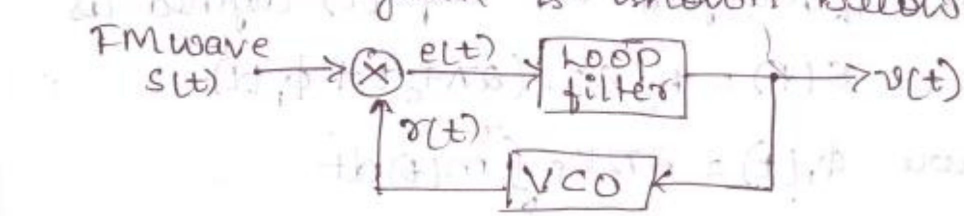
3

Phase Locked Loop (PLL)

* PLL is an electronic ckt with a voltage controlled oscillator that constantly adjusts to match the frequency of an input signal.
* PLLs are used to generate, stabilize, demodulate, or to modulate a signal.
* PLLs are used in telecommunications, computers, radio, wireless communications & many more.
* PLL is a negative feedback system works by constantly adjusting oscillator to match the phase and frequency of an input signal.
* It consists of three major components.

1. VCO: which performs frequency modulation on its own control signal.
2. Multiplier: which multiplies an incoming FM wave by the o/p of VCO
3. Loop filter of a low pass kind, the function is to remove high frequency components contained in the multiplier's o/p signal.

The block diagram is shown below.



* PLL operates in three modes.

1) Free running Mode:

PLL operates in this mode when there is no input input applied to it.

2) Capture mode:

As soon as the i/p frequency is applied, the VCO starts to change and begin producing an output frequency for comparision. This mode is called capture mode.

3) Phase lock mode

The frequency comparision stops as soon as the o/p frequency is adjusted to become equal to the input frequency. This refers to phase locked mode.

* When input signal (control signal) is zero,

a) The frequency of the signal generated by VCO is equal to unmodulated carrier frequency f_c of $s(t)$.

b) The VCO o/p has a 90° phase shift wrt the unmodulated carrier wave.

Let the i/p FM signal is ~~dsp~~ $s(t)$ defined as

$$s(t) = A_c \sin(2\pi f_c t + \phi_1(t)) \quad \text{--- (1)}$$

where $$\phi_1(t) = 2\pi k_f \int_0^t m(t)\,dt \quad \text{--- (2)}$$

A_c is the carrier amplitude

K_f is the frequency sensitivity factor of frequency modulator.

Let the o/p of VCO be $r(t)$, defined as

$$r(t) = A_v \cos\left[2\pi f_c t + \phi_2(t)\right] \quad \text{--- (3)}$$

where A_v is amplitude &

$\phi_2(t)$ is related to $v(t)$ as

$$\phi_2(t) = 2\pi K_v \int_0^t v(t)\, dt \quad \text{--- (4)}$$

K_v is the frequency sensitivity factor of VCO.

The function of PLL is to adjust the angle $\phi_2(t)$ so that it equals $\phi_1(t)$.

Let $e(t)$ be the output of Multiplier.

$$e(t) = s(t) \cdot r(t). \quad \text{--- (5)}$$

(1) & (3) in (5)

$$e(t) = A_c \sin\left(2\pi f_c t + \phi_1(t)\right) \cdot A_v \cos\left(2\pi f_c t + \phi_2(t)\right) \quad \text{--- (6)}$$

$$\left\{ \sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right] \right\}$$

$$e(t) = \frac{A_c A_v}{2}\left[\sin\left(4\pi f_c t + \phi_1(t) + \phi_2(t)\right)\right] + \frac{A_c A_v}{2}\left[\sin\left(\phi_1(t) - \phi_2(t)\right)\right] \quad \text{--- (7)}$$

eqn 7 has two terms

1^{st} term is a high frequency component defined as double freq term.

$$= \frac{A_c A_v}{2} \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$$

$$= K_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] \quad — (8)$$

where $K_m = \frac{1}{2}$ is the multiplier gain.

2^{nd} term is a low frequency component defined as difference freq term

$$K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)] \quad — (9)$$

Loop filter is designed to suppress the high frequency components hence discard the high (double) frequency term from 7.

$$\therefore e(t) = K_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$

$$e(t) = K_m A_c A_v \sin[\phi_e(t)] \quad — (10)$$

where $\phi_e(t)$ is the phase error, defined as

$$\phi_e(t) = \phi_1(t) - \phi_2(t) \quad — (11)$$

The error signal e(t) is passed through a loop filter with impulse response h(t). The o/p of filter is $v(t)$, defined as

$$v(t) = e(t) * h(t)$$

$$v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau \quad — (12)$$

Substitute eqn (10) in (12)

$$v(t) = \int_{-\infty}^{\infty} K_m A_c A_v \sin(\phi_e(\tau))\, h(t-\tau)\, d\tau \quad (13)$$

Block diagram representation of eqn (13) is

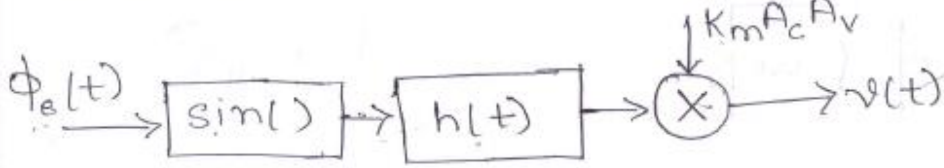


Fig (2)

But $\phi_e(t) = \phi_1(t) - \phi_2(t)$

Rewriting the Block diagram [Fig (2).

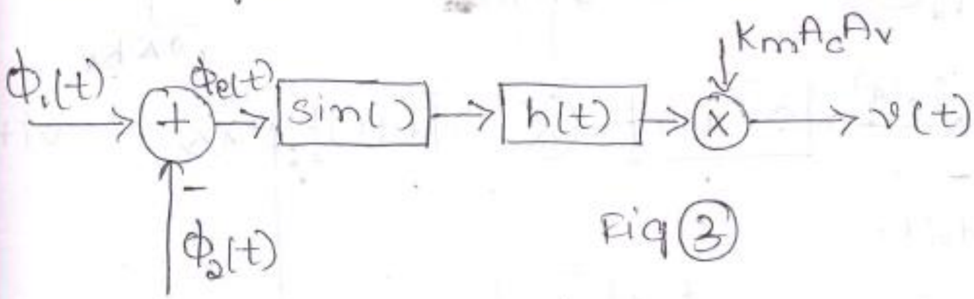


Fig (3)

But $\phi_2(t) = 2\pi K_v \int_0^t v(t)\, dt.$

Rewriting block diagram (3).

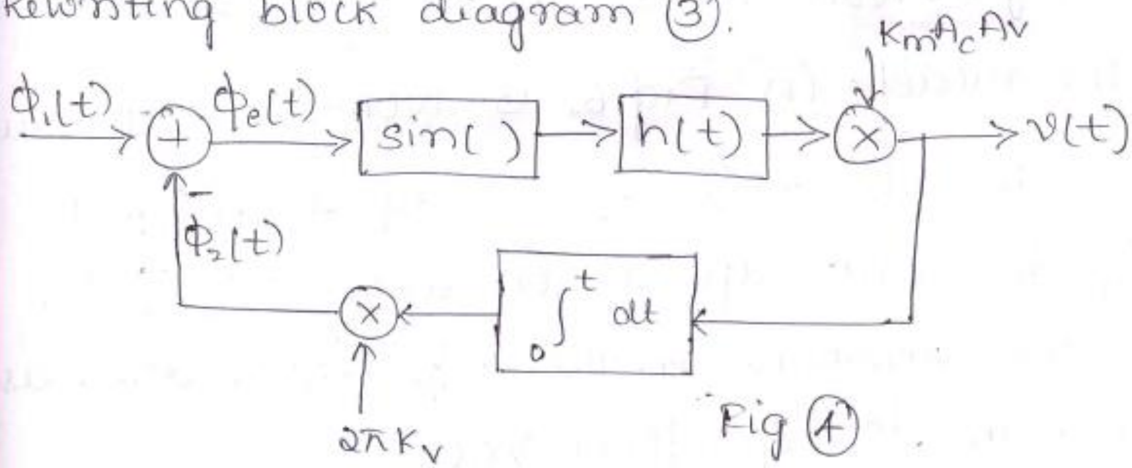


Fig (4)

Fig ④. Block diagram can be redrawn as

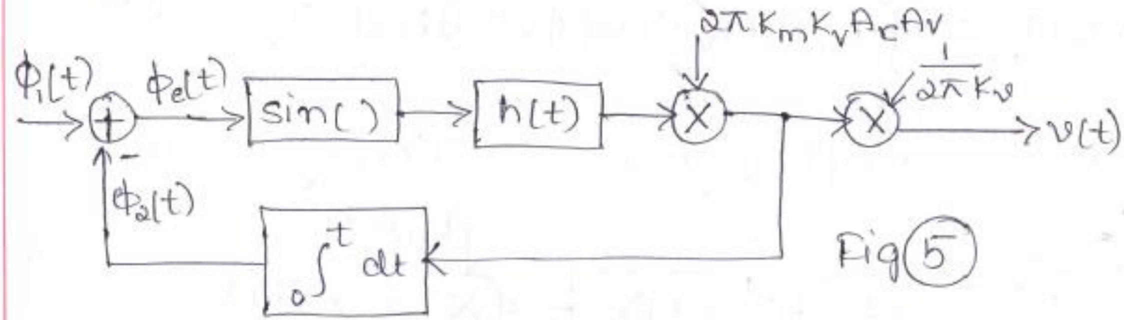


Fig ⑤

Let $K_0 = K_m K_v A_c A_v$

K_0 is a loop gain parameter.

Redraw fig ⑤

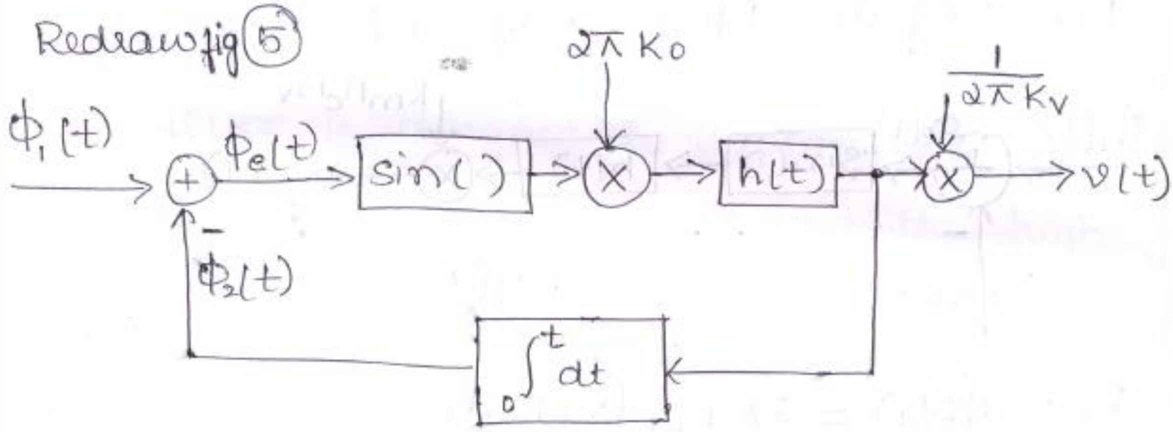


Fig 6: Non Linear Model of PLL

The model in Fig 6, is Non-linear, because the the o/p v(t) doesnot depend only on i/p but also depends on the o/p of VCO. i.e, the variations in the o/p signal depends on both i/p & o/p of VCO.

When the phase error $\phi_e(t)$ is zero, the phase locked loop is said to be in phase lock.

When $\phi_e(t)$ is small compared to 1 rad, then

$$\sin(\phi_e(t)) \simeq \phi_e(t) \quad \text{—— (14)}$$

$$\phi_e(t) \text{ is small} \Rightarrow \phi_e(t) = 0$$

$\therefore$ From eqn

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$0 = \phi_1(t) - \phi_2(t)$$

$$\therefore \phi_1(t) = \phi_2(t) \quad \text{—— (15)}$$

WKT,

$$\phi_2(t) = 2\pi K_v \int_0^t v(t)\,dt$$

$$v(t) = \frac{1}{2\pi K_v} \frac{d\phi_2(t)}{dt}$$

as $\phi_1(t) = \phi_2(t)$,

$$v(t) = \frac{1}{2\pi K_v} \frac{d\phi_1(t)}{dt}$$

WKT,

$$\phi_1(t) = 2\pi K_f \int_0^t m(t)\,dt$$

$$\therefore v(t) = \frac{1}{2\pi K_v} 2\pi K_f \, m(t)$$

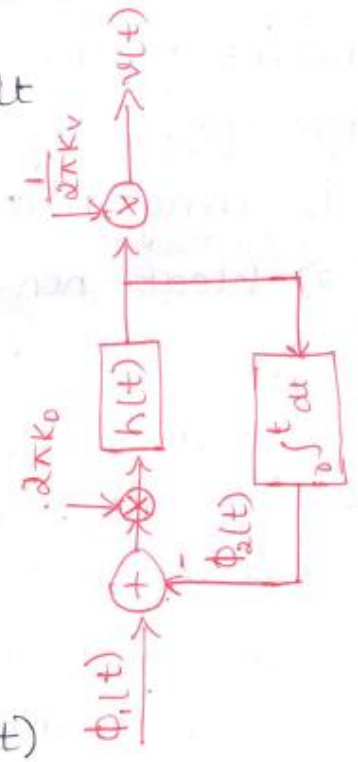


Fig 7: Linear model of PLL

$$\therefore v(t) = \frac{K_f}{K_v} m(t) \quad \text{— (16)}$$

The linear model of PLL is shown in fig 7.

eqn (16) states that when the loop operates in its phase locked ~~loo~~ mode, the o/p $v(t)$ is the original msg signal except for the scale factor $\frac{K_f}{K_v}$; Frequency demodulation of the incoming FM signal $s(t)$ is thereby accomplished.

4.

VSB modulation

Vestigial side band modulation (VSB) is a types of amplitude modulation in which one sideband and a part (vestige) of other side-band are transmitted.

VSB can be done for a higher frequency signal where there is no energy gap at the origin. It allows to have a non zero transition band, so this allows the use of non- ideal filter.

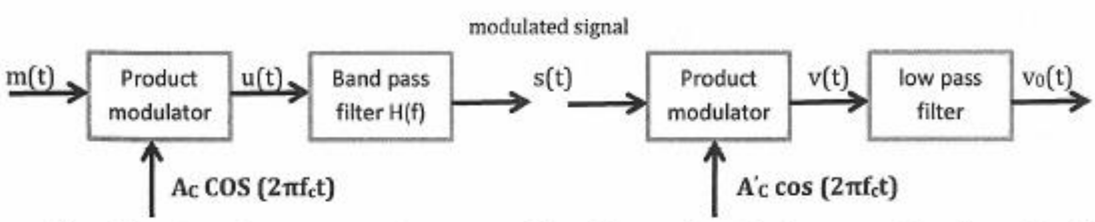


Fig: *Filtering scheme at transmitter* fig: *coherent detection for recovering of msg signal*

The generation of VSB includes product modulator, local oscillator and a band shaping filter with a transfer function **H(f)** as shown in the figure. The output of the product modulator **u(t)** is an frequency shifted signal which is passed through the filter to get modulated signal **s(t).**

$$U(t) = m(t) \cdot c(t)$$

$$U(t) = m(t).\, A_c \cos(2\pi f_c t)$$

Taking fourier transform

$$U(f) = \frac{A_c}{2}[M(f-f_c) + M(f+f_c)] \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The demodulated output s(t) in frequency domain is, **S(f)= u(f).H(f)**

using eq (1) $S(f) = \frac{A_c}{2}[M(f-f_c) + M(f+f_c)]\ H(f) \ldots\ldots\ldots(2)$

now $S(f-f_c) = \frac{A_c}{2}[M(f-f_c-f_c) + M(f-f_c+f_c)]\ H(f-f_c)$

$$S(f-f_c) = \frac{A_c}{2}[M(f-2f_c) + M(f)]\ H(f-f_c) \ldots\ldots\ldots\ldots(3)$$

$|||^{ly}$ $S(f+f_c) = \frac{A_c}{2}[M(f+f_c-f_c) + M(f+f_c+f_c)]\ H(f-f_c)$

$$S(f+f_c) = \frac{A_c}{2}[M(f) + M(f+2f_c)]\ H(f+f_c) \ldots\ldots\ldots\ldots(4)$$

In the demodulation process a coherent detector is used where modulated signal s(t) is multiplied with a locally generated sinusoidal wave $A'_c \cos(2\pi f_c t)$, which is synchronous with carrier wave used in modulation process, in both frequency and phase. Thus the output of product modulator is , $v(t) = A'_c \cos(2\pi f_c t)\ s(t)$, transforming this into frequency domain gives, $v(f) = \frac{A'_c}{2}[s(f-f_c) + s(f+f_c)] \ldots\ldots(5)$

Using eq (3)&(4) in eq (5) we get

$$v(f) = \frac{A'_c}{2}\left\{\frac{A_c}{2}[M(f-2f_c) + M(f)]\ H(f-f_c) + \frac{A_c}{2}[M(f) + M(f+2f_c)]\ H(f+f_c)\right\}$$

$$v(f) = \frac{A_cA'_c}{4}M(f-2f_c)\ H(f-f_c) + \frac{A_cA'_c}{4}M(f)\ H(f-f_c) + \frac{A_cA'_c}{4}M(f)\ H(f+f_c) + \frac{A_cA'_c}{4}M(f+2f_c)\ H(f+f_c)$$

Taking common,

$$v(f) = \frac{A_cA'_c}{4}M(f)\,[H(f-f_c) + H(f+f_c)] + \frac{A_cA'_c}{4}\{M(f-2f_c)\ H(f-f_c) + M(f+2f_c)\ H(f+f_c)\} \ldots\ldots(6)$$

the second term indicates the high frequency component, which is removed by the low pass filter to produce an output $v_0(t)$, so the spectrum remaining component's are

$$v_0(f) = \frac{A_cA'_c}{4}M(f)\,[H(f-f_c) + H(f+f_c)] \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

for a distortion less reproduction of original message signal at detector , the transfer function H(f) must satisfy condition $[H(f-f_c) + H(f+f_c)] = 2H(f_c)$

where $H(f_c)$, the value of H(f) @ $f=f_c$, is a constant equal to 0.5

thus $[H(f-f_c) + H(f+f_c)] = 2(0.5) = 1,$ $\quad -\omega \le f \le \omega$

then output will be $v_0(f) = \frac{A_cA'_c}{4}M(f)$

In time domain it is $v_0(t) = \frac{A_cA'_c}{4}M(t)$ hence output is a scaled value of msg signal

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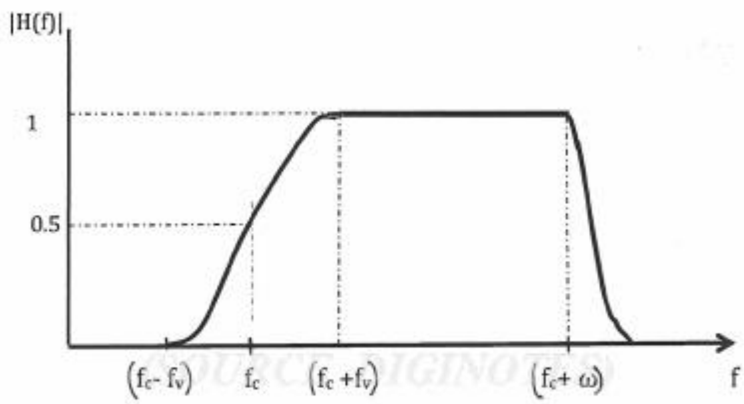

fig: amplitude response of VSB filter for positive frequency

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Single Side Band Suppressed Carrier (SSB-SC) Modulation

The transmission bandwidth of standard AM as well as DSB-SC modulated wave is 2 ω Hz i.e., twice the message bandwidth ω. Therefore, both these systems are **bandwidth inefficient systems** .

In both these systems, one half of the transmission bandwidth is occupied by the upper sideband (USB) and the other half is occupied by the lower sideband (LSB) as shown in fig.1 .

The information contained in the USB is exactly identical to that carried by the LSB . So, by transmitting both the sidebands we are transmitting the same information twice .

Hence, we can transmit only one sideband (USB or LSB) without any loss of information. So, it is possible to suppress the carrier and one sideband completely to conserve the bandwidth.

When only one sideband is transmitted, the modulation is called as **single sideband modulation** . It is also known as SSB or SSB-SC modulation .

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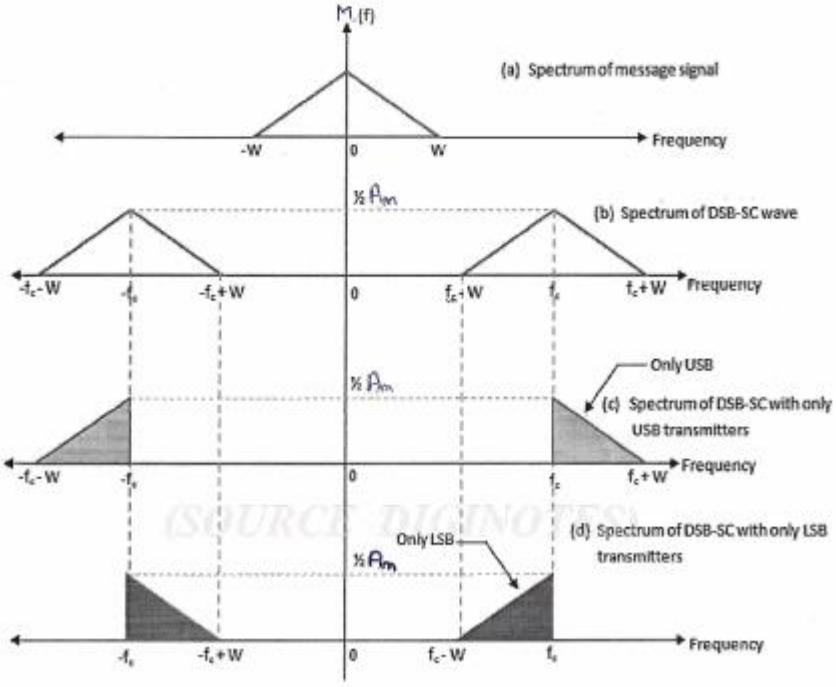


Fig 1

The generation of a SSB signal is a simple process, we first generate a DSB-SC and then apply an ideal band pass filter with a cutoff frequencies of f_c **and** $f_c + \omega$ for the upper side band. But construction of ideal filter is very difficult.

An **analog voice signal** has very little energy at **low frequencies (<300 Hz),** that is an energy gap in the spectrum near origin as shown in figure 2.a , the filter response is as in fig 2.b, the resulting band pass spectra is as shown in fig 2.c,

The filter must only satisfy the following :

1. The desired sideband lies inside the pass-band of filter.
2. The unwanted sideband lies inside the stop-band of the filter.

The separation between passband and stop band is twice the lowest frequency of msg signal ($2f_a$). this indicate the non-zero transition bandwidth, so design of filter is simplified , the analysis of SSB signal is done by using Hilbert transform. The demodulation is done by coherent detection so to provide synchronization a low power pilot carrier is used or an stable oscillators are used in both transmitter and receiver for generating carriers.

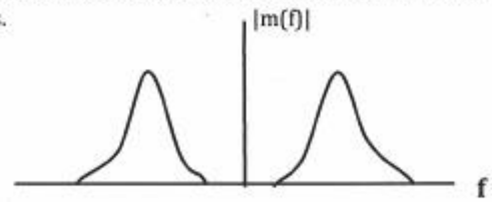
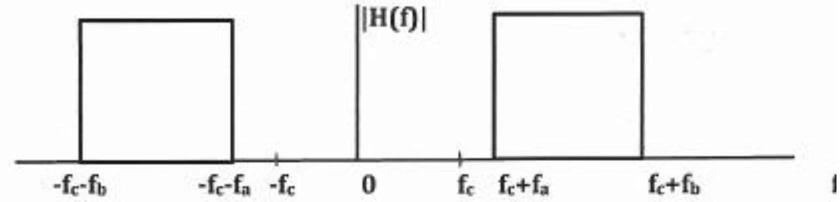
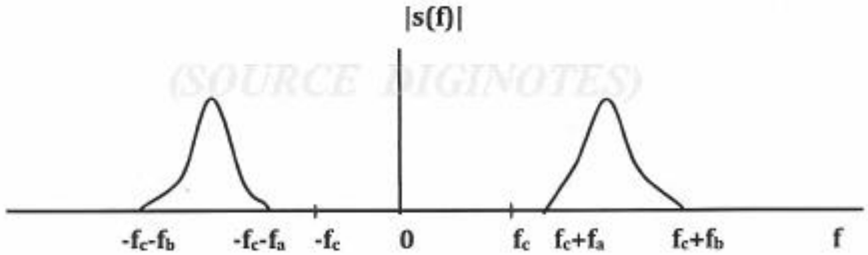
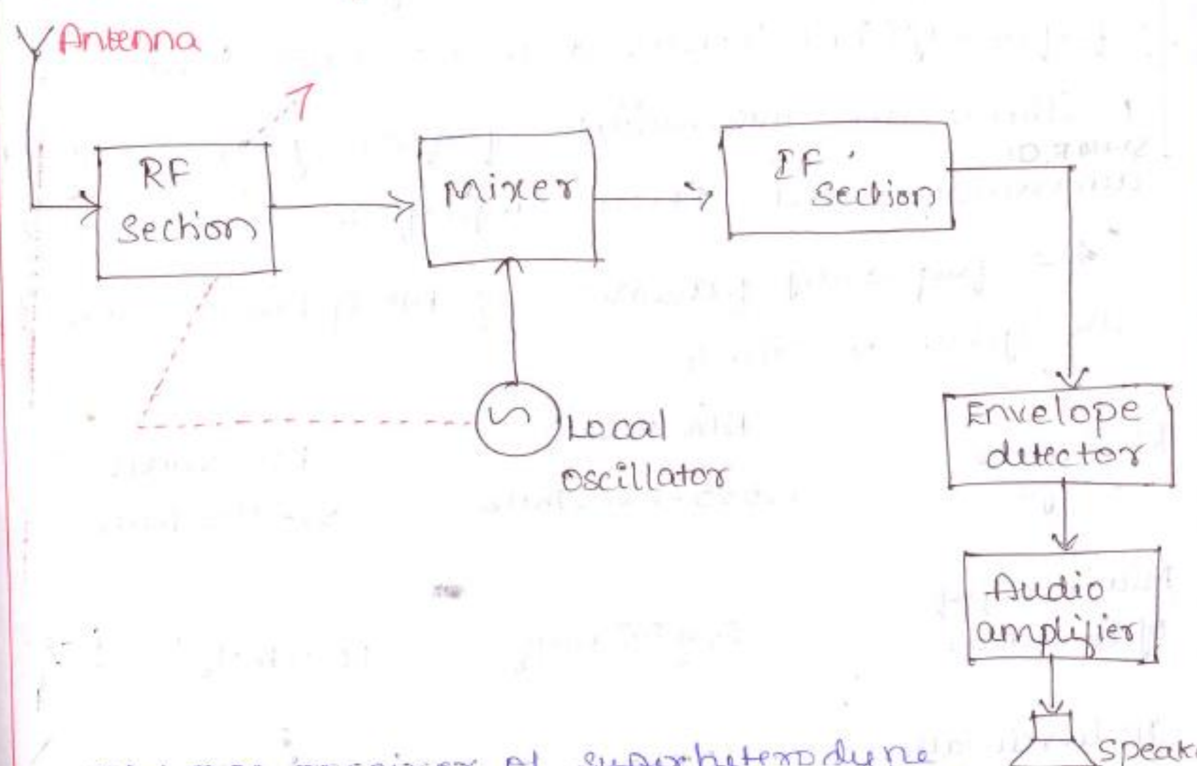


Fig 2.a

	 *Fig 2.b idealized frequency response of bandpass filter*  *Fig 2.c spectrum of SSB signal containing upper side band*
5b	Superheterodyne Receivers  Fig. AM receiver of superheterodyne type.

Superheterodyne receivers is a special type of receiver that fulfils several functions such as.

a) carrier frequency tuning
b) Filtering
c) Amplification.

The purpose of carrier frequency tuning is to select the desired signal.
Filtering is required to separate the desired signal from other modulated signals.
Amplification to compensate for the loss of signal power in the course of transmission.

Basically receivers consists of a radio-frequency (RF) section, a mixer and local oscillator, an intermediate frequency (IF) section, demodulator and Power amplifier.

The frequency parameters of AM & FM receivers are given in Table.

	AM radio	FM radio
RF carrier range	0.535-1.605 MHz	88-108 MHz
Mid band freq of IF section	0.455 MHz	10.7 MHz
IF bandwidth	10 KHz	200 KHz

The incoming amplitude modulated wave is picked by the receiving antenna.

The received signals are amplified in the RF section that is tuned to carrier frequency of the incoming wave. To avoid image interference, employ highly selective stages in the RF section in order to favor the desired signal & discriminate against the undesired signal.

The combination of Mixer and local oscillator (of adjustable frequency) provides a heterodyning junction, wherby the incoming signal is converted to a predermined intermediate frequency, usually lower than the incoming carrier frequency.

The result of heterodyning is to produce an intermediate frequency carrier defined by

$$f_{IF} = f_{RF} - f_{LO}$$

where f_{LO} is the frequency of the Local Oscillator & f_{RF} is the carrier freq. of incoming RF signal.

f_{IF} is refered as Intermediate frequency, because the signal is neither at the original input frequency nor at the final baseband frequency.

	The mixer – Local oscillator combination is referred as first detector & demodulator is referred as second detector. The IF section consists of one or more stages of tuned amplification which provides most of the amplification & selectivity in the receiver. The o/p of IF section is applied to a de-modulator to recover the msg signal. If coherent detection is used then a coherent signal source must be provided in the receiver The final operation in the receiver is the power amplification of the recovered msg signal.
6	$s(t) = 10\sin(5 \times 10^8 t + 4\sin(1520t)$. Determine (i) Carrier wave (ii) modulation index (iii) Frequency deviation 9iv) Power dissipated by FM wave across5Ω i) Carrier wave is $10\sin(5 \times 10^8 t)$. ii) Modulation index $\beta = 4$ iii) Frequency deviation = $\beta * f_m = 4 * \frac{1520}{2*pi} = 968.14\ Hz$ iv) Power dissipated by 5Ω resistor =$\frac{A_C^2}{2*R} = \frac{100}{2*5} = 10W$

7

Statement of Sampling Theorem.

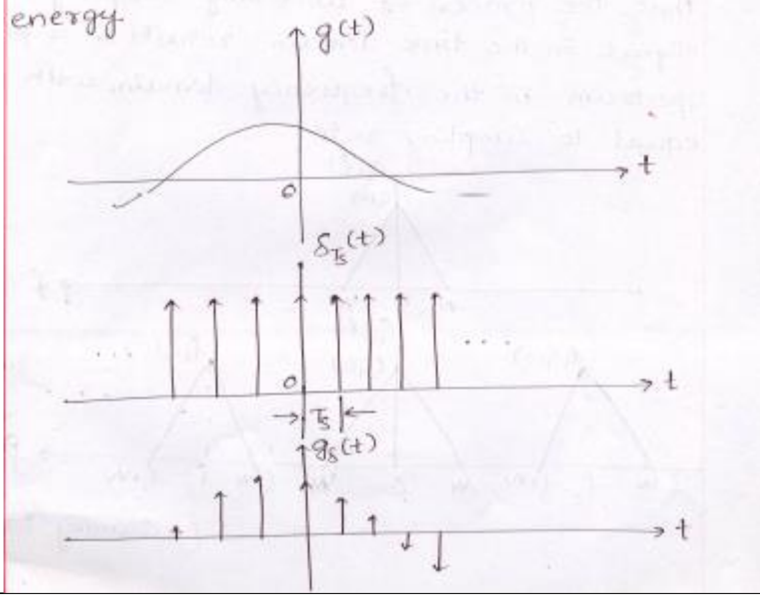
1. A band limited Signal of finite Energy which has all frequencies lesser than ω Hertz, it completely described by Specifying the Values of the Signal at instants of time Separated by $\frac{1}{2\omega}$ Seconds.

2. A band limited Signal of finite energy which has frequency Components lesser than ω Hertz, may be completely recovered from the knowledge of its Samples taken at the rate of 2ω Samples per Sec.

NOTE: A Continuous time Signal Can be Completely represented in its Samples and recovered back if the Sampling frequency is twice of the highest frequency Content of the Signal

i.e. $f_s \geq 2\omega$.

Consider an analog signal g(t) of finite energy



T_s - sampling period

$f_s = \frac{1}{T_s}$, sampling frequency

$g_s(t)$ - sampled version of $g(t)$

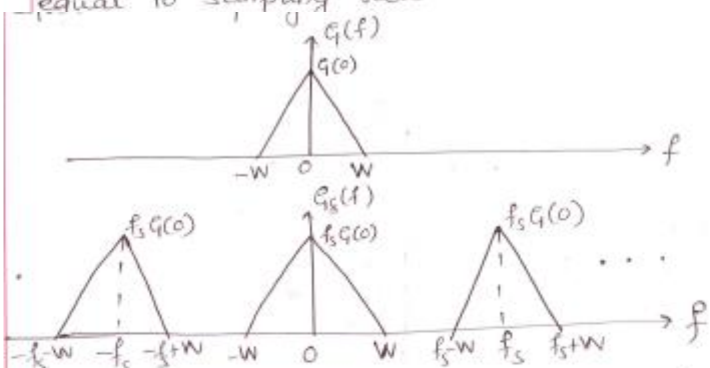
$$g_s(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad \cdots \quad (1)$$

$$FT\{g_s(t)\} = G(f) * \frac{1}{T_s} \sum_{K=-\infty}^{\infty} \delta\left(f - \frac{n}{T_s}\right)$$

ie, $$G_s(f) = G(f) * f_s \sum_{K=-\infty}^{\infty} \delta(f - nf_s)$$

$$= f_s \sum_{K=-\infty}^{\infty} G(f - nf_s) \quad \cdots \quad (2)$$

Thus, the process of uniformly sampling the signal in the time domain results in a periodic spectrum in the frequency domain, with period equal to sampling rate.



(assuming $f_s > 2W$.)