CMR INSTITUTE OF TECHNOLOGY	USN	CMRIT

InternalTest– III

Sub:	Control Systems					Code:	15EC43
Date:	14/ 05 / 2019	Duration:	90 mins	Max Marks:	50	Sem: IV	Branch: ECE-B

Answer any FIVE FULL Questions

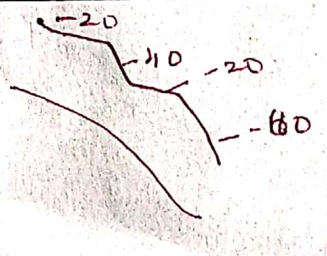
		Marks	OBE CO	OBE RBT
1	Represent the electrical circuit shown in Fig.1 by a state mode. $A=\begin{bmatrix}0 & 0 & -1/L_1\\ 0 & -R_1/L_2 & 1/L_2\\ 1/C & -1/C & 0\end{bmatrix}$ $B=\begin{bmatrix}1/L_1\\0\\1\end{bmatrix}$ $C=[0\ \ R_1\ \ 0]$	[10]	CO6	L3
2	Obtain the state transition matrix for a given system matrix $A=\begin{bmatrix}0 & 1\\ -3 & -4\end{bmatrix}$ $=\begin{bmatrix}1.5e^{-t}-0.5e^{-3t} & 0.5e^{-t}-0.5e^{-3t}\\ -1.5e^{-t}+1.5e^{-3t} & -0.5e^{-t}+1.5e^{-3t}\end{bmatrix}$	[10]	CO6	L3
3 (a)	List the properties of state transition matrix.	[2]	CO6	L1
(b)	Determine the state model for a system whose differential equation is given by $$\frac{d^3y(t)}{dt^3}+4\frac{d^2y(t)}{dt^2}+7\frac{dy(t)}{dt}+2y(t)=5U(t)$$ $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -7 & -4\end{bmatrix}$ $B=\begin{bmatrix}0\\0\\5\end{bmatrix}$ $C=[1\ \ 0\ \ 0]$	[8]	CO6	L3
4	The forward transfer function of a control system is given by $$G(S)H(S)=\frac{K}{S(S+1)(0.1S+1)}$$ Draw the asymptotic Bode plot and hence find the value of K for which (i) The gain margin is 10 dB. ⇒ K = 3.16 (ii) The phase margin is 24°. K = 2.511 PM = 180 − 140 = 40°; GM = 20 dB	[10]	CO5	L3
5	Estimate the transfer function from Bode plot shown in Fig.5. (Slopes are in dB/dec). K = 100 $T.F=\frac{100S(1+0.001S)^2}{(1+S)(1+0.1S)}$	[10]	CO5	L3
6	For a given unity feedback system $G(S)=\frac{242(S+5)}{S(S+1)(S^2+5S+121)}$, sketch the Bode Plot and find Gain margin, Phase Margin.	[10]	CO5	L3

ϕ → −90 to −270°

$\omega_{gc}=3$ GM = 17

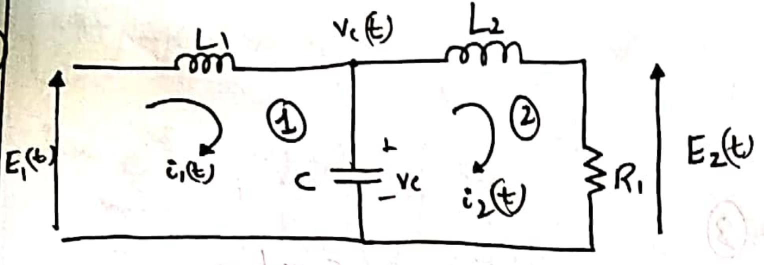
$\omega_{pc}=10.1$ PM = 42

$\phi_{gc}=138°$



7	For a feedback control system $G(S)H(S) = \frac{K}{S(S+2)(S+10)}$ Sketch the Nyquist Plot and hence calculate the range of values of K for stability. $P = 0$  Mag: $\frac{K}{\omega\sqrt{4+\omega^2}\sqrt{100+\omega^2}}$ $\phi = -90 - \tan^{-1}\omega/2 - \tan^{-1}\omega/10$	[10]	CO5	L3
8	For a feedback control system $G(S)H(S) = \frac{40}{(S+4)(S^2+2S+2)}$ Find Gain Margin and stability from Nyquist plot.	[10]	CO5	L3

Internal Assessment test - III

1) 

There are three energy storing elements ; L_1, L_2 & C

∴ no. of state variables = 3.

let the state variable be $x_1(t)$; $x_2(t)$ and $x_3(t)$.

here; $x_1(t) = i_1(t) \rightarrow \{L_1\}$.

$x_2(t) = i_2(t) \rightarrow \{L_2\}$.

$x_3(t) = V_c \rightarrow \{c\}$.

applying KVL in loop 1;

$$\Rightarrow E_1 - L_1 \frac{di_1}{dt} - \frac{1}{C}\int i_1 - i_2 \, dt = 0$$

$$\Rightarrow E_1 = L_1 \cdot \frac{di_1}{dt} + V_c$$

$$\Rightarrow E_1 = L_1 \dot{x}_1 + x_3$$

$$\Rightarrow \dot{x}_1 = -\frac{1}{L_1} x_3 + \frac{1}{L_1}(E_1) \quad \text{—(1)}$$

applying KVL in loop 2;

$$\Rightarrow L_2 \frac{di_2}{dt} + i_2 R_1 + \frac{1}{C}\int i_2 - i_1 \, dt = 0$$

$$\Rightarrow L_2 \cdot \dot{x}_2 + x_2 R_1 + V_c = 0$$

$$\Rightarrow L_2 \dot{x}_2 = -R_1 x_2 - x_3$$

$$\Rightarrow \dot{x}_2 = -\frac{R_1}{L_2} x_2 - \frac{1}{L_2} x_3 \quad \text{—(2)}$$

applying KCL at node $V_c(t)$;

$$\Rightarrow -i_1 + i_2 + C\frac{dV_c}{dt} = 0$$

$$\Rightarrow i_1 = C\cdot\dot{x}_3 + i_2$$

$$\Rightarrow \dot{x}_3 = \frac{1}{C}(x_1) - \frac{1}{C}(x_2) \quad \text{—(3)}$$

from eq (1), (2) & (3); ∴ they're time dependent;

wogd,

$$\dot{x}_1(t) = -\frac{1}{L_1}x_3(t) + \frac{1}{L_1}E_1(t)$$

$$\dot{x}_2(t) = -\frac{R_1}{L_2}x_2(t) - \frac{1}{L_2}x_3(t)$$

$$\dot{x}_3(t) = \frac{1}{C}x_1(t) - \frac{1}{C}x_2(t)$$

hence; we get;

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 0 & -1/L_1\\ 0 & -R_1/L_2 & -1/L_2\\ 1/C & -1/C & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}1/L_1\\ 0\\ 0\end{bmatrix}E_1(t) \quad \text{— (A)}$$

at o/p response;

$$E_2(t) = i_2 \cdot R_1$$

$$= x_2 \cdot R_1$$

$$\therefore E_2 = \begin{bmatrix}0 & R_1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \quad \text{—(B)}$$

eq (A) & (B) are the two state equation; together they represent the state model.

2) State transition matrix

[WKT; zero i/p response => ZIR = $e^{At} \cdot x(0)$]

here; $e^{At} = \phi(t) = L^{-1}\{[SI-A]^{-1}\}$

Given: $A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$

WKT, $s = \begin{bmatrix} s & s \\ s & s \end{bmatrix}$

& $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; identity matrix

$\therefore SI - A = \begin{bmatrix} s & s \\ s & s \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$

$= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$

$= \begin{bmatrix} s & -1 \\ 3 & s+4 \end{bmatrix}$ //

now; $[SI-A]^{-1} = \dfrac{adj.[SI-A]}{|SI-A|}$

here; adj $[SI-A]$ = transpose of co-factor matrix $= \begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}$

$|SI-A| = s(s+4) - (3\times -1)$

$= s(s+4) + 3$

$= s^2 + 4s + 3$ //

$$\therefore [SI-A]^{-1} = \begin{bmatrix} \frac{s+4}{s^2+4s+3} & \frac{1}{s^2+4s+3} \\ \frac{-3}{s^2+4s+3} & \frac{s}{s^2+4s+3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+4}{(s+3)(s+1)} & \frac{1}{(s+3)(s+1)} \\ \frac{-3}{(s+3)(s+1)} & \frac{s}{(s+3)(s+1)} \end{bmatrix}$$

partial fraction for each term;

$$\frac{s+4}{(s+3)(s+1)} = \frac{A}{(s+3)} + \frac{B}{(s+1)}$$

$\Rightarrow s+4 = A(s+1) + B(s+3)$

$\Rightarrow s+4 = (A+B)s + (A+3B)$

$\therefore A+B = 1$

$A+3B = 4$

upon solving the simultaneous eq.

we get,

$A = -1/2$

$B = 3/2$

$$\therefore \frac{s+4}{(s+3)(s+1)} = \frac{-1/2}{(s+3)} + \frac{3/2}{(s+1)}$$

$$= \frac{1}{2}\left[\frac{3}{(s+1)} - \frac{1}{(s+3)}\right]$$

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$$\frac{1}{(s+3)(s+1)} = \frac{A}{(s+3)} + \frac{B}{(s+1)}$$

$\Rightarrow 1 = A(s+1) + B(s+3)$

$\Rightarrow 1 = (A+B)s + (A+3B)$

$\therefore A+B = 0$

$A+3B = 1$

wogd $A = -1/2$

$B = 1/2$

$$\therefore \frac{1}{(s+3)(s+1)} = \frac{-1/2}{(s+3)} + \frac{1/2}{(s+1)}$$

$$= \frac{1}{2}\left[\frac{1}{s+1} - \frac{1}{s+3}\right]$$

$$\frac{-3}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1}$$

$\Rightarrow -3 = (A+B)s + (A+3B)$

$\therefore A+B = 0$

$A+3B = -3$

wogd, $A = 3/2$

$B = -3/2$

$$\therefore \frac{-3}{(s+3)(s+1)} = \frac{3/2}{(s+3)} + \frac{-3/2}{(s+1)}$$

$$= \frac{1}{2}\left[\frac{3}{(s+3)} - \frac{3}{(s+1)}\right]$$

$$\frac{s}{(s+3)(s+1)} = \frac{A}{(s+3)} + \frac{B}{(s+1)}$$

$\Rightarrow s = (A+B)s + (A+3B)$

$\therefore A+B = 1$

$A+3B = 0$

wogd

$A = 3/2$

$B = -1/2$

$$\therefore \frac{s}{(s+3)(s+1)} = \frac{3/2}{(s+3)} + \frac{-1/2}{(s+1)}$$

$$= \frac{1}{2}\left[\frac{3}{(s+3)} - \frac{1}{(s+1)}\right]$$

$\therefore$ wogd,

$$[sI-A]^{-1} = \frac{1}{2}\begin{bmatrix} \frac{3}{s+1} - \frac{1}{s+3} & \frac{1}{s+1} - \frac{1}{s+3} \\ \frac{3}{s+3} - \frac{3}{s+1} & \frac{3}{s+3} - \frac{1}{s+1} \end{bmatrix}$$

PTO

now upon inverse laplace transform

$\Rightarrow$ we get, $\phi(t) = L^{-1}\left[\{sI - A\}^{-1}\right]$

$$= L^{-1}\left[\frac{1}{2}\begin{bmatrix} \frac{3}{s+1} - \frac{1}{s+3} & \frac{1}{s+1} - \frac{1}{s+3} \\ \frac{3}{s+3} - \frac{3}{s+1} & \frac{3}{s+3} - \frac{1}{s+1} \end{bmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} 3 \cdot e^{-t} - e^{-3t} & e^{-t} - e^{-3t} \\ 3(e^{-3t} - e^{-t}) & 3e^{-3t} - e^{-t} \end{bmatrix}$$

$\left[\because L\{e^{-\alpha t}\} = \frac{1}{s+\alpha}\right]$

$$= \begin{bmatrix} \frac{3}{2}e^{-t} - \frac{e^{-3t}}{2} & \frac{e^{-t}}{2} - \frac{e^{-3t}}{2} \\ \frac{3}{2}e^{-3t} - \frac{3}{2}e^{-t} & \frac{3}{2}e^{-3t} - \frac{e^{-t}}{2} \end{bmatrix}$$

$\rightarrow$ state transition matrix

PTO

Q.3. Determine the state model using phase variables if the system is described by the determine wheather the system is stable or NOT differential eqn :-

$$\frac{d^3 y(t)}{dt^3} + 4\frac{d^2 y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 2y(t) = 5u(t)$$

and Draw the state diagram;

→ STATE VARIABLES ARE;

$$x_1(t) = y(t)$$

$$x_2(t) = \frac{dy(t)}{dt} = \dot{x}_1(t)$$

$$x_3(t) = \frac{d^2y(t)}{dt^2} = \dot{x}_2(t)$$

$$\dot{x}_3(t) = \frac{d^3y(t)}{dt^3}$$

substitute state variables in the differential eqn;

$$\dot{x}_3(t) + 4x_3(t) + 7x_2(t) + 2x_1(t) = 5u(t)$$

$$\dot{x}_3(t) = 5u(t) - 2x_1(t) - 7x_2(t) \cdot 4x_3(t)$$

$$\dot{x}_2(t) = x_3(t)$$

$\dot{x}_1(t) = x_2(t)$

$y(t) = x_1(t)$

The state model eqn we've;

$\dot{X}(t) = Ax(t) + BU(t)$

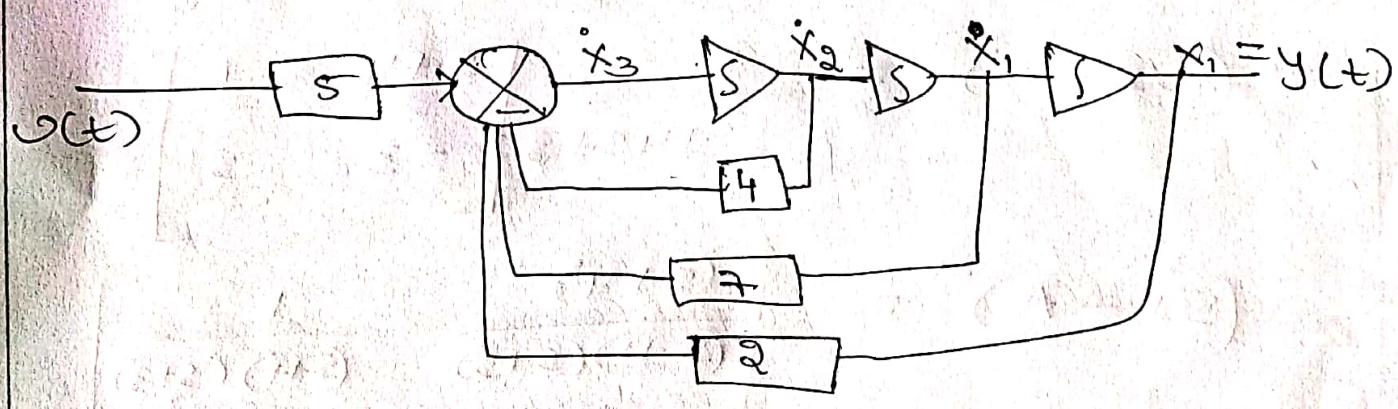
$y(t) = C(x(t)) + DU(t).$

so;

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -7 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

STATE DIAGRAM:-



5 4) Open loop TF of system is $G(s) = \frac{K}{s(1+s)(1+0.1s)}$. Determine values of K

(i) $GM = 10\,dB$

(ii) $PM = 24^\circ$.

Step 1: Consider K=1

$$\Rightarrow G(s) = \frac{1}{s(1+s)(1+0.1s)}$$

$$\therefore G(j\omega) = \frac{1}{(j\omega)(1+j\omega)(1+0.1j\omega)} = \frac{1}{j\omega} \cdot \frac{1}{(1+j\omega)} \cdot \frac{1}{(1+0.1j\omega)}$$

$\frac{1}{j\omega} \rightarrow$ NIL; $\frac{1}{1+j\omega} \rightarrow \omega_{c1} = 1$; $\frac{1}{1+0.1j\omega} \rightarrow \omega_{c2} = \frac{1}{0.1} = 10$

Step 2: magnitude plot

Terms	ω_c (rad/sec)	slope dB/decade	change in slope dB/decade
$1/j\omega$	—	−20 dB	−20
$\frac{1}{1+j\omega}$	1	−20dB	−40
$\frac{1}{1+0.1j\omega}$	10	−20 dB	−60

step 3: Determining starting point of magnitude plot.

$$20\log|Y(j\omega)|_{@\omega=0.1} = 20\log\left|\frac{1}{j\omega}\right|_{@\omega=0.1}$$
$$= 20\log 10 = 20\,dB$$

step 4: Phase plot

$$\phi = -90 - \tan^{-1}(\omega) - \tan^{-1}(0.1\omega)$$

ω	ϕ (degrees)
0.1	-96.28
1	-140.71
2	-164.74
3	-178.26
5	-195.25
10	-219.28
15	-232.49
20	-240.57

$GM = -1 \times 20\log|G(j\omega)|_{@\omega=\omega_{pc}}$

$= -1 \times -20$

$= 20$

$PM = 180^\circ + \phi_{gc} = 180^\circ - 140^\circ$

$= 40^\circ$

(i) $GM = 10dB$

$\Rightarrow 20\log K = 10$

$\therefore K = 10^{1/2}$

$= 3.162$

(ii) $PM = 24^\circ$

$\Rightarrow \phi_{gc} = 24^\circ - 180^\circ$

$= -156^\circ$

hence; $20\log K = 8$

$K = 10^{0.4}$

$= 2.51$

→ Dec 2017 – Jan 2018

2) The open loop TF of a system is $G(s) = \dfrac{K}{s(1+0.5s)(1+0.2s)}$

find K so that (i) Gm is 6dB

(ii) $PM = 25^\circ$

step1: let K=1

$$G(s) = \frac{1}{s(1+0.5s).(1+0.2s)}$$

$$\Rightarrow G(j\omega) = \frac{1}{(j\omega)(1+0.5j\omega)(1+0.2j\omega)} = \frac{1}{j\omega} \times \frac{1}{1+0.5j\omega} \times \frac{1}{1+0.2j\omega}$$

step2: magnitude plot

Terms	ω_c (rad/sec)	slope dB/dec.	change in slope dB/dec
$1/j\omega$	—	−20	−20
$1/(1+0.5j\omega)$	2	−20	−40
$1/(1+0.2j\omega)$	5	−20	−60

step3: starting point of magnitude plot

$$20\log_{10}|Y(j\omega)|_{@\omega=0.1} = 20\log_{10}\left|\frac{1}{\omega}\right|_{@\omega=0.1}$$

$$= 20dB$$

step4: phase plot

$$\phi = -90 - \tan^{-1}(0.5\omega) - \tan^{-1}(0.2\omega)$$

ω	ϕ (degree)
0.1	−94.0
0.5	−109.74
1	−127.87
3	−172.27
5	−203.198
7	−218.516
9	−228.416
10	−232.125
15	−243.9

$$GM = -1 \times 20\log|G(j\omega)|_{@\omega=\omega_{pc}}$$

$$= -1 \times -15$$

$$= 15$$

$$PM = 180^\circ + \phi_{gc}$$

$$= 180^\circ - 130^\circ$$

$$= 50^\circ$$

(i) GM = 6dB

20 log K = 9

$\therefore K = 10^{0.45}$

$= 2.818$

(ii) PM = 25°

$= 180^\circ + \phi_{gc}$

$\therefore \phi_{gc} = 25 - 180^\circ$

$= -155^\circ$

$\therefore 20 \log K = 6$

$\Rightarrow \log K = 0.3$

$\therefore K = 10^{0.3} = 1.99$

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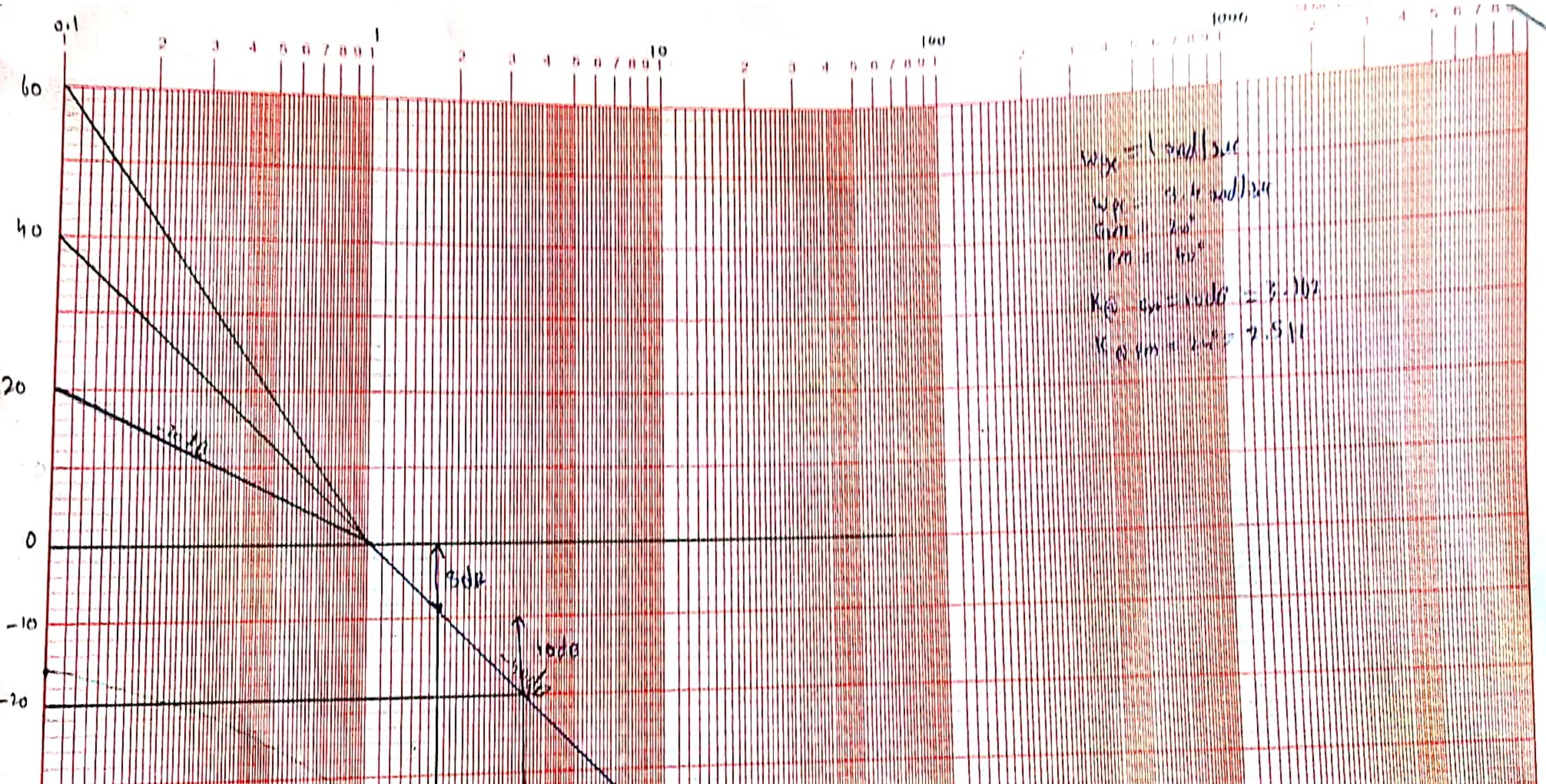
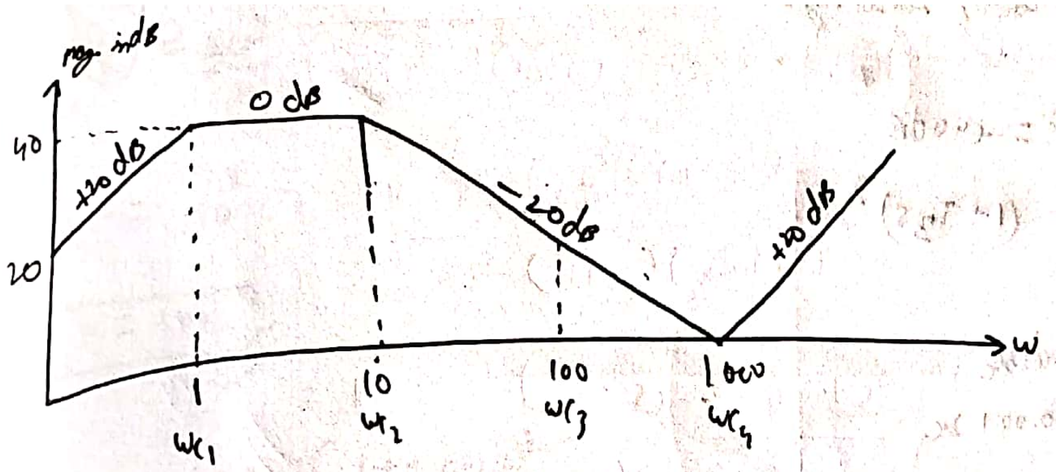

0.1
1
10
100
1000
60
40
20
0
-10
-20
180°
-20db
-60db
10dB
wpc



has starting term ⇒ change in slope = 20 dB

$\therefore 1^{st}$ term = KS —①

at w_{c_1}; change in slope = −20 dB

⇒ 2nd term = $\frac{1}{1+T_1 S}$

$\because w_{c_1} = 1$ rad/sec

$\Rightarrow T_1 = \frac{1}{1} = 1$ sec

$\therefore$ 2nd term = $\frac{1}{1+S}$ —②

at w_{c_2}; change in slope = −20 dB

⇒ 3rd term = $\frac{1}{1+T_2 S}$

$\because w_{c_2} = 10$ rad/sec

$\Rightarrow T_2 = \frac{1}{10} = 0.1$ sec

$\therefore$ 3rd term = $\frac{1}{1+0.1S}$ —③

→ at ω_{c_3}; change in slope = 0 dB

∴ 4th term = constant; can be neglected

→ at ω_{c_4}; change in slope = +40 dB

∴ 5th term = $(1+T_3 s)^2$

∵ $\omega_{c_4} = 1000$ rad/sec

⇒ $T_4 = \frac{1}{1000} = 0.001$ sec

∴ 5th term = $(1+0.001s)^2$ —— ④

∴ from eq ①, ②, ③ & ④,

we get;

Transfer function ⇒ $TF = \frac{KS(1+0.001s)^2}{(1+s)(1+0.1s)}$ //

now;

initial condition;

$20\log_{10}(KS)_{@\omega=0.1} = 20$

$\Rightarrow 20\log_{10}(K) + 20\log_{10}(s)_{@\omega=0.1} = 20$

$\Rightarrow 20\log_{10}(K) + 20\log_{10}(0.1) = 20$

$\Rightarrow 20\log_{10}(K) + 20\log_{10}(1) - 20\log_{10}(10) = 20$

CMR

$\Rightarrow 20 \log_{10}(K) + 0 - 20 = 20$

$\Rightarrow 20 \log_{10}(K) = 40$

$\Rightarrow \log_{10}(K) = \frac{40}{20} = 2$

$\Rightarrow K = 10^2$

$\therefore K = 100$

3) For a unity feedback system $G(s) = \frac{242(s+5)}{s(s+1)(s^2+5s+121)}$. Sketch bode plot

& find ω_{gc}, ω_{pc}, gain margin & PM.

Step 1:

$$G(s) = \frac{242 \times 5\left(\frac{s}{5}+1\right)}{s(s+1)\,121\left(0.0082s^2+0.041s+1\right)}$$

$$= \frac{10\left(\frac{s}{5}+1\right)}{s(s+1)(0.0082s^2+0.041s+1)}$$

$$\Rightarrow G(j\omega) = \frac{10\left(\frac{j\omega}{5}+1\right)}{j\omega(j\omega+1)(j\omega+2.5-10.75j)(s+2.5+10.75j)}$$

$$= \frac{10}{j\omega} \times \frac{1}{(j\omega+1)} \times \frac{\left(\frac{j\omega}{5}+1\right)}{1} \times \frac{1}{(j\omega+2.5-10.75j)}$$

$$\times \frac{1}{(j\omega+2.5+10.75j)}$$

PTO

step-2 magnitude table

terms	ω_c rad/sec	slope (dB/dec)	change in slope (dB/dec)
$10/j\omega$	—	−20	−20
$\frac{1}{j\omega+1}$	1	−20	−40
$\left(\frac{j\omega}{5}+1\right)$	5	+20	−20
$\frac{1}{j\omega+2.5-10.75j} \times \frac{1}{(j\omega+2.5+10.75j)}$	11	−40	−60

step-3: starting pt. of magnitude plot

$$20\log|Y(j\omega)|_{@\omega=0.1} = 20\log|10/\omega|_{@\omega=0.1}$$
$$= 40\,dB$$

step-4: phase plot

$$\phi = -90^\circ + \tan^{-1}(0.2\omega) - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega-10.75}{2.5}\right) - \tan^{-1}\left(\frac{\omega+10.75}{2.5}\right)$$

ω	ϕ (deg)
0.1	−94.8
0.5	−112.03
1	−126.05
3	−138.12
5	−138.11
7	−143.0
10	−177.2
11	−198.4
13	−232.6
15	−248.6

$\omega_{gc} = 3$ rad/sec

$\omega_{pc} = 10.1$ rad/sec

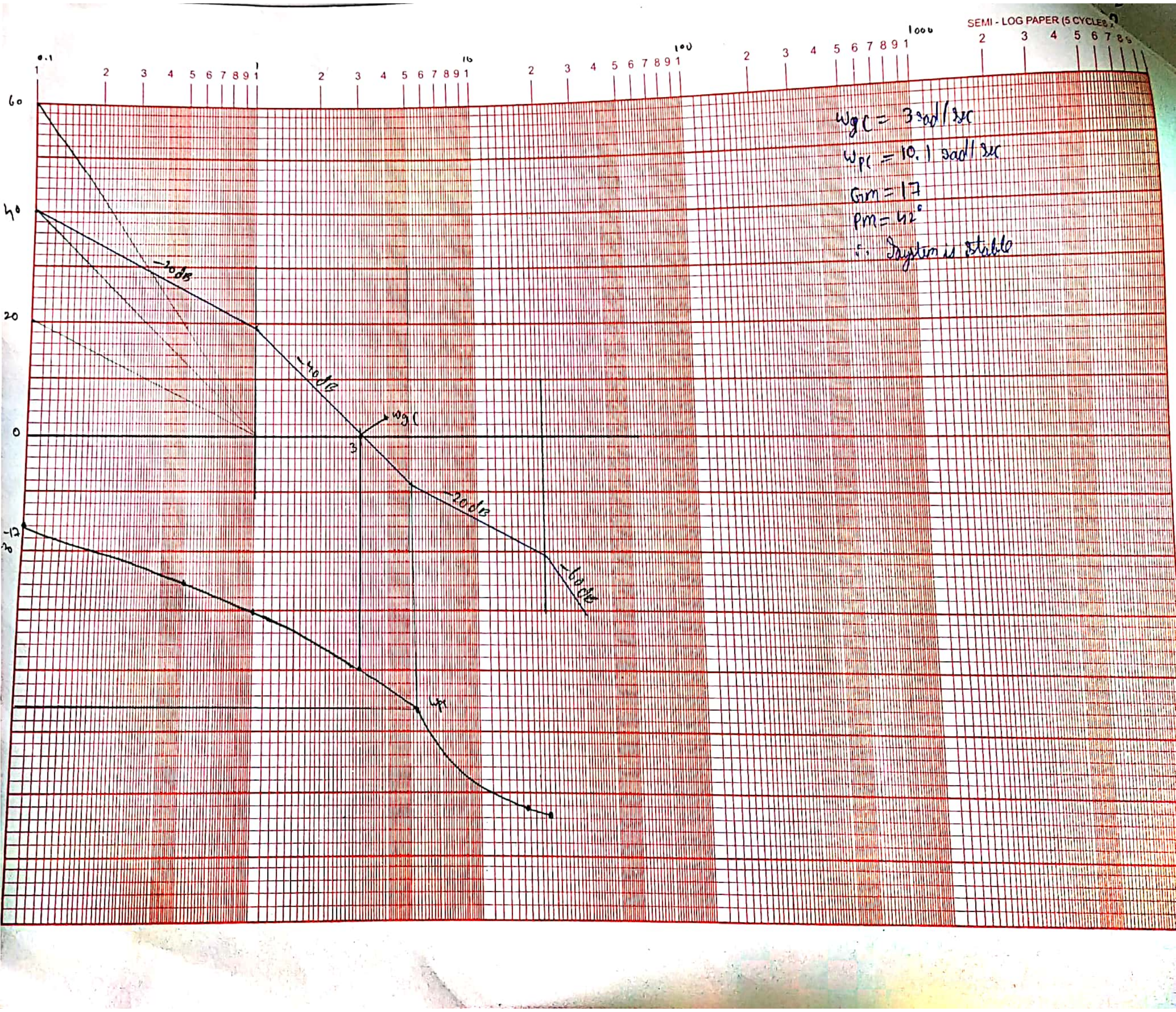
$GM = -1 \times 20\log|G(j\omega)|_{@\omega=\omega_{pc}}$

$= -1 \times -17 = 17$

$PM = 180^\circ + \phi_{gc}$

$= 180^\circ - 138^\circ = 42^\circ$

∴ given system is stable



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7) $G(s)\,H(s) = \dfrac{K}{s(s+2)(s+10)}$ — (1) [; here first assuming $K=1$]

step 1: P : no. of poles to RHS of the F plane ⇒ $P=0$

Z : " " zeros " " " " ⇒ $Z=0$

$N = Z - P$

$= 0 - 0$

$\therefore N = 0$

step 2: check for stability is ; $N = -P$

here; $N = -0 = 0$ {hence; system is stable}

step 3: $G(j\omega)\,H(j\omega) = \dfrac{1}{(j\omega)(2+j\omega)(10+j\omega)}$ — (A)

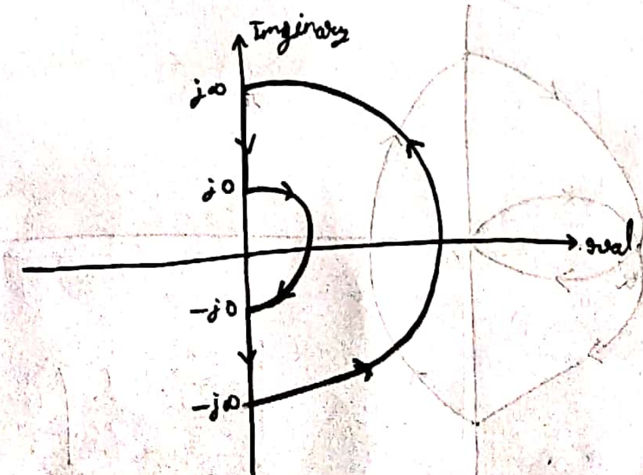
now;

$$|G(j\omega)\,H(j\omega)| = \frac{1}{\omega\sqrt{(4+\omega^2)(100+\omega^2)}}$$

and

$$\angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

step 4:



section I: $j\infty$ to $j0$.

section II: $j0$ to $-j0$.

section III: $-j0$ to $-j\infty$.

section IV: $-j\infty$ to $j\infty$.

→ **sec I:**

starting point	$\omega \to \infty$	$0\angle -270°$	$(-90) - (-270)$ $= -90 + 270$ $= +180$ → anti-clockwise
terminal point	$\omega \to 0$	$\infty \angle -90°$	

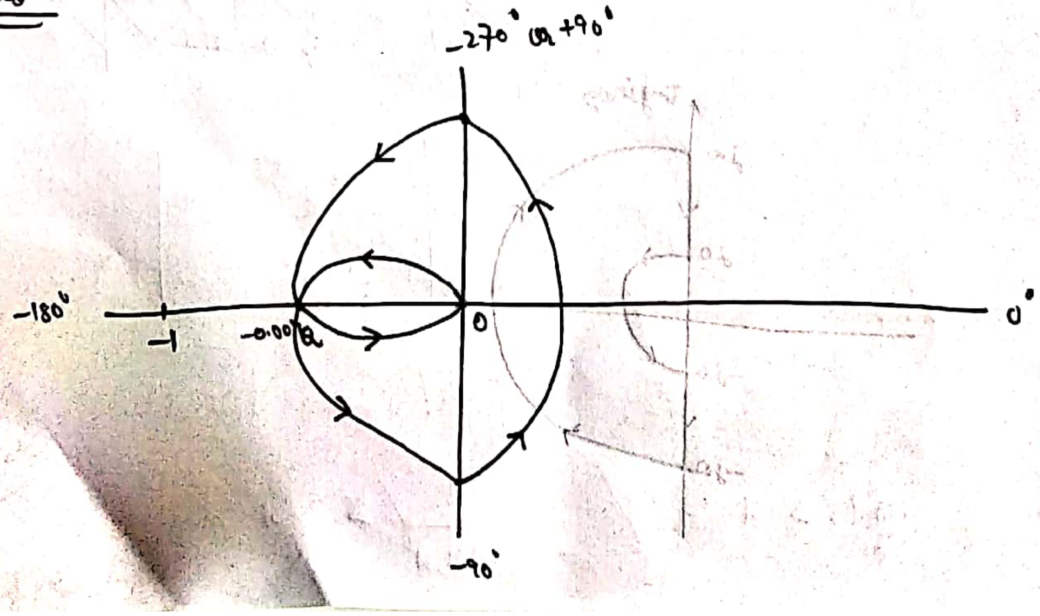
→ **sec II:**

starting point	$\omega \to 0$	$\infty \angle -90°$	$90 - (-90)$ $= 90 + 90 = 180$ → anti clockwise
terminal point	$\omega \to -0$	$\infty \angle 90°$	

→ **sec III:** it's the mirror image of sec I

→ **sec IV:** point at origin.

Nyquist plot:



step 5: from eq ① ; when $s = j\omega$;

$$G(j\omega)H(j\omega) = \frac{K}{(j\omega)(2+j\omega)(10+j\omega)} \times \frac{(2-j\omega)(10-j\omega)(-j\omega)}{(2-j\omega)(10-j\omega)(-j\omega)}$$

$$= \frac{K(-j\omega)(2-j\omega)(10-j\omega)}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$= \frac{K(-j\omega)(20-2j\omega-10j\omega+j^2\omega^2)}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$= \frac{K(-20j\omega+2j^2\omega^2+10j^2\omega^2-j^3\omega^3)}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$= \frac{K(-2\omega^2-10\omega^2+j\omega^3-j20\omega)}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$= \frac{K(-12\omega^2)}{\omega^2(4+\omega^2)(100+\omega^2)} + \frac{j\omega(+\omega^2-20)K}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$\therefore G(j\omega)H(j\omega) = \frac{-12K}{(4+\omega^2)(100+\omega^2)} + j\frac{K(+\omega^2-20)}{\omega(4+\omega^2)(100+\omega^2)} \quad \text{— ②}$$

in eq. ②; substituting imaginary part to zero.

$$\Rightarrow \frac{K(+\omega^2-20)}{\omega(4+\omega^2)(100+\omega^2)} = 0 \quad \Rightarrow \omega^2 = 20 \quad \Rightarrow \omega = \sqrt{20} \quad \therefore \omega = \pm 2\sqrt{5}$$

$$\Rightarrow (+\omega^2-20) = 0$$

~~$\Rightarrow -(\omega^2+20) = 0$~~

~~$\Rightarrow \omega^2 = -20 \Rightarrow \omega = \sqrt{-20}$~~

~~$\therefore \omega = \pm\sqrt{20}\,i$~~

now: length of OQ = substitute ω_{pc} in real part of eq (2),

$\omega_{pc} = \sqrt{20}$

$$\Rightarrow \text{length } OQ = \cancel{\frac{+2K(\sqrt{20}+5)}{(\sqrt{20})(4+20)(100+20)}} = \frac{-12K}{(4+20)(100+20)}$$

$OQ = -K/240$

$$\Rightarrow \cancel{-1.427\times10^{-3}K} \qquad = -4.166\times10^{-3}K$$

$|OQ| < 1$

$\boxed{0 < K < 240}$

when $K = 1$

$\Rightarrow$ length of $OQ = -4.166\times10^{-3}$

$= -0.00416$

now: $-1+j0$ is the unit circle which lies outside the nyquist plot

$\therefore$ the system is stable.

to find range of K:

wkt, $|\text{length of } OQ| < 1$

$$\Rightarrow \cancel{\frac{K}{(\omega^2)\sqrt{4+\omega^2}\sqrt{100+\omega^2}} < 1} \quad @\omega=\omega_{pc}$$

$$\Rightarrow \left|\frac{-12K}{(40+20)(100+20)}\right| < 1$$

$$\Rightarrow |-4.166\times10^{-3}K| < 1$$

$$\Rightarrow K < \frac{1}{4.166\times10^{-3}}$$

$$\Rightarrow K < 240.038 \approx 240$$

3) $G(s)\,H(s) = \dfrac{40}{(s+4)\,(s^2+2s+2)} = \dfrac{40}{(s+4)\,(s-(-1+j))\,(s-(-1-j))} = \dfrac{40}{(s+4)\,(s+1-j)\,(s+1+j)}$

step1: $P = 0$; weight imaginary roots which lie on vertical axis;

$Z = 0$

$\therefore N = Z - P$
$= 0$

step2: check for stability ; $N = -P$

here, $N = 0$

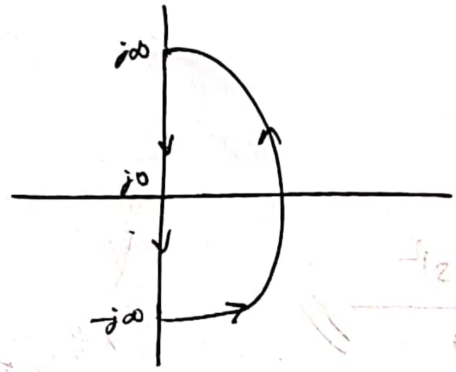
step3: $G(j\omega)\,H(j\omega) = \dfrac{40}{(4+j\omega)\,(1-j+j\omega)\,(1+j+j\omega)}$

$= \dfrac{40}{(4+j\omega)\,(1+j(\omega-1))\,(1+j(\omega+1))}$

$$\therefore |G(j\omega)H(j\omega)| = \frac{40}{\sqrt{16+\omega^2}\cdot\sqrt{1+(\omega-1)^2}\cdot\sqrt{1+(\omega+1)^2}}$$

$$\& \angle G(j\omega)H(j\omega) = -\tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}(\omega-1) - \tan^{-1}(\omega+1)$$

step 4:



section I : j∞ to j0

section II : j0 to -j∞

section III : -j∞ to j∞

→ sec I :

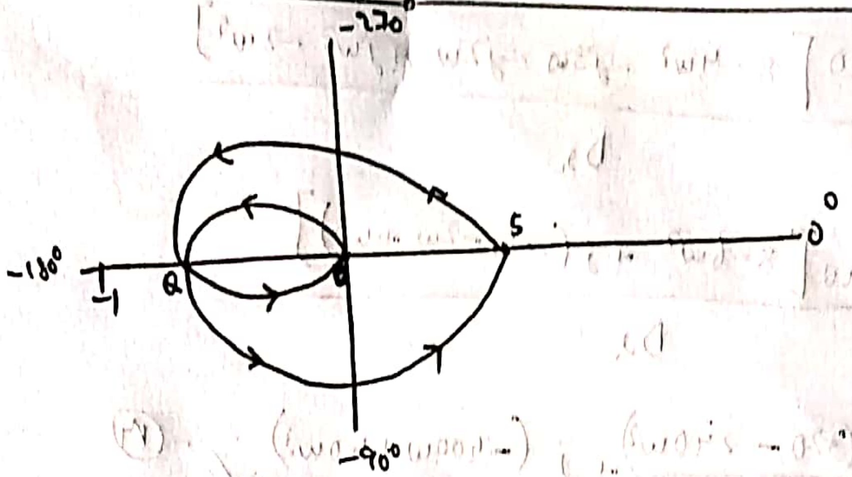
starting point	$\omega \to \infty$	$0\angle -270°$	$0-(-270°)$ $=270°$ → anti clockwise
terminal point	$\omega \to 0$	$5\angle 0°$	

→ sec II : mirror image of sec I.

→ sec III : point at origin.

PTO

Nyquist plot :



steps: here;

$$G(j\omega)H(j\omega) = \frac{40}{(4+j\omega)(1+j(\omega-1))(1+j(\omega+1))} \times \frac{(4-j\omega)(1-j(\omega-1))(1-j(\omega+1))}{(4-j\omega)(1-j(\omega-1))(1-j(\omega+1))}$$

$$= \frac{40(4-j\omega)(1-j(\omega-1))(1-j(\omega+1))}{(16+\omega^2)(1+(\omega-1)^2)(1+(\omega+1)^2)}$$

let $D_r = (16+\omega^2)(1+(\omega-1)^2)(1+(\omega+1)^2)$

$$\Rightarrow G(j\omega)H(j\omega) = \frac{40(4-j\omega)\left[(1-j(\omega+1)) - [j(\omega-1) - j^2(\omega-1)(\omega+1)]\right]}{D_r}$$

$$= \frac{40(4-j\omega)\left[1-j(\omega+1)-j(\omega-1)-(\omega^2-1)\right]}{D_r}$$

$$= \frac{40(4-j\omega)\left[(2-\omega^2)-j((\omega+1)+(\omega-1))\right]}{D_r}$$

$$= \frac{40(4-j\omega)\left[(2-\omega^2)-j2\omega\right]}{D_r}$$

$$= \frac{40\left[4(2-\omega^2)-j8\omega-j\omega(2-\omega^2)+j^2 2\omega^2\right]}{D_r}$$

$$\Rightarrow G(j\omega)H(j\omega) = \frac{40\left[8 - 4\omega^2 - j8\omega - j2\omega + j\omega^3 - 2\omega^2\right]}{Dr}$$

$$= \frac{40\left[8 - 6\omega^2 + j(-8\omega - 2\omega + \omega^3)\right]}{Dr}$$

$$\therefore G(j\omega)H(j\omega) = \frac{(320 - 240\omega^2)}{Dr} + j\frac{(-400\omega + 40\omega^3)}{Dr} \quad -(M)$$

when; imaginary pt. of eq (M) is zero

$$\Rightarrow (-400\omega) + (40\omega^3) = 0$$

$$\Rightarrow 40\left[-10\omega + \omega^3\right] = 0$$

$$\Rightarrow \omega\left[-10 + \omega^2\right] = 0$$

$$\Rightarrow \omega^2 = 10$$

$$\therefore \omega_{pc} = \pm\sqrt{10}$$

substitute ω_{pc} in real part, we get; length of OA $= \frac{320 - 240 \times 10}{(16+10)(1+(\sqrt{10}-1)^2)(1+(\sqrt{10}+1)^2)}$

$$= -0.769$$

here; $-0.769 < -1$ the unit circle lies outside the nyquist plot

$\therefore$ system is stable

now gain margin $\Rightarrow G_m = \left|\frac{1}{\text{length of OA}}\right| = \left|\frac{1}{-0.769}\right| = 1.3$

or $G_m(\text{in dB}) = 20\log_{10}\left|\frac{1}{\text{length of OA}}\right|$

$$= 20\log_{10}\left|\frac{1}{-0.769}\right|$$

$$= 20\log_{10}(1.3) = 2.28\,dB$$

$\therefore G_m = 1.3$ or $2.28\,dB$