Scheme Of Evaluation

Internal Assessment Test III – May.2019

Sub:	Principles Of Communication Systems							Code:	17EC44
Date:	14/05/2019	Duration:	90mins	Max Marks:	50	Sem:	IV	Branch:	ECE(A,B & D)

Note: Answer Any Five Questions

Question #	Description	Marks Distribution	Max Marks
1	(a)Explain the TDM system with necessary block diagram. • Block Diagram • Theory (b) A signal $m_1(t)$ is bandlimited to 3.6khz and 2 other signals $m_2(t)$, and $m_3(t)$ are bandlimited to 2.4khz each. These signals re to be transmitted by TDM. Set up a scheme for realizing this. What must be the speed of the commutator? Determine the minimum bandwidth required? • TDM Scheme • Speed • Bandwidth	- 3 3 - 2 1 1	10
2	(a) What is Quantization noise? Derive the output signal to noise ratio of a uniform quantizer. • Definition • Derivation b) What is Quantization process? Explain the different types of Quantization with their input output characteristics. • i/o characteristics • Theory	- 1 4 - 3 2	10
3	Explain the generation and reconstruction of PCM signals. • Block diagram • Theory	- 4 6	10
4	(a)Explain Shot noise and Thermal noise with relevant expressions. • Shot noise • Thermal noise (b)Define white noise. Plot power spectral density (PSD) and autocorrelation function of white noise • Definition of white noise • Theory, PSD and autocorrelation	- 2 4 - 1 3	10

Question #	Description	Marks Distribution	Max Marks
5	(a)Define Noise equivalent bandwidth and derive the expression for the same. • Definition • Derivation (b)Prove that the total volume under the surface of a probability density function is always 1.Equations • Proof	- 1 5 - 4	10
6	(a)Define Cross correlation and Autocorrelation function. Explain properties of autocorrelation. • Definitions • Properties (b)Define mean, variance, standard deviation and covariance function of a random process. • Definitions	- 2 4 - 4	10

Solution

1

Time Division Multiplexing is a method of transmitting and receiving independent signals over a common channel by means of synchronised switches at each end of transmission line so that each signal appears on the line only a fraction of time in an alternating pattern.

* Fig(5) shows the block diagram of TDM system.

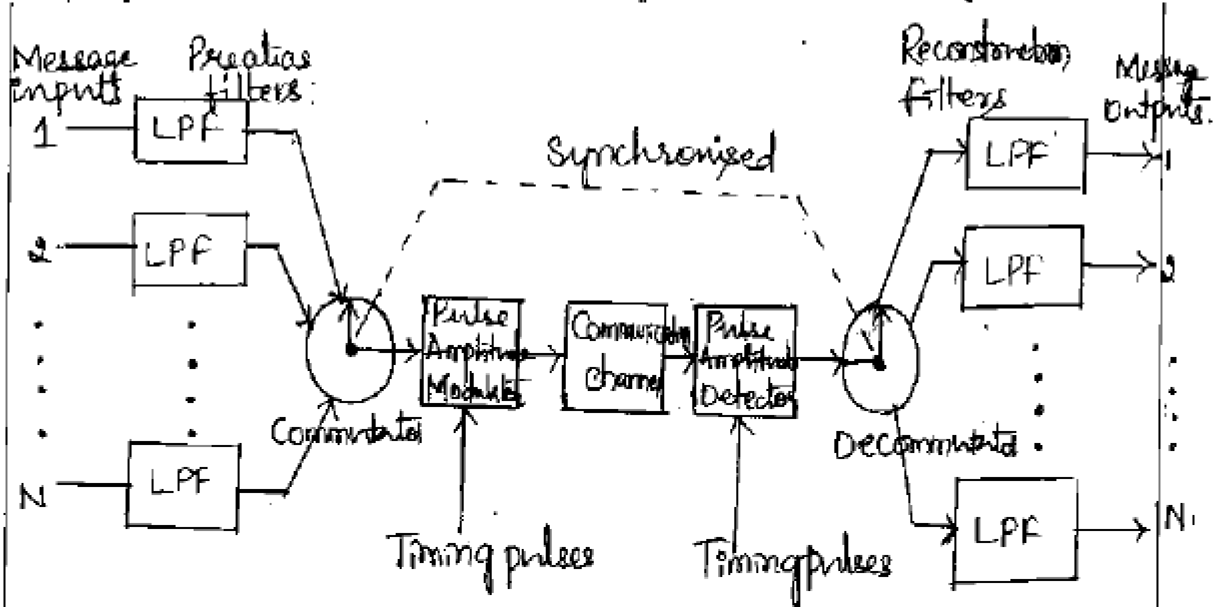
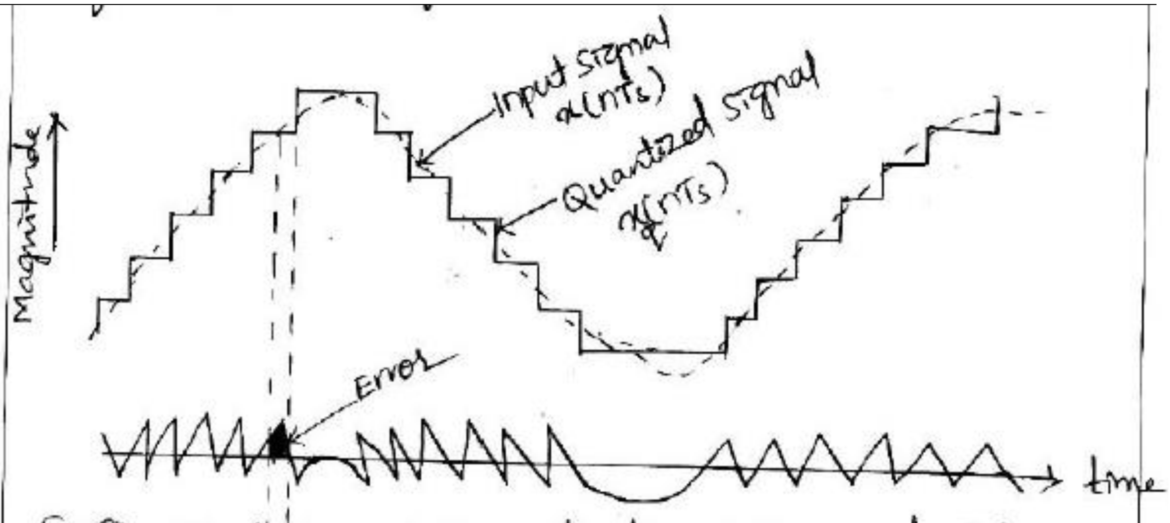


Fig 5 : Block Diagram of TDM System.

* The concept of TDM is illustrated in the fig(5). The Lowpass filters are used to remove high frequency components present in the message signal. The output of the pre-alias filters are then fed to a commutator which is usually implemented using electronic switching circuitry.

* The function of commutator is as follows:

	1) To take a narrow sample of each of the 'N' samples of input at a rate of $f_s \geq 2W$. 2) To sequentially interleave (multiplex) these 'N' samples inside a sampling interval $T_s = 1/f_s$. * The multiplexed signal is then applied to a pulse amplitude modulator whose purpose is to transform the multiplexed signal into a form suitable for transmission over a common channel. * At the receiving end, the pulse amplitude demodulator performs the reverse operation of PAM and the decommutator distributes the signals to the appropriate low pass reconstruction filters. The decommutator operates in synchronisation with the commutator.
2a	* The use of quantization introduces an error defined as the difference between the input signal 'm' and the output signal 'V'. This error is called Quantization noise. Fig(B), illustrates a typical variation of the Quantization noise as a function of time, assuming the use of a uniform quantizer of the midtread type.



Fig(8) : Illustration of Quantization process and Noise

* Let the random variable 'Q' denotes the quantization error and 'q' its sample value.

$$q = m - v \qquad (1)$$

* Consider then an input 'm' of continuous amplitude in the range $(-m_{max}, m_{max})$ then, the step-size of the quantizer is given by,

$$\Delta = \frac{2m_{max}}{L} \qquad (2)$$

* Now, the probability density function of the quantization error 'Q' as follows,

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < q \leq \frac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases} \qquad (3)$$

* Now, the variance of Quantization error is

$$\sigma_Q^2 = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2 f_Q(q)\, dq$$

$$= \frac{1}{\Delta}\left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{3\Delta}\left[\left(\frac{\Delta}{2}\right)^3 - \left(\frac{-\Delta}{2}\right)^3\right] = \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} - \frac{(-\Delta)^3}{8}\right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right] = \frac{1}{3\Delta} \times 2\left(\frac{\Delta^3}{8}\right) = \frac{1}{3} \cdot \frac{\Delta^2}{4}$$

$$\therefore \boxed{\sigma_Q^2 = \frac{\Delta^2}{12}}$$

This is known as "Mean squared quantization error" or Normalized Noise power or Quantization error interms of pow — (A)

* Let us consider "R" which denote the number of bits per sample then the Quantized level is given by,

$$L = 2^R \qquad (4)$$

substituting Eq^n (4) in Eq^n (2) we get,

$$\Delta = \frac{2\, m_{max}}{2^R} \qquad (5)$$

Now substitute Eq^n (5) in Eq^n (A) we get

$$\therefore \sigma_Q^2 = \left[\frac{\frac{2m_{max}}{2^R}}{12}\right]^2 = \frac{4\, m_{max}^2}{2^{2R}} \times \frac{1}{12}$$

$$\therefore \sigma_Q^2 = \frac{1}{3} m_{max}^2 2^{-2R} \qquad (6)$$

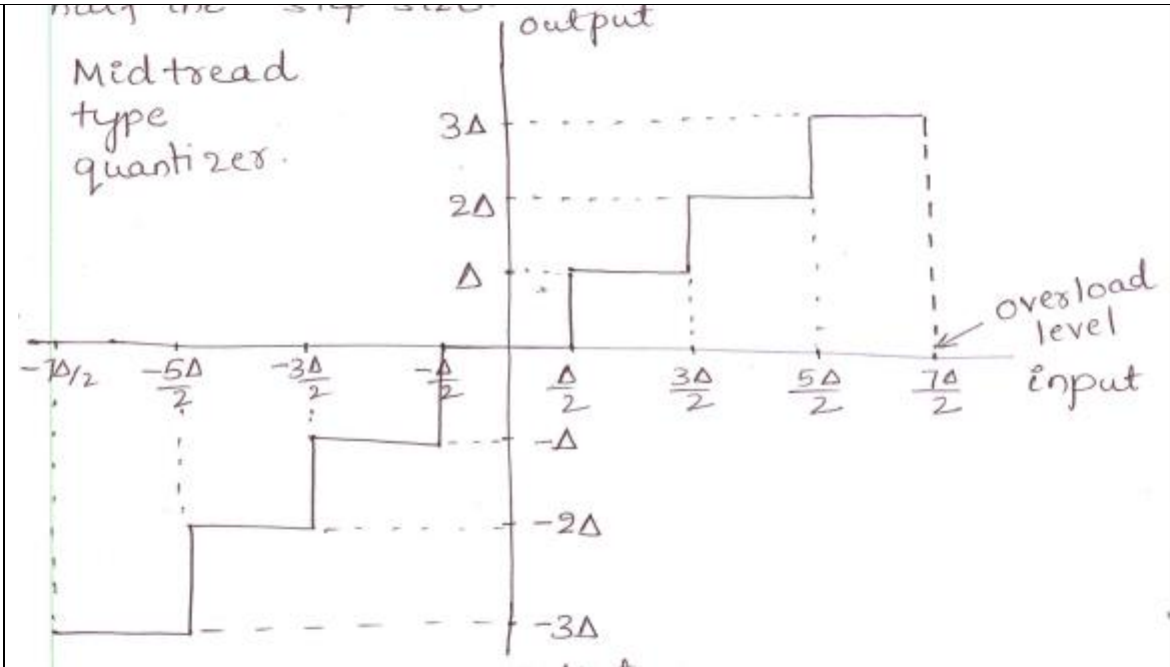
Let 'P' denote the avg power of message signal $m(t)$. We may express the output signal to noise ratio of a uniform quantizer as,

$$(SNR)_o = \frac{P}{\sigma_Q^2}$$

$$= \frac{P}{\frac{1}{3} m_{max}^2 2^{-2R}}$$

$$\therefore \boxed{(SNR)_o = \left(\frac{3P}{m_{max}^2}\right) 2^{2R}}$$

2 b	An analog signal such as voice has a continuous range of amplitudes. However, it is not necessary to transmit the exact amplitudes of the samples. Any human sense (the ear or the eye) has ultimate receiver can detect only finite intensity differences. This means that the original analog signal may be approximated by a signal constructed of discrete amplitudes. The conversion of continuous range of analog sample values into a digital form is called quantization. As a result of quantization; i. The peak to peak range of input sample values is subdivided into a finite set of dicision levels or decision thresholds that are aligned with the risers of the staircase ii. The output is assigned a discrete value selected from a finite set of representation levels that are aligned with the risers of the staircase. In case of a uniform quantizer, the separation between the decision thresholds and separation between representation levels have a common value called the step size. 4 [4] As a result of quantization, a quantization error is introduced, the maximum value of which is half the step size.


Midtread type quantizer.
output
3Δ
2Δ
Δ
overload level
input
−7Δ/2
−5Δ/2
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−Δ
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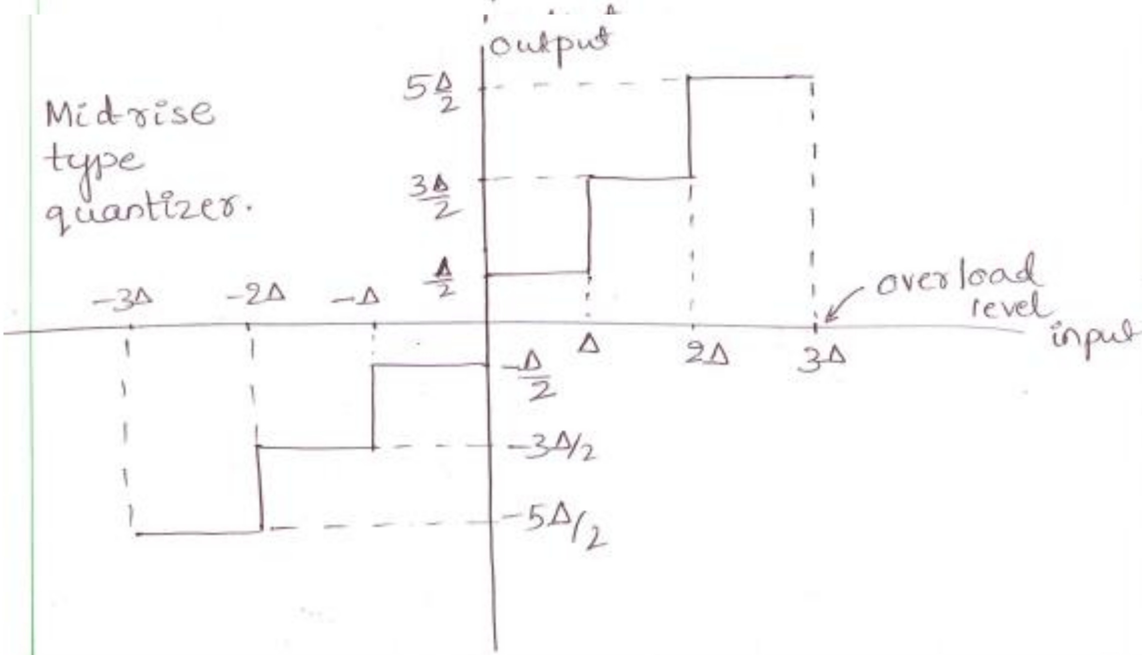
Midrise type quantizer.
output
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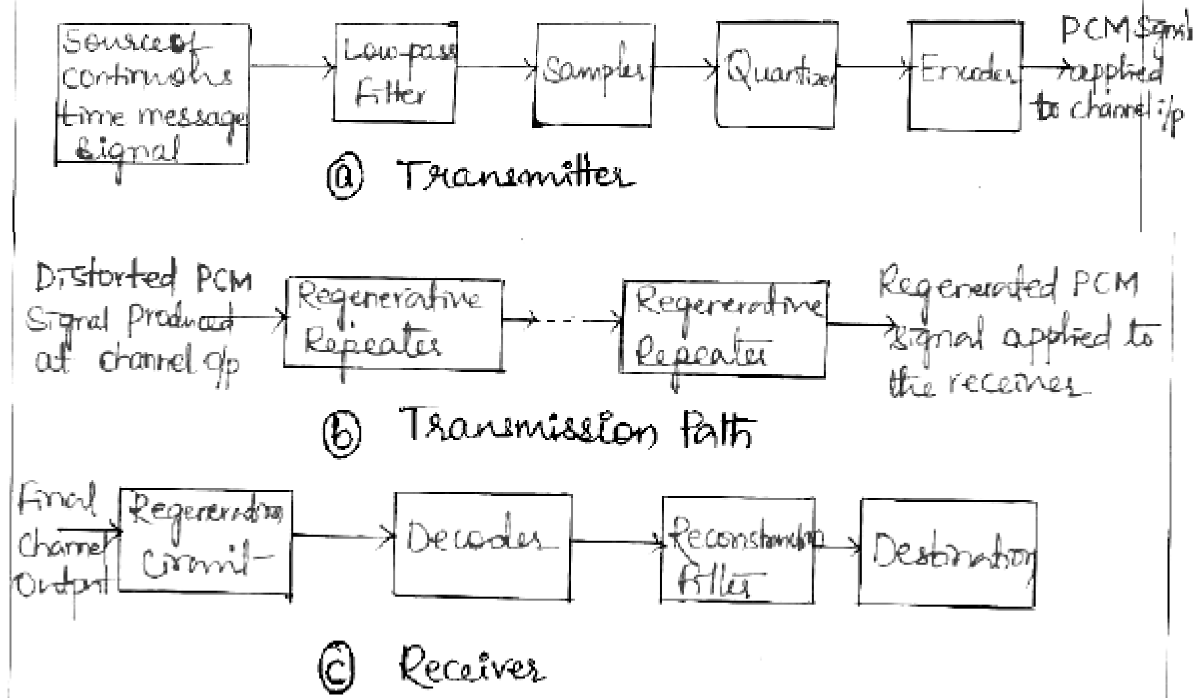
Δ/2
overload level
input
−3Δ
−2Δ
−Δ
Δ
2Δ
3Δ
−Δ/2
−3Δ/2
−5Δ/2

	In a mid tread type quantizer, the decision thresholds are located at $\pm\frac{\Delta}{2}, \pm\frac{3\Delta}{2}, \ldots$ and representation levels are located at $0, \pm\Delta, \pm2\Delta, \ldots$ where Δ is the step size. The origin lies in the middle of a tread of the staircase, hence the name mid-tread. 5/41 In a midriser type quantizer, the decision thresholds are located at $0, \pm\Delta, \pm2\Delta, \ldots$ and the representation levels are located at $\pm\Delta/2, \pm\frac{3\Delta}{2}, \pm5\Delta/2, \ldots$ where Δ is the step size. The origin lies in the middle of a riser, hence the name mid riser type.
3	* In pulse code Modulation (PCM), a message signal is represented by a sequence of coded pulses, which is accomplished by representing the signal in discrete form in both time and amplitude. * The basic operations performed in the transmitter of a PCM system are sampling, quantizing and encoding as shown in fig 6(a). The Lowpass filter prior to sampling

is included to prevent aliasing of the message signal. The quantizing and encoding operations are usually performed in the same circuit, which is called an analog-to-digital converter.

* The basic operations in the receiver are regeneration of impaired signals, decoding and reconstruction of the train of quantized samples as shown in fig 6(c). Regeneration also occurs at intermediate points along the transmission path as necessary as indicated in fig(6)(b).



Fig(6) : The basic elements of a PCM system.

* SAMPLING :

The incoming message signal is sampled with a train of narrow rectangular pulses so as to closely approximate the instantaneous sampling process. In order to ensure perfect reconstruction of the message signal at the receiver, the sampling rate must be greater than or equal to the highest frequency component 'W' of the message signal in accordance with the sampling theorem.

$$f_s \geq 2W.$$

* Quantization :

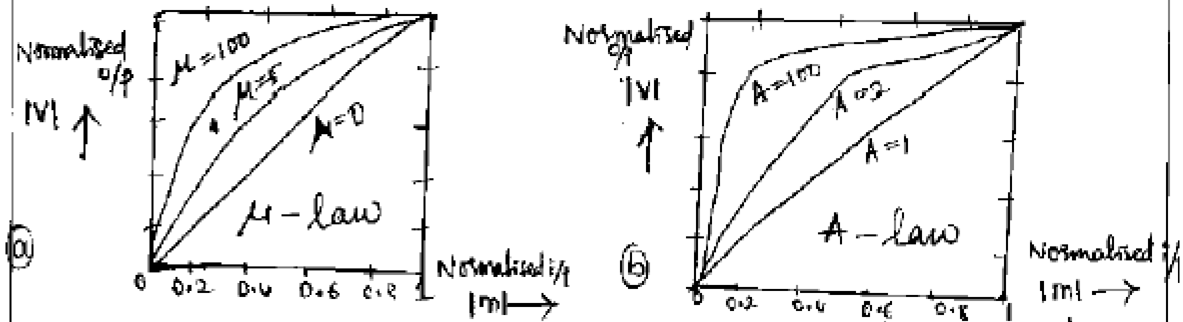
The sampled version of the message signal is then quantized, thereby providing a new representation of the signal that is discrete in both time and amplitude.

* For uniform quantization, we have mid-tread and mid-rise quantizer and for non-uniform quantization, we have two compression laws μ-law and A law.

* The use of a non-uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. A particular form of compression law that is used in practice is the so called μ-law, defined by

$$|v| = \frac{\log(1+\mu|m|)}{\log(1+\mu)} \quad \text{———} (1)$$

where m and v are normalized input & output vtg and 'μ' is positive constant.



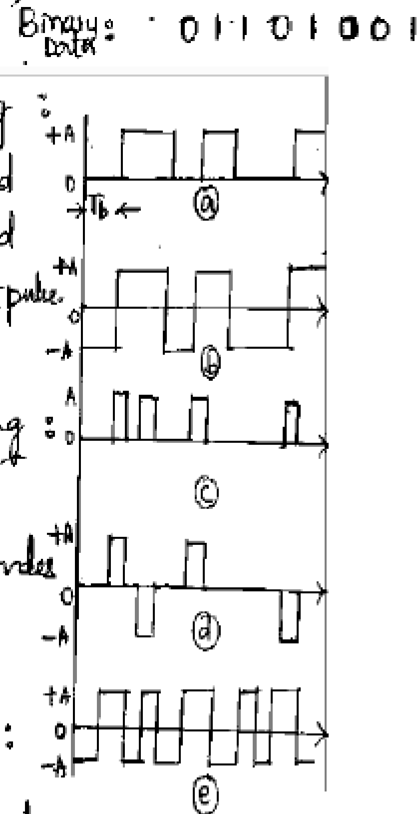
* Another compression law that is used in practice is the so called 'A' law as shown above.

$$|v| = \begin{cases} \dfrac{A|m|}{1+\log A}, & 0 \le |m| \le \dfrac{1}{A} \\ \dfrac{1+\log(A|m|)}{1+\log A}, & \dfrac{1}{A} \le |m| \le 1 \end{cases}$$

* Encoding :

* In combining the processes of sampling and quantizing the specification of a continuous message (baseband) signal becomes limited to a discrete set of values, but not in the form best suited to transmission over a line or radio path.
* In a binary code, each symbol may be either of two distinct values or kinds, such as the presence or absence of a pulse. The two symbols of a binary code are customarily denoted as 0 and 1.
* Line code : It is a line code that a binary stream

of data takes on an electrical representation. The five line codes are illustrated in fig(7).



1) Unipolar Nonreturn to zero (NRZ) Signaling :

In this line code symbol '1' is represented by transmitting a pulse of amplitude 'A' and symbol '0' is represented by switching off the pulse.

2) Polar Non-return to zero (NRZ) Signalling :

In this line code, symbol 1 and 0 are represented by transmitting pulses of amplitudes +A and -A respectively.

3) Unipolar Return to zero (RZ) signalling :

Here, symbol 1 is represented by a rectangular pulse of amplitude A and half-symbol width and symbol 0 is represented by transmitting no pulse.

(a) → Unipolar NRZ
(b) → Polar NRZ
(c) → Unipolar RZ
(d) → Bipolar RZ
(e) → Manchester code

4) Bipolar Return to zero (BRZ) Signalling :

This line code uses three amplitude levels as shown in fig. specifically, +ve & -ve pulses of equal amplitude are used alternatively for symbol 1. 0 for no pulse.

5) Split-phase (Manchester code)

Symbol 1 is represented a positive pulse of amplitude 'A' followed by a negative pulse of amplitude -A with both pulses being a half-symbol wide. For symbol '0', the polarities of these two pulses are reversed.

* REGENERATION :

The distorted PCM wave obtained from the transmitter is sent to the amplifier equilizer. The output of equilizer device is passed to the Decesion making device to decide the signal interms of 1 or 0 (coded o/p).

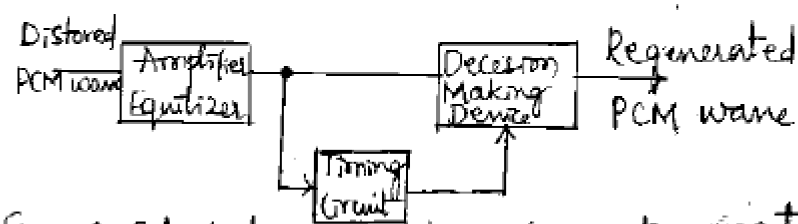


Fig : Block diagram of a regenerative repeater.

4a **Shot noise:**

- Shot noise is produced due to discrete nature of current flow in electric circuits.
- It is produced in all amplifying device because of random variation in the arrival of electron at the output electrode of an amplifying device.
- It has uniform power spectral density
- For PN junction diodes the mean square value of fluctuating shot noise current is $E[I_{SN}^2] = 2q(I + I_S)B_N$
  Where $q = 1.6 * 10^{-19}$ $Coulumbs, B_N$ *is noise equivalent bandwidth,* I *is required DC – current,* I_S *is reverse leakage current*

Thermal Noise

- It is generated due to random movement of thermally induced carriers in a conductor(noise generated in resistors due to random motion of electrons).
- The random motion of thermally induced electrons produces electric current which is random in nature. Thus random current is called "thermal noise"
- The mean-square value of the thermal noise voltage V_{TN}, appearing across the terminals of a resistor, measured in a bandwidth of Δf Hertz, is given by:

$$\mathbf{E}\left[V_{TN}^2\right] = 4kTR\Delta f \ \text{volts}^2$$

- k : *Boltzmann's constant*=1.38×10^{-23} joules per degree Kelvin.
- T : *Absolute temperature* in degrees Kelvin.

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- R: The resistance in ohms.

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b</td><td>

- The noise which has Gaussian distribution and have flat power spectral density over a wide range of frequencies is called white noise.
- It is denoted by w(t).
- A white noise is generally assumed to be zero-mean.
- A white noise is a mathematical abstraction; it cannot be physically realized since it has infinite average power.
- White noise is analogous to the term "white light" in the sense that it has all frequency components in equal amounts.
- The power spectral density of white noise process, w(t) is $S_W(f) = \frac{N_o}{2}$ where $N_o = kT_e, N_O$ *is* a real constant and called the *intensity* of the white noise, k is the boltzmann's constant, T_e is the equivalent noise temperature of the receiver. (Note: A power specrtal density is a measure of signal's power content versus frequency. A PSD is used to characterize broadband random signals.)

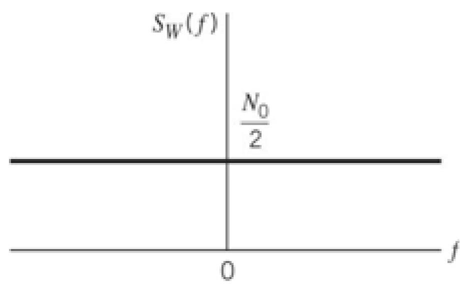

Fig :1 Power spectral density of white noise

- The autocorrelation function is the inverse Fourier transform of the power spectral density:
- $R(\tau) = \frac{N_o}{2}\delta(\tau)$

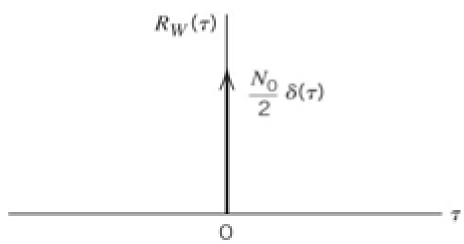

Fig2: Autocorrelation function of white noise

- That is the autocorrelation function of white noise consists of a delta function weighted by the factor $\frac{N_o}{2}$ *and occuring at* $\tau = 0$.

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<tr><td>5a</td><td>

- **"Noise equivalent bandwidth is defined as the bandwidth of an ideal filter such that the power at the output of this filter, if excited by white noise, is equal to that of the real filter given the same input signal" (In simple words, noise equivalent bandwidth is defined as the band width of an ideal filter which produce same integrated noise power as that of an arbitrary filter).**

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- Bandwidth enables computation of the power required to transmit the signal.
- Consider an ideal low pass filter of bandwidth B and passband amplitude response of one.

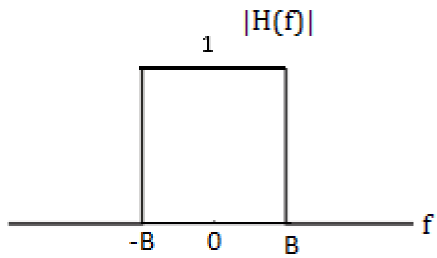

Fig 1: Amplitude or magnitude response of ideal low pass filter

- Apply a white Gaussian noise w (t) of zero mean and power spectral density $\frac{N_o}{2}$ to an ideal low pass filter.

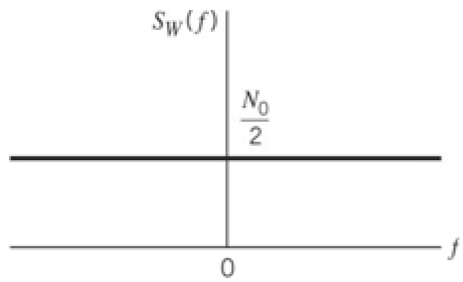

Fig 2: Power spectral density of white noise

- The power spectral density of the noise appearing at the filter output is therefore

$$S_N(f) = \begin{cases} \frac{N_o}{2}, & -B < f < B \\ 0, & |f| > B \end{cases}$$

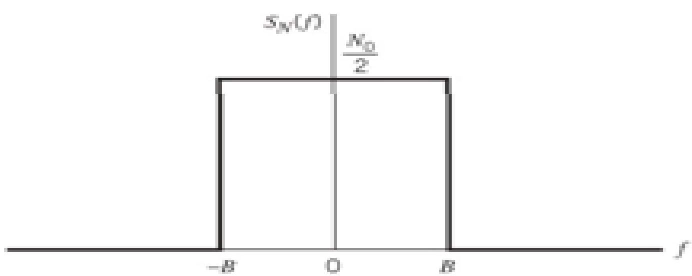

Fig 3: The power spectral density of ideal low pass filtered noise

- The average output noise power is $N_{out} = \frac{N_O}{2} 2B|H(0)|^2 = N_O B|H(0)|^2$
- Consider an arbitrary low pass filter whose amplitude response is as shown below

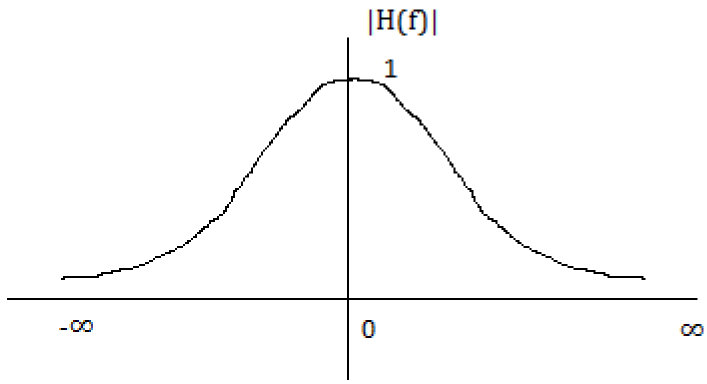

Fig4: Response of arbitrary low pass filter

- The average output noise power is $N_{out} = \frac{N_O}{2}\int_{-\infty}^{\infty}|H(f)|^2 df = N_O\int_0^{\infty}|H(f)|^2\, df$

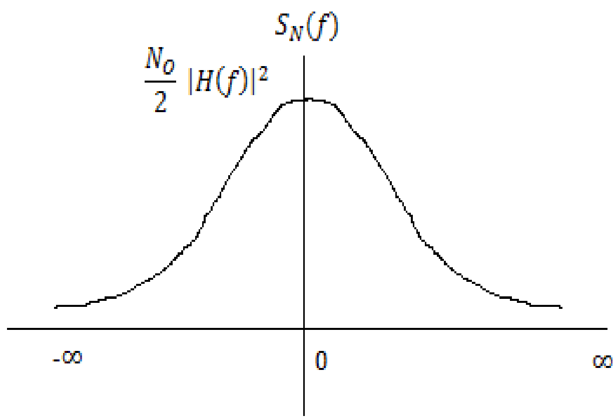

Fig 5: The power spectral density of arbitrary low pass filtered noise

Thus equating equations we get

- $$B = \frac{\int_0^{\infty}|H(f)|^2 df}{|H(0)|^2}$$

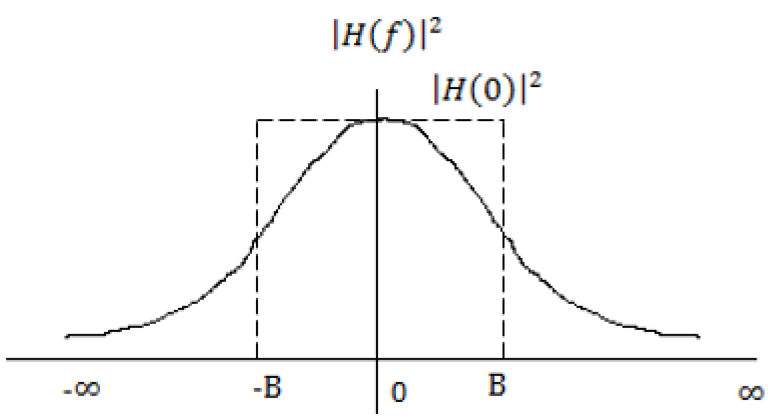

Fig 6: Illustration of noise equivalent bandwidth

- Thus, the procedure for calculating the noise equivalent bandwidth consists of replacing the arbitrary low pass filter of transfer function H(f) by an equivalent ideal low pass filter of zero frequency response H(0) and bandwidth B, as shown below.

5 b

↳ Definition: The Derivative of probability distribution function is called as probability density function (pdf). It is denoted by. $f_X(x)$.

Mathematically :

$$f_X(x) = \frac{d F_X(x)}{dx},$$

where, $F_X(x) \Rightarrow$ probability Distribution function.

$X \Rightarrow$ Random Variable that can take any real value 'x'.

* The total area under the probability density curve is always equal to 1 (unity).

i.e., $$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

Proof :- By definition,

$$f_X(x) = \frac{d F_X(x)}{dx} \quad \text{--- (1)}$$

Apply Integral on both sides between the limits $-\infty$ to ∞.

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{-\infty}^{\infty} \frac{d F_X(x)}{dx}\,dx.$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \Big[F_X(x)\Big]_{x=-\infty}^{\infty} = F_X(+\infty) - F_X(-\infty)$$

$$\therefore \int_{-\infty}^{\infty} f_X(x)\,dx = F_X(\infty) - F_X(-\infty) = 1 - 0 = 1$$

(because, $F_X(\infty) = 1$ and $F_X(-\infty) = 0$ (properties of distribution function)

6a (a)Define Cross correlation and Autocorrelation function. Explain properties of autocorrelation.

↳ Definition: The auto correlation function of the random variable 'X', is defined as the expectation of the product of two random variables, $X(k)$ and $X(l)$, obtained by observing the random variable 'X' at times 'k' and 'l' respectively.

It is denoted by $\gamma_X(k,l)$ (or) $\gamma_x(\tau)$

i.e., Auto Correlation function is given by

$$\boxed{\gamma_X(k,l) = E[X(k) \cdot X(l)] = \gamma_X(k-l)}$$

∴ Auto correlation function gives correlation of a signal wi[th] itself but delayed in time.

Auto correlation function, $\gamma_x(k,l)$ is a function of time diffe-rence $(k-l)$.

Properties :-

Let $\gamma_x(\tau)$ be the auto correlation function of Random process (variable) 'X'. then

⟨i⟩ $\gamma_x(\tau) = \gamma_x(k-l)$: It is a function of time differen[ce] $(k-l)$

⟨ii⟩ $\gamma_x(0) = E[X^2]$; when $k=l$.

⟨iii⟩ $\gamma_x(\tau)$ is maximum value at $\tau=0$.

i.e., $\gamma_x(0) \geq \gamma_x(\tau)$ for any value of 'τ'

(iv) $\gamma_x(\tau)$ is a even function of (τ)

i.e, $\gamma_x(\tau) = \gamma_x(-\tau)$

Cross Correlation

	VTU Q.P. ↳ Definition:- Consider two random variables X and Y, observed at time instants 'K' and 'l' respectively, then the cross correlation function between random variables X and Y is given by $$\gamma_{XY}(K,l) = E[X(K) \cdot Y(l)]$$ (or) $$\gamma_{YX}(l,K) = E[Y(l) \cdot X(K)]$$ ∴ cross correlation function gives the correlation between two different random processes. ↳ Properties:- 1. It is symmetric function. i.e., $\gamma_{XY}(\tau) = \gamma_{XY}(-\tau)$ 2. It does not have maximum at origin 3. $\lvert\gamma_{XY}(\tau)\rvert \leq \frac{1}{2}[\gamma_X(0) + \gamma_Y(0)]$ where $\gamma_X(0)$ & $\gamma_Y(0)$ are auto-correlation functions of X and Y respectively at $\boxed{\tau=0}$
6 b	Mean: Mean value is also known as Expected value of random variable 'X'. It is the first order moment about the origin. It is denoted by $E[X]$ (or) m_X Where $E \rightarrow$ Expectation operator. Mathematically, mean of continuous random variable 'X' is given by, $$m_X = E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$$ Note: If Random Variable 'X' represents Voltage 'v', then mean ⟶ represents Average DC-value (V_{dc})

• Covariance :- It gives joint expectation of two-random -variables X and Y. It is denoted by symbol 'λ_{XY}'.

i.e., $\text{cov}(X,Y) = \lambda_{XY} = E\left[(X-m_x)(Y-m_y)\right]$

where m_x = Mean of random Variable 'X', m_y = Mean of random Variable 'Y' } Constants.

↳ Simplification:- W.K.T Covariance between X, Y is

$$\text{cov}(X,Y) = \lambda_{XY} = E\left[(X-m_x)(Y-m_y)\right]$$

$$= E\left[XY - Xm_y - m_xY + m_xm_y\right]$$

$$= E[XY] - E[X\underbrace{m_y}_{\text{Constant}}] - E[\underbrace{m_x}_{\text{Constant}}Y] + E[\underbrace{m_x m_y}_{\text{Const}}]$$

$$= E[XY] - m_y E[X] - m_x E[Y] + m_x m_y$$

$$= E[XY] - m_y m_x - m_x m_y + m_x m_y$$

$$= E[XY] - 2m_x m_y + m_x m_y$$

$$\boxed{\text{Cov}(X,Y) = \lambda_{XY} = E[XY] - m_x m_y} \quad \text{— ①}$$

Note: • If $E[XY] = 0$, then the random Variables X, Y are said to be Orthogonal.

• If two random Variables X and Y are Independent then they are said to be "Uncorrelated".

• For Independent Random variables, X and Y $E[XY] = E[X] \cdot E[Y] = m_x \cdot m_y$ and Covariance is zero (using equation ①)