Scheme of Evaluation
Internal Assessment Test 3 – May.2019

Sub:	System Modeling and Simulation							**Code:**	15CS834
Date:	18/05/2019	Duration:	90mins	Max Marks:	50	**Sem:**	VIII	**Branch:**	ISE

Note: Answer Any Five Questions

Question #		Description	Marks Distribution		Max Marks
1a.		• Explanation about binomial and Uniform distributions. • Formulaes for pmf, mean, variance.	3 M 2 M	5 M	10 M
1b.		• Discrete random variable explanation • Continuous random variable explanation	2.5 M 2.5M	5 M	
2		Different steps in the useful model of input data: • Collect data from the real system of interest • Identify a probability distribution • Choose parameters • Evaluate the chosen distribution along with explanation	2.5*4	10 M	10 M
3		Types of simulations w.r.t output analysis. • Explanation of Terminating simulation with examples • Explanation of Non-Terminating simulation with examples	5M 5M	10M	
4		Characteristics of Queuing System: • The Calling Population explanation • System Capacity • Arrival process • Queue behavior and Queue Discipline • Service Time and Service Mechanism Queuing Notations	8M (2+1+1+2+2) 2M	10 M	10 M
5	a)	Four methods to select input model without data: i)Engineering data ii) Expert Option iii) Physical or Conventional Limitations iv)The nature of Process along with explanation	1.5*4M	6 M	10 M
	b)	Types of Time-series input models: i) AR(1) Model ii) EAR(1) Model along with Explanation.	2M 2M	4M	

6	• Finding a1,a2……a7 • Finding E0 to E11 • Finding observed values • Chi-Square test table • Justification of Acceptance/Rejection	1 M 1M 2M 5M 1M	10 M	10 M
7	• Poisson variate calculation P(0),P(1),…P(11) • Finding E0 to E11 • Chi-Square test table • Justification of Acceptance/Rejection	1 M 1M 6M 2M	10 M	10 M

Internal Assessment Test 3 Solutions– May.2019

Sub:	System Modeling and Simulation							**Code:**	15CS834
Date**:**	20/05/2019	Duration:	90mins	Max Marks:	50	**Sem:**	VIII	**Branch:**	ISE

Note: Answer Any Five Questions

1. **Explain: i) Binomial Distribution ii)Uniform Distribution**

Solution:

i)Binomial Distribution

The no of successes in n Bernoulli trials is said to follow binomial distribution.

$P(x) = (n)\ p^x q^{n-x}$ for x=0,1,2,….n

Mean: E(x) = np

Variance : V(x) = npq

ii)Uniform Distribution

In probability theory and statistics, the **continuous uniform distribution** or **rectangular distribution** is a family of symmetric probability distributions such that for each member of the family, all intervals of the same length on the distribution's support are equally probable. The support is defined by the two parameters, a and b, which are its minimum and maximum values.

Pdf : f(x) = 1/b-a , a<x<b

Cdf : F(x) = x-a/b-a , a<x<b

Mean: E(x) = a+b/2

Variance: $v(x) = (b-a)^2/12$

1.b. Explain Discrete and continuous random variable with an example.

Solution:

Discrete Random Variables

- *X* is a discrete random variable if the number of possible values of *X* is finite, or countably infinite.

Example: Consider jobs arriving at a job shop.

Let *X* be the number of jobs arriving each week at a job shop.

Rx= possible values of *X* (range space of *X*) = *{0,1,2,…}*

p(xi) = probability the random variable is *xi isP(X=xi), i = 1,2, …* must satisfy:

1. $p(x_i) \geq 0$, for all *i*

2. $\sum p(x_i) = 1$

The collection of pairs $[x_i, p(x_i)]$, $i = 1,2,\ldots$, is called the probability distribution of X, and $p(x)$ is called the probability mass function (pmf) of X.

Continuous Random Variables

X is a continuous random variable if its range space R is an interval or a collection of intervals.

The probability that X lies in the interval $[a,b]$ is given by:

b

$\int P(a = X = b) = f(x)dx$

a

$f(x)$, denoted as the pdf of X, satisfies:

1. $f(x) = 0$, for all x in R
2. $f(x)dx = 1$
3. $f(x) = 0$, if x is not in R

2. Explain different steps in the development of a useful model of input data

Solution:

There are 4 steps in the ...
Sol:-
useful model of input data.

1.) collect data from the real system of interest.

L this requires a substantial time and resource. In some cases it is not possible to collect data for ex when time is limited (or) when the i/p process does not exist.

— when data are not available expert opinion and knowledge of the process must be used.

2) Identify a probability distribution to represent the input process.

— when data are available this step begins with the development of frequency distrn (or) histogram of the data.

3) choose parameters that determine a specific instance of the distrn family.

4) Evaluate the choosen distribution and the associated parameters for goodness of fit.

— chi-square and kolmogorov-smirnov tests are goodness of fit tests. If the choosen distrn is not satisfied from these tests then the analyst returns to second step and chooses a different family of distributions.

Data collection :- (2) Explain data collection in details

Data collection is very important but hard to achieve there are 2 approaches

1. classical approach

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3. **Explain the types of simulation w.r.t output analysis**

Solution:

Types of Simulations with respect to output analysis

There are 2 types of simulations

1. Terminating simulations
2. Steady State Simulations

A terminating simulation is one that runs for some duration of time T_E where E is the specified event that stops the simulation.

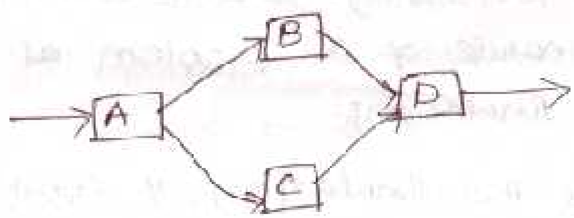
Ex:- A retail shop closes every evening from which it opens from 9am to 5 pm.

↳ Here E = atleast 7 hours of time has been used

2) A company which sells a product would like to decide how many items to have in inventory during planning for 100 months.

↳ Here E = 100 months

3) A communication system consists of several components plus several backup components, as in fig.



Here E = consider the system over a period of time T_E until system fails.

E = {A fails, (or) D fails (or) B and C fails}

- In this case we cannot predict E in advance bcz we do not know when the component fails.

4. Explain the characteristics of queuing system. Briefly explain queuing notations

characteristics of Queuing system

The key elements of queuing system are the customers and servers.

customers :— refers to people, m/c, trucks, mechanics, Patients airplanes, email etc..

servers :— refers to receptionists, repair person, cpu, any resource that provides requested service

the following table gives few examples of queuing system

system	customer	server
Airport	Airplanes	Runway
Hospital	patients	Nurses, doctor
Reception desk	people	Receptionist
~~theatre~~		

The term customer means anything that arrives at a facility and requires service.

→ the characteristics of queuing system are

- calling population
- system capacity
- Arrival process
- Queue behaviour and queue discipline
- service time and service mechanism

The calling population

the population of potential customers is referred as calling population.

→ the calling population can be either finite (or) infinite

→ the main difference b/n finite and infinite is based on how arrival rate is defined.

→ In infinite population the arrival rate is not affected by the no of customers who have left the calling population and joining the queuing system.

Ex:- Hotel

→ In finite population the arrival rate depends on the number of customers being served and waiting.

Ex:- ~~Ba~~ In a ~~bank~~ hospital where there are prior appointments the customers are patients here and the arrival rate of patients depends on finite arrivals.

2. System capacity :-

System capacity is defined as the maximum no of customers allowed in system (or) in waiting queue.

Ex:- An automatic car wash might have room for only 10 cars to wait in line to enter the mechanism. An arriving customer who finds the system full does not enter but returns immediately to the calling population.

— Some systems such as concert ticket sales for students may be considered to have unlimited capacity

3. The arrival process :-

Arrival process describe how customer arrival, how the arrival are distributed in time and whether there is finite population model (or) infinite population model.

→ Arrivals may occur at scheduled times (or) random times.

→ The most important model for random arrivals is the poisson arrival process.

↳ If A_n represents IAT b/n customer $n-1$ and customer n then for poisson arrival process A_n is exponentially distributed with mean $1/\lambda$ time units.

4) Queue behaviour and Queue discipline

Queue behaviour refers to the action of customer while in queue waiting for service to begin.

Queue behaviour are

→ Balk — leave when they see that the line is too long.

→ Renege — leave after being in the line when they see that the line is moving too slowly.

→ Jockey — move from one line to another if they think they have choosen a slow line.

— Queue discipline refers to how the customers are severed in the queue

- FIFO, LIFO,
- SIRO — service in random

5) Service times and service mechanism

Service times are denoted by $S_1, S_2, S_3, \ldots$

They may be constant (or) of random duration.

the exponential, weibull, gamma, lognormal distributions are used to model the service times.

A queuing system consists of no of service centers and interconnecting queues each service center consists of no of servers c working in parallel. parallel service mechanisms are either single server ($c=1$), multiple server (or) unlimited servers.

5.a.Explain four methods of selecting input models without data

<u>Solution:</u>

b) :- engineering data :-

- often a product (or) process has performance ratings provided by the manufacturer.

Ex :- mean time to failure of a disk drive is 10000 hrs a laser printer can produce 8 pages/min etc.

- company rules might specify time (or) production standards.

- Expert option :-

- Talk to people who are experienced with the process. they can provide optimistic, pessimistic and most likely times.

- they might also able to say whether the process is nearly constant (or) highly variable and they can define the source of variability.

- physical or conventional limitations :-

- most real processes have physical limits on performance

Ex :- computer data entry cannot be faster than a person can type.

- Because of company policies there could be upper limits on how long a process may take and do not ignore it.

- the nature of the process :-

when data are not available then uniform, triangular, and beta distributions are often used as I/P models.

- uniform distn - poor choice bcz upper and lower bounds are likely as central values in real process.

- Triangular distn - in addition to upper and lower bounds a most likely value is given then use triangular distn.

- Beta distn - density function has to be plotted

5.b)Explain the types of time series input models.

<u>Solution:</u>

Two models are

- Auto regressive order-1 model AR(1)
- Exponential autoregressive order-1 model EAR(1)

<u>AR(1) model</u> :-

consider the time series model

$$x_t = \mu + \phi(x_{t-1} - \mu) + \varepsilon_t$$

for $t = 2, 3, \ldots$ where $\varepsilon_2, \varepsilon_3$ are independent and identically distributed with mean $= 0$ and variance $= \sigma_\varepsilon^2$.

If the initial value x_1 is choosen appropriately then $x_1, x_2, \ldots$ are normally distributed with mean μ variance $\sigma_\varepsilon^2/(1-\phi^2)$ and

$$\rho_h = \phi^h$$

for $h = 1, 2, \ldots$

- the time series model is called the AR(1) model
- Estimation of ϕ can be obtained from

$$\phi = \rho^1 - corr(x_t, x_{t+1})$$

The following algorithm generates a stationary AR(1) time series given ϕ, μ and σ_ε^2.

1. Generate x_1 from normal distn with mean μ and variance $\sigma_\varepsilon^2/(1-\phi^2)$ set $t = 2$
2. Generate ε_t from normal distribution with mean 0 and variance σ_ε^2.
3. Set $x_t = \mu + \phi(x_{t-1} - \mu) + \varepsilon_t$
4. Set $t = t+1$ and goto step 2.

<u>EAR(1) model</u> :-

Consider the time series model

$= \phi x_{t-1} + \varepsilon_t$, with prob $1-\phi$

for $t = 2, 3, \ldots$ where $\varepsilon_2, \varepsilon_3, \ldots$ are independent and identically distributed with mean $1/\lambda$.

-- If the initial value of x is chosen appropriately then $x_1, x_2, \ldots$ are exponentially distributed with mean $1/\lambda$ and

$$\rho_h = \phi^h$$

for $h = 1, 2, \ldots$

The time series model is called EAR(1) model

- the following algorithm generates a stationary EAR(1) time series given ϕ and λ.

1. generate x_1 from exponential distribution with mean $= 1/\lambda$ and set $t=2$.

2. generate U from uniform distribution on (0,1].

If $U \leq \phi$ then set

$$x_t = \phi x_{t-1}$$

otherwise generate ε_t from exponential distn with mean $1/\lambda$ and set

$$x_t = \phi x_{t-1} + \varepsilon_t$$

3. set $t = t+1$ and go to step 2.

6) **Chi-Square test for continuous data**

Solution:

- Finding P value
    - P=1/k = 1/6 = 0.1667

- Finding Expected Values
    - Ei=nPi=50*0.1667 = 8.33

- Finding A0,A1,….values

  Ai=-1/λln(1-iP)

  A0=-1/1.206(1-0*0.1667)=0

A1=-1/1.206ln(1-1*0.1667) = 0.1512
A2= -1/1.206 ln(1-2*0.1667) = 0.3362
A3= -1/1.206 ln(1-3*0.1667) = 0.5749
A4= -1/1.206 ln(1-4*0.1667) = 0.9112
A5 = -1/1.206 ln(1-5*0.1677) = 1.4865

- Chi-square table

Interval	**Oi**	**Ei**	**Oi-Ei**	**(Oi-Ei)2**	**(Oi-Ei)2/Ei**
0- 0.1512	6	8.33	2.33	5.428	0.6517
0.1512 - 0.3362	6	8.33	2.33	5.428	0.6517
0.3362 - 0.5749	10	8.33	1.67	2.788	0.334
0.5749 - 0.9112	7	8.33	1.33	1.768	0.2123
0.9112 - 1.4865	5	8.33	3.33	11.088	1.331
1.4865 - ∞	16	8.33	7.67	58.82	7.0622
Total					**10.24**

- Justification of acceptance
  - 10.24 > 9.49 so the data given are rejected.

7) Chi-Square test for discrete data
Solution:
For Poisson distribution $P(x) = e^{-\alpha}\alpha^{x} / x!$ for x=0,1,2,…..

Compute P(0),P(1),P(2),……P(11) as follows
$P(0)= e^{-3.64}(3.64)^{0} / 0! = 0.026$
$P(1)= e^{-3.64}(3.64)^{1} / 1! = 0.096$
$P(2)= e^{-3.64}(3.64)^{2} / 2! = 0.174$
$P(3)= e^{-3.64}(3.64)^{3} / 3! = 0.211$
……till P(11) as follows

Chi-square test Table

$$P(4) = \frac{e^{-3.64}(3.64)^4}{4!} = 0.192$$

$$P(8) = \frac{e^{-3.64}(3.64)^8}{8!} = 0.020$$

$$P(5) = \frac{e^{-3.64}(3.64)^5}{5!} = 0.140$$

$$P(9) = \frac{e^{-3.64}(3.64)^9}{9!} = 0.008$$

$$P(6) = \frac{e^{-3.64}(3.64)^6}{6!} = 0.085$$

$$P(10) = \frac{e^{-3.64}(3.64)^{10}}{10!} = 0.003$$

$$P(7) = \frac{e^{-3.64}(3.64)^7}{7!} = 0.044$$

$$P(11) = \frac{e^{-3.64}(3.64)^{11}}{11!} = 0.001$$

chi-square test table

x_i	observed frequency O_i	expected frequency E_i $(n \times p_i)$	$\frac{(O_i-E_i)^2}{E_i}$
0	12 } 22	100×0.026 = 2.6 } 12.2	7.87
1	10	100×0.096 = 9.6	
2	19	100×0.174 = 17.4	0.15
3	17	100×0.211 = 21.1	0.80
4	10	100×0.192 = 19.2	4.41
5	8	100×0.140 = 14.0	2.57
6	7	100×0.085 = 8.5	0.26
7	5 } 17	100×0.044 = 4.4 } 7.6	11.62
8	5	100×0.020 = 2.0	
9	3	100×0.008 = 0.8	
10	3	100×0.003 = 0.3	
11	1	100×0.001 = 0.1	
			27.68

– Always we have to see that the expected freq values should be >5. If not then combine with previous (or) next value until it becomes >5.

Here $X^2_{\alpha,K-S-1} = 7-1-1 = 5$

K = No of intervals divided, S= No of parameters estimated i.e only α is given

$X^2_{0.05,5} = 11.1$

- Justification of acceptance

The computed value 27.61 > 11.1 so the hypothesis is rejected