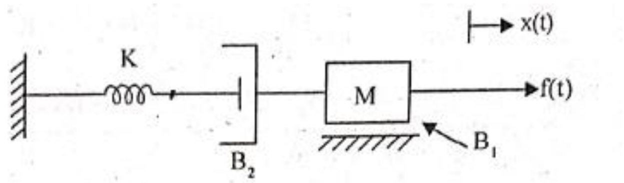
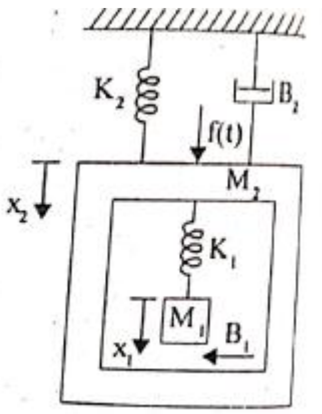
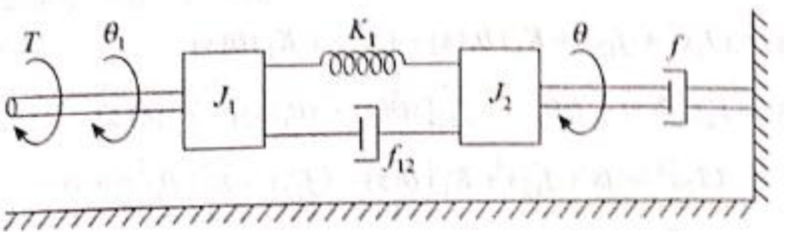
CMR
INSTITUTE OF
TECHNOLOGY

USN

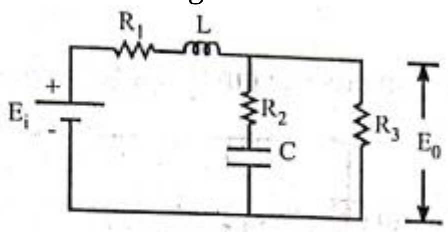
Internal Assesment Test - I

Sub:	**CONTROL SYSTEMS**							Code:	15EE61
Date:	05/03/2019	Duration:	90 mins	Max Marks:	50	Sem:	6th	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	a. Differentiate open loop and closed loop control system with examples.	5	CO1	L1
	b. What are fundamental components of mechanical rotational systems? Explain with equations	5	CO2	L1
2	For the mechanical system shown in Fig 2 obtain the transfer function X(s)/F(s)  Fig 2	10	CO2	L3
3	Write the differential equations governing the following system shown in Fig 3 and obtain the analogous F-V circuit and F-I circuit.  Fig 3	10	CO2	L3
4	Write the differential equations governing the following system shown in fig 4 and obtain the transfer function Θ(s)/T(s)  Fig 4	10	CO2	L3

P.T.O

5	Obtain the transfer function of the given network in the Fig 5  Fig 5	10	CO2	L3
6	Using relevant equations obtain the mathematical model armature controlled dc motor.	10	CO2	L2

Answers

1. a

Open-loop control system	*Closed-loop control system*
1. The open-loop systems are simple and economical.	1. The closed-loop systems are complex and costlier.
2. They consume less power.	2. They consume more power.
3. The open-loop systems are easier to construct because of less number of components required.	3. The closed-loop systems are not easy to construct because of more number of components required.
4. Stability is not a major problem in open-loop control systems. Generally, the open-loop systems are stable.	4. Stability is a major problem in closed-loop control systems and more care is needed to design a stable closed-loop system.
5. The open-loop systems are inaccurate and unreliable.	5. The closed-loop systems are accurate and more reliable.
6. The changes in the output due to external disturbances are not corrected automatically. So they are more sensitive to noise and other disturbances.	6. The changes in the output due to external disturbances are corrected automatically. So they are less sensitive to noise and other disturbances.
	7. The feedback reduces the overall gain of the system.
	8. The feedback in a closed-loop system may lead to oscillatory response, because it may over correct errors, thus causing oscillations of constant or changing amplitude.

b.

angular acceleration α. The reaction torque T_j is equal to the product of J and angular acceleration. That is

$$T_j = J\alpha = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt} \tag{1.65}$$

where J = Moment of inertia, kg-m^2
θ = angular displacement, rad
$\omega = \frac{d\theta}{dt}$ = angular velocity, rad/sec
$\alpha = \frac{d^2\theta}{dt^2}$ = angular acceleration, rad/sec^2
T_j = reaction torque, N-m

By Newton's second law, reaction torque is equal to applied torque.

$$T = T_j \tag{1.66}$$

$$\therefore T = J\frac{d^2\theta}{dt^2} \tag{1.67}$$

The elastic deformation of the body can be represented by a spring constant. When a torque 'T' is applied to a spring as shown in Fig. 1.27, it is twisted by an angle θ. The spring will produce an opposing torque 'T_k' which is proportional to angular displacement

$$T_k \propto \theta \tag{1.68}$$

$$T_k = k\theta \tag{1.69}$$

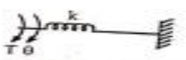


Fig. 1.27 A Spring (one end fixed).

Where k is the spring stiffness constant.

By Newton's second law

$$T = T_k \tag{1.70}$$

$$\therefore T = k\theta \tag{1.71}$$

When the spring has angular displacement at both ends as shown in Fig. 1.28, the opposing Torque is proportional to the difference between the angular displacement. That is

$$T_k \propto (\theta_1 - \theta_2) \tag{1.72}$$

$$T_k = k(\theta_1 - \theta_2) \tag{1.73}$$

Using Newton's law we have,

$$\Longrightarrow T = T_k \tag{1.74}$$

$$\therefore T = k(\theta_1 - \theta_2) \tag{1.75}$$

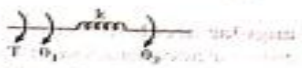


Fig. 1.28 A spring (both ends free).

Damping occurs whenever a body moves through a fluid. The damping is represented by a dash-pot with a viscous friction coefficient B. Whenever a torque T is applied as shown in Fig. 1.29, it is opposed by the damping torque T_b which is equal to the product of B and the angular velocity of the dash-pot.

$$T_b \propto \omega \tag{1.76}$$

$$\Longrightarrow T_b = B\omega \text{ and } T = T_b \tag{1.77}$$

$$\therefore T = B\omega = B\frac{d\theta}{dt} \tag{1.78}$$

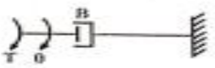


Fig. 1.29 A Dash pot (one end fixed).

If both ends of the dash-pot are not fixed as shown in Fig. 1.30, then the angular velocity is measured at both ends of the dash-pot.

$$T_b = B(\omega_1 - \omega_2) \tag{1.79}$$

$$= B\left[\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right] \tag{1.80}$$

$$\therefore T = B\left[\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right] \tag{1.81}$$

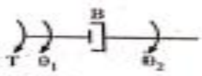


Fig. 1.30 A Dash pot (Both ends free).

Table 1.3 Variables and Parameters of Mechanical Rotational System.

Symbol	Quantity	Units
$T(t)$	Torque	Newton-meter
$\theta(t)$	Angular displacement	radians
$\omega(t)$	Angular velocity	rad/sec
$\alpha(t)$	Angular acceleration	rad/sec^2
J	Moment of inertia	kg-m^2
k	Stiffness constant	N-m/rad
B	Damping-torque	N-m/rad/sec

2.

The free body diagram of mass M is shown in fig 2. The opposing forces are marked as f_m, f_{b1} and f_{b2}.

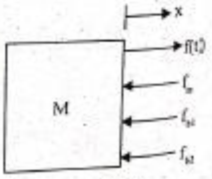


Fig 2

$$f_m = M\frac{d^2x}{dt^2} \;;\; f_{b1} = B_1\frac{dx}{dt} \;;\; f_{b2} = B_2\frac{d}{dt}(x - x_1)$$

By Newton's second law the force balance equation is,

$$f_m + f_{b1} + f_{b2} = f(t)$$

$$\therefore M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B_2\frac{d}{dt}(x - x_1) = f(t)$$

On taking Laplace transform of the above equation we get,

$$Ms^2\,X(s) + B_1\,s\,X(s) + B_2\,s\,[X(s) - X_1(s)] = F(s)$$

$$\left[Ms^2 + (B_1 + B_2)\,s\right]X(s) - B_2\,s\,X_1(s) = F(s) \qquad .....(1)$$

... is shown in fig 3. The oppos...

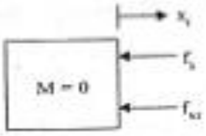


Fig 3

$$f_{b2} = B_2\frac{d}{dt}(x_1 - x)\;;\quad f_k = K\,x_1$$

By Newton's second law, $f_{b2} + f_k = 0$

$$\therefore B_2\frac{d}{dt}(x_1 - x) + K\,x_1 = 0$$

On taking Laplace transform of the above equation we get,

$$B_2\,s\,[X_1(s) - X(s)] + K\,X_1(s) = 0$$

$$(B_2\,s + K)\,X_1(s) - B_2\,s\,X(s) = 0$$

$$\therefore X_1(s) = \frac{B_2\,s}{B_2\,s + K}\,X(s) \qquad .....(2)$$

Substituting for $X_1(s)$ from equation (2) in equation (1) we get,

$$\left[M\,s^2 + (B_1 + B_2)\,s\right]X(s) - B_2\,s\left[\frac{B_2\,s}{B_2\,s + K}\right]X(s) = F(s)$$

$$X(s)\,\frac{\left[[M\,s^2 + (B_1 + B_2)\,s]\,(B_2\,s + K) - (B_2\,s)^2\right]}{B_2\,s + K} = F(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{B_2\,s + K}{[M\,s^2 + (B_1 + B_2)\,s]\,(B_2\,s + K) - (B_2\,s)^2}$$

RESULT

The differential equations governing the systems are,

1. $M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B_2\frac{d}{dt}(x - x_1) = f(t)$
2. $B_2\frac{d}{dt}(x_1 - x) + K\,x_1 = 0$

The equations of motion in s-domain are,

1. $[M\,s^2 + (B_1 + B_2)\,s]\,X(s) - B_2\,s\,X_1(s) = F(s)$
2. $(B_2\,s + K)\,X_1(s) - B_2\,s\,X(s) = 0$

The transfer function of the system is,

$$\frac{X(s)}{F(s)} = \frac{B_2\,s + K}{[M\,s^2 + (B_1 + B_2)\,s]\,(B_2\,s + K) - (B_2\,s)^2}$$

3.

The free body diagram of M_1 is shown in fig 2. The opposing forces are marked as f_{m1}, f_{b1} and f_{k1}.

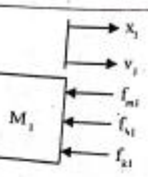


Fig 2

$$f_{m1} = M_1 \frac{d^2x_1}{dt^2} \;;\; f_{b1} = B_1 \frac{d(x_1 - x_2)}{dt} \;;\; f_{k1} = K_1(x_1 - x_2)$$

By Newton's second law, $f_{m1} + f_{b1} + f_{k1} = 0$

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d(x_1 - x_2)}{dt} + K_1(x_1 - x_2) = 0 \quad .....(1)$$

The free body diagram of M_2 is shown in fig 3. The opposing forces are marked as f_{m2}, f_{b2}, f_{b1}, f_{k2} and f_{k1}.

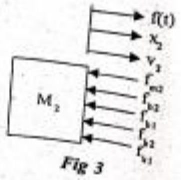


Fig 3

$$f_{m2} = M_2 \frac{d^2x_2}{dt^2} \;;\; f_{b2} = B_2 \frac{dx_2}{dt} \;;\; f_{b1} = B_1 \frac{d}{dt}(x_2 - x_1)$$

$$f_{k2} = K_2 x_2 \;;\; f_{k1} = K_1(x_2 - x_1)$$

By Newton's second law, $f_{m2} + f_{b2} + f_{k2} + f_{b1} + f_{k1} = f(t)$

$$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + B_1 \frac{d}{dt}(x_2 - x_1) + K_1(x_2 - x_1) = f(t) \quad .....(2)$$

On replacing the displacements by velocity in the differential equations (1) and (2) governing the mechanical system we get,

$$\left(\text{i.e., } \frac{d^2x}{dt^2} = \frac{dv}{dt}, \quad \frac{dx}{dt} = v \text{ and } x = \int v\,dt\right)$$

$$M_1 \frac{dv_1}{dt} + B_1(v_1 - v_2) + K_1 \int (v_1 - v_2)dt = 0 \quad .....(3)$$

$$M_2 \frac{dv_2}{dt} + B_2 v_2 + K_2 \int v_2 dt + B_1(v_2 - v_1) + K_1 \int (v_2 - v_1)dt = f(t) \quad .....(4)$$

FORCE-VOLTAGE ANALOGOUS CIRCUIT

The given mechanical system has two nodes (masses). Hence the force voltage analogous electrical circuit will have two meshes. The force applied to mass, M_2 is represented by a voltage source in second mesh.

The elements M_1, K_1 and B_1 are connected to first node. Hence they are represented by analogous element in mesh 1 forming a closed path. The elements M_2, K_2, B_2, B_1 and K_1 are connected to second node. Hence they are represented by analogous element in mesh 2 forming a closed path.

The elements B_1 and K_1 are common between node 1 and 2 and so they are represented as common elements between mesh 1 and 2. The force-voltage electrical analogous circuit is shown in fig 4.

The electrical analogous elements for the elements of mechanical system are given below.

$f(t) \rightarrow e(t)$	$v_1 \rightarrow i_1$	$M_1 \rightarrow L_1$	$K_1 \rightarrow 1/C_1$	$B_1 \rightarrow R_1$
	$v_2 \rightarrow i_2$	$M_2 \rightarrow L_2$	$K_2 \rightarrow 1/C_2$	$B_2 \rightarrow R_2$

The mesh basis equations using Kirchoff's voltage law for the circuit shown in fig 4 are given below (refer fig 5 and 6)

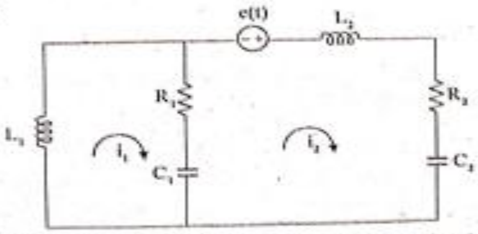


Fig 4 : Force-voltage electrical analogous circuit

FORCE-CURRENT ANALOGOUS CIRCUIT

The given mechanical system has two nodes (masses). Hence the force-current analogous electrical circuit will have two nodes. The force applied to mass M_2 is represented as a current source connected to node2 in analogous electrical circuit.

The elements M_1, K_1 and B_1 are connected to first node. Hence they are represented by analogous elements as elements connected to node 1 in analogous electrical circuit. The elements M_2, K_2, B_2 & K_1 are connected to second node. Hence they are represented by analogous elements as elements connected to node 1 in analogous electrical circuit.

The elements K_1 and B_1 is common to node 1 and 2 and so they are represented by analogous element as common elements between two nodes in analogous circuit. The force-current electrical analogous circuit is shown in fig 7.

The electrical analogous elements for the elements of mechanical system are given below

$f(t) \to i(t)$ $v_1 \to v_1$ $M_1 \to C_1$ $B_1 \to 1/R_1$ $K_1 \to 1/L_1$

$v_2 \to v_2$ $M_2 \to C_2$ $B_2 \to 1/R_2$ $K_2 \to 1/L_2$

The node basis equations using Kirchoff 's current law for the circuit shown in fig (7) are given below [Refer fig (8) and (9)].

$$C_1 \frac{dv_1}{dt} + \frac{1}{R_1}(v_1 - v_2) + \frac{1}{L_1}\int (v_1 - v_2)\, dt = 0$$

$$C_2 \frac{dv_2}{dt} + \frac{1}{R_2} v_2 + \frac{1}{L_2}\int v_2 dt + \frac{1}{R_1}(v_2 - v_1) + \frac{1}{L_1}\int (v_2 - v_1) dt = i(t)$$

It is observed that the node basis equations (7) and (8) are similar to the differential equations and (4) governing the mechanical system.

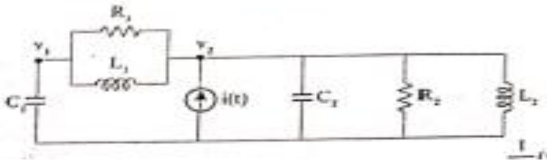


Fig 7 : Force-current electrical analogous circuit

4.

Taking the Laplace transform of Eqs. (i) and (ii) with zero initial conditions,

$$T(s) = J_1 s^2 \theta_1(s) + f_{12}[s\theta_1(s) - s\theta(s)] + K_1[\theta_1(s) - \theta(s)]$$

$$T(s) = (J_1 s^2 + f_{12}s + K_1)\,\theta_1(s) - (f_{12}s + K_1)\theta(s)$$

$$0 = J_2 s^2 \theta(s) + fs\theta(s) + f_{12}[s\theta(s) - s\theta_1(s)] + K_1[\theta(s) - \theta_1(s)]$$

$$(J_2 s^2 + fs + f_{12}s + K_1)\,\theta(s) - (f_{12}s + K_1)\,\theta_1(s) = 0$$

$$\theta_1(s) = \frac{(J_2 s^2 + fs + f_{12}s + K_1)\,\theta(s)}{f_{12}s + K_1}$$

Substituting the value of $\theta_1(s)$ from Eq. (iv) in Eq. (iii), we get

$$T(s) = (J_1 s^2 + f_{12}s + K_1)\frac{[J_2 s^2 + (f + f_{12})s + K_1]\,\theta(s)}{f_{12}s + K_1} - (f_{12}s + K_1)\,\theta(s)$$

$$T(s) = \left[\frac{J_1 J_2 s^4 + (J_2 f_{12} + J_1 f + J_1 f_{12})s^3 + (J_1 K_1 + J_2 K_1 + ff_{12} + f_{12}^{\,2})s^2 + (2K_1 f_{12} + K_1 f)s + K_1^2 - f_{12}^{\,2}s^2 - K_1^2 - 2f_{12}sK_1}{f_{12}s + K_1}\right]\theta(s)$$

$$T(s) = \left[\frac{J_1 J_2 s^4 + (J_2 f_{12} + J_1 f + J_1 f_{12})s^3 + (J_1 K_1 + J_2 K_1 + ff_{12})s^2 + K_1 fs}{f_{12}s + K_1}\right]\theta(s)$$

Therefore, the transfer function is

$$\frac{\theta(s)}{T(s)} = \frac{f_{12}s + K_1}{s[J_1 J_2 s^3 + (J_2 f_{12} + J_1 f + J_1 f_{12})s^2 + (J_1 K_1 + J_2 K_1 + ff_{12})s + K_1 f]}$$

5.

Loop 1:

$$E_i = R_1 i_1 + L\frac{di_1}{dt} + R_2(i_1 - i_2) + \frac{1}{C}\int (i_1 - i_2)dt \qquad (1.194)$$

Loop 2:

$$R_3 i_2 + \frac{1}{C}\int (i_2 - i_1)dt + R_2(i_2 - i_1) = 0 \qquad (1.195)$$

$$E_0 = R_3 i_2 \qquad (1.196)$$

Taking Laplace transform of the equations by assuming zero initial conditions on both sides, we have

$$E_i(s) = R_1 I_1(s) + LsI_1(s) + R_2 I_1(s) - R_2 I_2(s) + \frac{1}{Cs}[I_1(s) - I_2(s)] \qquad (1.197)$$

$$R_3 I_2(s) + \frac{1}{Cs}[I_2(s) - I_1(s)] + R_2[I_2(s) - I_1(s)] = 0 \qquad (1.198)$$

$$E_o(s) = R_3 I_2(s) \qquad (1.199)$$

$$E_i(s) = I_1(s)\left[R_1 + R_2 + Ls + \frac{1}{Cs}\right] - I_2(s)\left[R_2 + \frac{1}{Cs}\right]$$

$$0 = -\left(R_2 + \frac{1}{Cs}\right) I_1(s) + \left(R_3 + R_2 + \frac{1}{Cs}\right) I_2(s)$$

Set of equations governing the system can be presented in a matrix form as follows:

$$\begin{bmatrix} R_1 + R_2 + Ls + \frac{1}{Cs} & -(R_2 + \frac{1}{Cs}) \\ -(R_2 + \frac{1}{Cs}) & (R_3 + R_2 + \frac{1}{Cs}) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} E_i(s) \\ 0 \end{bmatrix}$$

Simplifying Eq. (1.194), Eq. (1.195) and Eq. (1.196) we have,

$$I_2(s) = \frac{E_i(s)\left(R_2 + \frac{1}{Cs}\right)}{\left(R_1 + R_2 + Ls + \frac{1}{Cs}\right)\left(R_3 + R_2 + \frac{1}{Cs}\right) - \left(R_2 + \frac{1}{Cs}\right)^2}$$

$$\frac{E_0(s)}{E_i(s)} = \frac{R_3\left(R_2 + \frac{1}{Cs}\right)}{\left(R_1 + R_2 + Ls + \frac{1}{Cs}\right)\left(R_2 + R_3 + \frac{1}{Cs}\right) - \left(R_2 + \frac{1}{Cs}\right)^2}$$

6.

$$e_b = k_b n = k_b \omega$$
$$= k_b \frac{d\theta}{dt} \quad (1.184)$$

where

k_b = back emf constant

θ = angular displacement (rad)

Applying Laplace transform with zero initial conditions yield

$$E_b(s) = k_b s\theta(s) \quad (1.185)$$

The differential equation of the armature circuit is

$$e_a = L_a \frac{di_a}{dt} + R_a i_a + e_b \quad (1.186)$$

Applying Laplace transform on both sides we get

$$E_a(s) = sL_a I_a(s) + R_a I_a(s) + E_b(s) \quad (1.187)$$

The torque developed by the motor T_M is a function of the flux developed by the field current and armature current. Since the field current is constant, the torque can be expressed as

$$T_M = k_T i_a \quad (1.188)$$

where k_T = Torque constant of the motor having units of N - m / A

$$\Rightarrow \quad T_M(s) = k_T I_a(s) \quad (1.189)$$

Torque T_M drives the mechanical load and is given by

$$T_M = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \quad (1.190)$$
$$\Rightarrow \quad T_M(s) = Js^2\theta(s) + Bs\theta(s)$$

From Eq. (1.189) and Eq. (1.190) we obtain

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = k_T i_a \quad (1.191)$$

Taking Laplace transform on both sides yields

$$Js^2\theta(s) + Bs\theta(s) = k_T I_a(s)$$
$$\theta(s)[Js^2 + Bs] = k_T I_a(s) \quad (1.192)$$
$$\Rightarrow I_a(s) = \theta(s)\frac{(Js^2 + Bs)}{k_T} \quad (1.193)$$

Substituting Eq. (1.193) and Eq.(1.185) in Eq. (1.187) yields

$$E_a(s) = (sL_a + R_a)\frac{(Js^2 + Bs)}{k_T}\theta(s) + k_b s\theta(s)$$
$$= \theta(s)\left[\frac{(sL_a + R_a)(Js^2 + Bs) + k_T k_b s}{k_T}\right] \quad (1.194)$$

$$\Rightarrow \quad \frac{\theta(s)}{E_a(s)} = \frac{k_T}{JL_a s^3 + (R_a J + L_a B)s^2 + (R_a B + k_b k_T)s} \quad (1.195)$$

Dividing the numerator and denominator by $k_b k_T$ we get

$$\frac{\theta(s)}{E_a(s)} = \frac{1/k_b}{s\left[\frac{JL_a}{k_b k_T}s^2 + \left(\frac{R_a J}{k_b k_T} + \frac{L_a B}{k_b k_T}\right)s + \left(\frac{R_a B}{k_b k_T} + 1\right)\right]}$$
$$= \frac{1/k_b}{s\left[T_a T_m s^2 + (T_m + \tau T_a)s + (\tau + 1)\right]} \quad (1.196)$$

where

$$T_a = \frac{L_a}{R_a}; \quad T_m = \frac{JR_a}{k_b k_T}; \quad \tau = \frac{R_a B}{k_b k_T} \quad (1.197)$$

The block diagram of the armature controlled DC motor is shown in Fig.1.84.

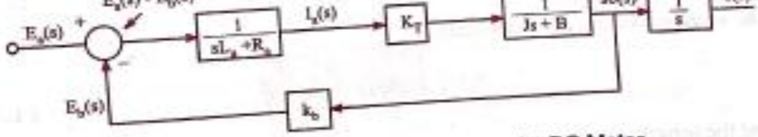


Fig. 1.84 Block diagram representation of a DC Motor.