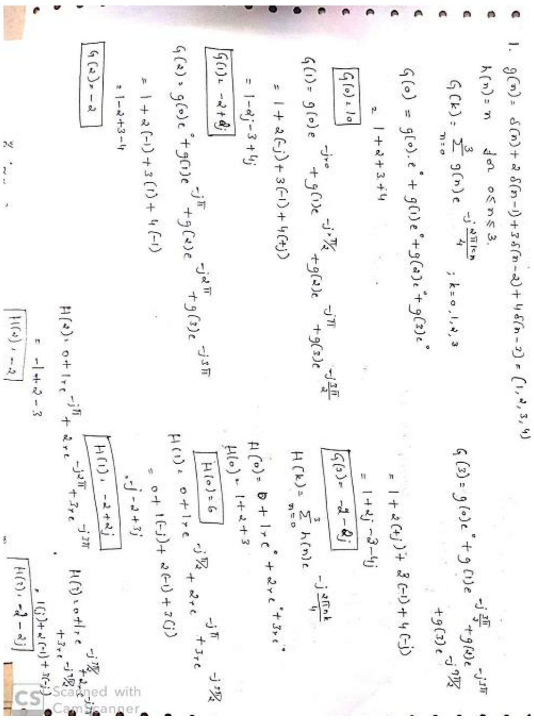
DSP IAT 2 QP and Scheme and Solution

1. Using STOCK HAM's method find circular convolution of the sequences, $g(n)=\delta(n) + 2\,\delta(n-1) + 3\,\delta(n-2) + 4\,\delta(n-3)$ and $h(n) = n$ for $0\le n\le 3$.

DFT of g(n) – 3 marks

DFT of h(n) – 3 marks

IDFT of y(n) – 4 marks



1. $g(n) = \delta(n)+2\delta(n-1)+3\delta(n-2)+4\delta(n-3) = (1,2,3,4)$

$h(n) = n$ for $0\le n\le 3$.

$G(k) = \sum_{n=0}^{3} g(n)e^{-j\frac{2\pi kn}{4}}$; $k=0,1,2,3$

$G(0) = g(0)e^0 + g(1)e^0 + g(2)e^0 + g(3)e^0$

$= 1+2+3+4$

$\boxed{G(0)=10}$

$G(1) = g(0)e^{-j0} + g(1)e^{-j\pi/2} + g(2)e^{-j\pi} + g(3)e^{-j\frac{3\pi}{2}}$

$= 1+2(-j)+3(-1)+4(+j)$

$= 1-2j-3+4j$

$\boxed{G(1) = -2+2j}$

$G(2) = g(0)e^0 + g(1)e^{-j\pi} + g(2)e^{-j2\pi} + g(3)e^{-j3\pi}$

$= 1+2(-1)+3(1)+4(-1)$

$= 1-2+3-4$

$\boxed{G(2) = -2}$

$G(3) = g(0)e^0 + g(1)e^{-j\frac{3\pi}{2}} + g(2)e^{-j3\pi} + g(3)e^{-j\frac{9\pi}{2}}$

$= 1+2(+j)+3(-1)+4(-j)$

$= 1+2j-3-4j$

$\boxed{G(3) = -2-2j}$

$H(k) = \sum_{n=0}^{3} h(n)e^{-j\frac{2\pi nk}{4}}$

$H(0) = 0+1e^0+2e^0+3e^0$

$H(0) = 1+2+3$

$\boxed{H(0)=6}$

$H(1) = 0+1e^{-j\pi/2}+2e^{-j\pi}+3e^{-j\frac{3\pi}{2}}$

$= 0+1(-j)+2(-1)+3(j)$

$= -j-2+3j$

$\boxed{H(1) = -2+2j}$

$H(2) = 0+1e^{-j\pi}+2e^{-j2\pi}+3e^{-j3\pi}$

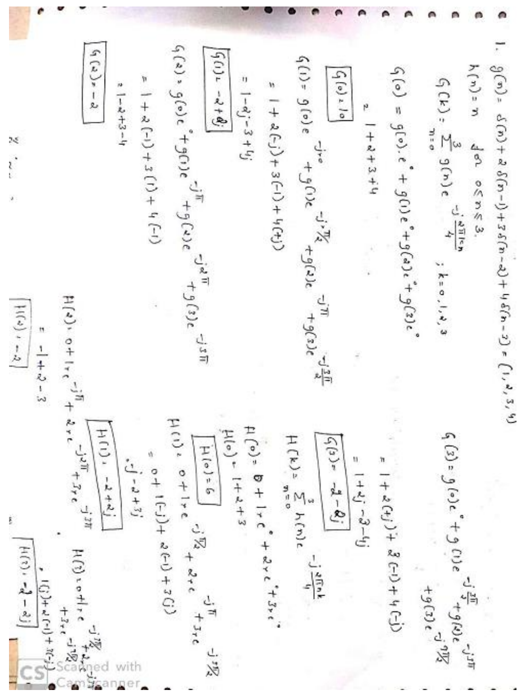
$= -1+2-3$

$\boxed{H(2) = -2}$

$H(3) = 0+1e^{-j\frac{3\pi}{2}}+2e^{-j3\pi}+3e^{-j\frac{9\pi}{2}}$

$= 1(j)+2(-1)+3(-j)$

$\boxed{H(3) = -2-2j}$



2. Find the output y(n) of a filter whose impulse response is given by h(n)=(3,2,1,1) and input signal is given by x(n)=(1, 2, 3, 3, 2, 1, -1, -2, -3, 5, 6, -1, 2 0, 2, 1) using overlap-add method. Use 7 point circular convolution in your approach.

   Circular convoluted sequence – 5marks
   Overlap add table – 5marks

→ Here, $h(n) = \{3, 2, 1, 1\}$ i.e. $M=4$ and given $\boxed{N=7}$

$\therefore L = N-M+1 = 7-4+1 = 4$; $\boxed{L=4}$ i.e. we have to take '4' samples at a time from the input block and padding (M-1) zeros i.e. '3' zeros to form a input block.

Now, $h(n) = \{3, 2, 1, 1, 0, 0, 0\}$

and

$x_1(n) = \{1, 2, 3, 3, \underbrace{0, 0, 0}_{(M-1)\ \text{zeros}}\}$

$x_2(n) = \{2, 1, -1, -2, 0, 0, 0\}$

$x_3(n) = \{-3, 5, 6, -1, 0, 0, 0\}$ $x_4(n) = \{2, 0, 2, 1, 0, 0, 0\}$

Now apply circular convolution.

$y_1(n) = x_1(n) \circledast h(n) = \{3, 8, 14, 18, 11, 6, 3\}$

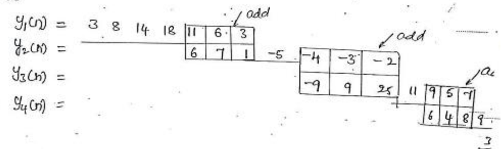
$y_2(n) = x_2(n) \circledast h(n) = \{6, 7, 1, -5, -4, -3, -2\}$

$y_3(n) = x_3(n) \circledast h(n) = \{-9, 9, 25, 11, 9, 5, -1\}$

$y_4(n) = x_4(n) \circledast h(n) = \{6, 4, 8, 9, 4, 3, 1\}$.

Now, $y(n)$ is written by adding last (M-1) samples from of $y_1(n)$ with first (M-1) samples of $y_2(n)$ and last (M-1) samples of $y_2(n)$ with first (M-1) samples of $y_3(n)$ and so on.

i.e.



$y(n) = \{3, 8, 14, 18, 17, 13, 4, -5, -13, 6, 23, 11, 15, 9, 7, 9, 4, 3,$

3. Find the 8-point DFT of the sequences x(n)=2^n ; 0≤n≤7 using RADIX 2-DIT FFT algorithm.

Twiddle factors- 2 marks
FFT Signal flowgraph – 8 marks

3. $x(n) = 2^n$; $0 \le n \le 7$

$x(n) = (2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7)$

$x(n) = (1, 2, 4, 8, 16, 32, 64, 128)$

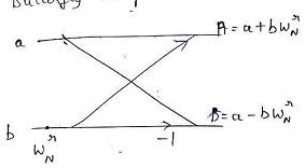
Twiddle factors : $W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

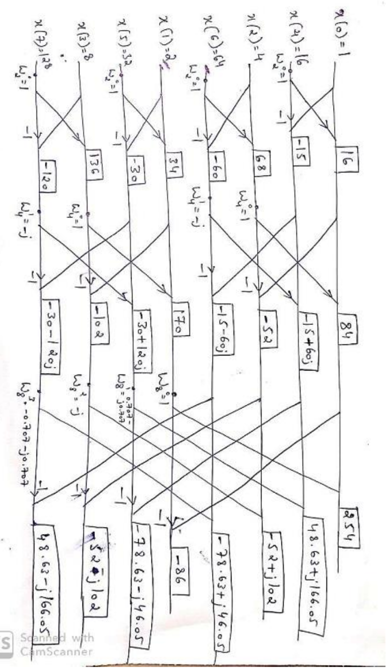
$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$

Butterfly computation:



$X(k) = \{254, 48.63 + j166.05, -52 + j102, -78.63 + j46.05, -86, -78.63 - j46.05, -52 - j102, 48.63 - j166.05\}$



4. The first five points of DFT of a sequence are given as {7, -0.707-j0.707, -j, 0.707-j0.707, 1}. Obtain the corresponding time domain sequence of length-8 using RADIX 2-DIF FFT algorithm.

Twiddle factors- 2 marks
FFT Signal flowgraph – 8 marks

4. $X(k) = \{\underset{\uparrow}{7}, -0.707 - j0.707, -j, 0.707 - j0.707, 1\}$

By symmetry property,

$X(N-k) = X^*(k)$ or $X^*(N-k) = X(k)$

Hence $X(5) = X^*(8-5)$

$= X^*(3)$

$\therefore X(5) = 0.707 + j0.707$

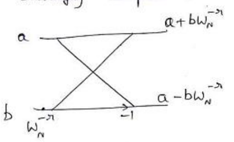
$X(6) = X^*(8-6) = X^*(2)$

$X(6) = j$

$X(7) = X^*(8-7) = X^*(1)$

$X(7) = -0.707 + j0.707$

Butterfly Computation:

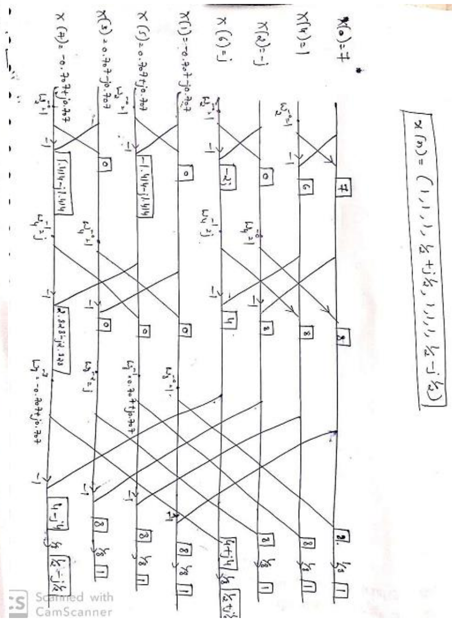


Twiddle Factors:

$W_8^{-0} = 1$

$W_8^{-1} = 0.707 + j0.707$

$W_8^{-2} = j$; $W_8^{-3} = -0.707 + j0.707$



5. Design a los pass filter with an approximate frequency response given below using rectangular window, w(n).

$H_d(e^{j\omega}) = e^{-j2\omega}$; $|\omega| \leq \pi/4$

$= 0$; $\pi/4 \leq |\omega| \leq \pi$

Determine the filter co-efficient h(n) if the window function is defined as below,

$w(n) = 1$, $0 \leq n \leq 4$

=0, otherwise

Hd(n) – 5 marks

H(n) – 5 marks

Solution

The magnitude response and phase response of the desired lowpass IIR filter is sketched below:

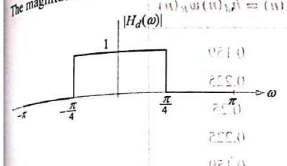


Fig. Ex.5.1(a) Magnitude response of the desired lowpass filter.

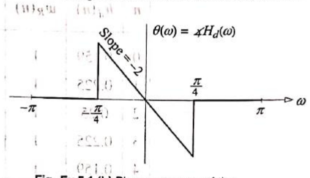


Fig. Ex.5.1 (b) Phase response of the desired lowpass filter.

Since, the magnitude response is zero over the frequency band: $\frac{\pi}{4} < |\omega| < \pi$, we infer from Paley-Wiener theorem that $h_d(n)$ is noncausal.

We know that,
$$h_d(n) \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)e^{j\omega n}d\omega$$

$$\Rightarrow h_d(n) = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-j2\omega}e^{j\omega n}d\omega$$

$$= \frac{\sin\left[\frac{\pi}{4}(n-2)\right]}{\pi(n-2)}, \ n \neq 2$$

Also,
$$h_d(2) = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^0\, d\omega = \frac{1}{2\pi}\times\frac{\pi}{2} = \frac{1}{4}$$

Given
$$w_R(n) = \begin{cases} 1, & 0 \leqslant n \leqslant 4 \\ 0, & \text{otherwise} \end{cases}$$

Hence,
$$h(n) = h_d(n)\, w_R(n)$$
$$= \begin{cases} \frac{\sin\left[\frac{\pi}{4}(n-2)\right]}{\pi(n-2)} \times 1, & n = 0, 1, 3, 4 \\ \frac{1}{4}\times 1, & n = 2 \end{cases}$$

The filter coefficients $h_d(n)$ and $h(n)$ are tabulated below:

n	$h_d(n)$	$w_R(n)$	$h(n) = h_d(n)w_R(n)$
0	0.159	1	0.159
1	0.225	1	0.225
2	0.25	1	0.25
3	0.225	1	0.225
4	0.159	1	0.159

Since, N is odd, the frequency response of the [4]centre symmetric FIR filter is computed as follows:

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} 2h(n)\cos\left[\omega\left(n - \left(\frac{N-1}{2}\right)\right)\right]\right]$$

Here, $N = 5$.

Hence,
$$H(\omega) = e^{-j2\omega}\left(h(2) + \sum_{n=0}^{1} 2h(n)\cos[\omega(n-2)]\right)$$
$$= e^{-j2\omega}\left[h(2) + 2h(0)\cos 2\omega + 2h(1)\cos\omega\right]$$
$$= e^{-j2\omega}\left(0.25 + 0.318\cos 2\omega + 0.45\cos\omega\right)$$

5. Explain why windows are necessary in FIR filter design. What are the different windows in practice? Explain in brief.

Necessity of windows- 2 marks

Each window- 2 marks

* The easiest way to obtain an FIR filter is to simply truncate the impulse response.
* If $h_d(n)$ represents the impulse response of a filter.
* Then the impulse response of an FIR filter $h(n)$ [finite length] can be obtained as follows

$$h(n) = \begin{cases} h_d(n), & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$$

* In general, $h(n)$ can be thought of as being formed by the product of $h_d(n)$ & a "window function", $w(n)$, as follows:

$$h(n) = h_d(n) \cdot w(n)$$

Some of the most commonly used windows are as follows:

1. Rectangular window

$$W_R(n) = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases}$$

* The plot of window function is shown in Fig 1(a) for $N = 51$.
* The frequency response of the rectangular window is shown in Fig 1(a).
* The rectangular window clearly has the narrowest main lobe.
* The first side lobe is about 13dB below the main peak.
* In case of rectangular window the, sidelobes are larger in size since the discontinuity is abrupt.

5. Blackman Window

$$w_{BL}(n) = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, \text{ otherwise} \end{cases}$$

* A plot of Blackman window with its frequency response is shown in Fig 1(e).
* The first side lobe is 58 dB below the main peak.

2. Bartlett window [Triangular window]

$$w_B[n] = \begin{cases} 1 - \frac{2\left|n - \frac{N-1}{2}\right|}{N-1}, & 0 \le n \le N-1 \\ 0, \text{ otherwise} \end{cases}$$

* A plot of Bartlett window & it's frequency response is shown in Fig 1(b)
* The first side lobe is 27dB below the main peak.

4. Hamming Window

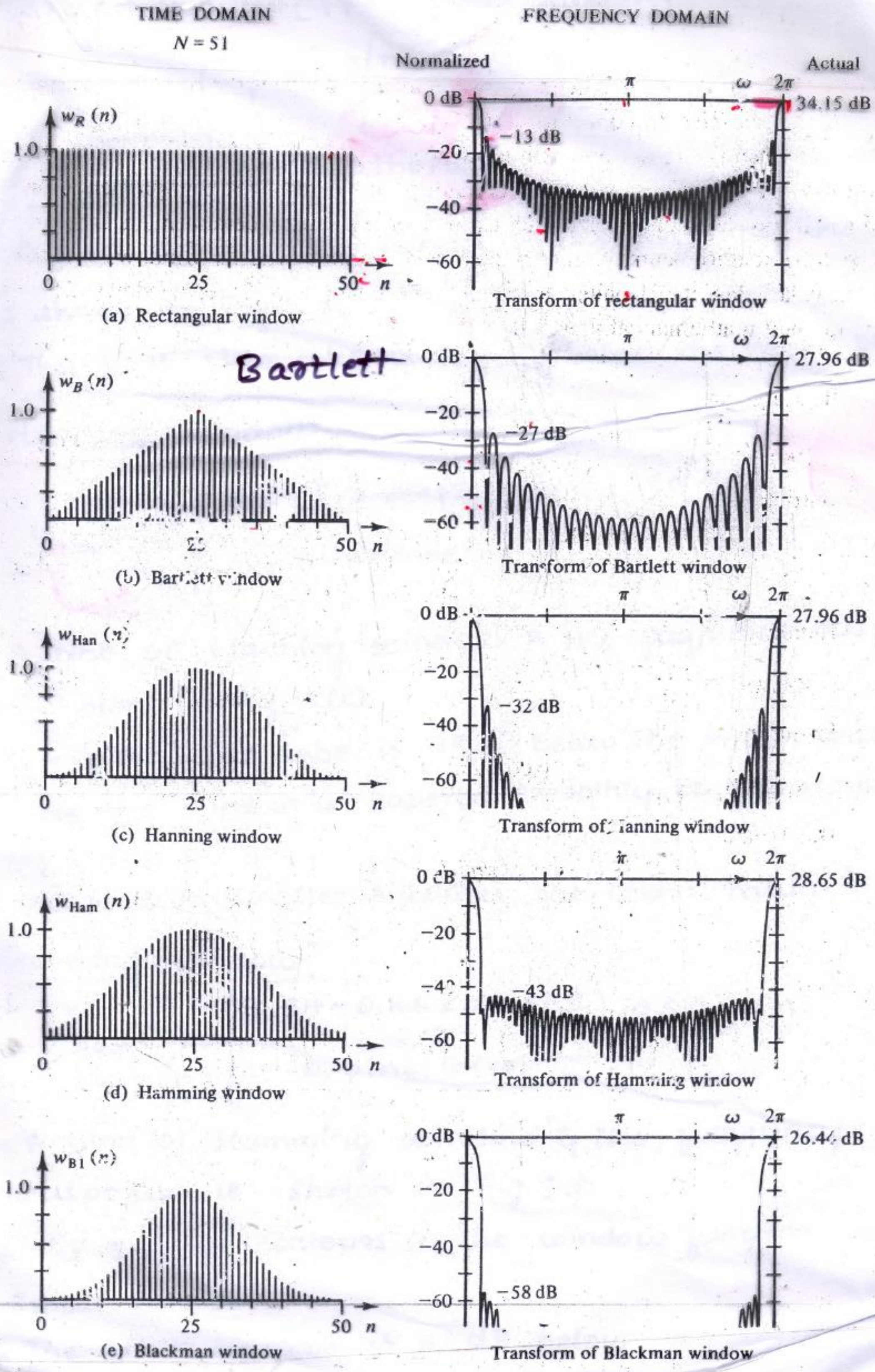
$$w_{Ham}(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, \text{ otherwise} \end{cases}$$

* A plot of Hamming window & it's frequency response is shown in Fig 1(d)
* The smooth contours in the window function lead to small sidelobes.
* The first sidelobe is 43 dB below the main peak.

3. Hanning Window

$$w_{Han}(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right), & 0 \le n \le N-1 \\ 0, \text{ otherwise} \end{cases}$$

* A plot of Hanning window & it's frequency response is shown in Fig 1(c).
* The first side lobe is 32 dB below the main peak.
* Here the window is tapered smoothly to zero (not abrup-tly).
* This leads to smaller sidelobes but wider main lobe.



(a) Rectangular window

(b) Bartlett window

(c) Hanning window

(d) Hamming window

(e) Blackman window

6. Calculate the IDFT of X(k) = {0, 2.828-j2.828, 0, 0, 0, 0, 0, 2.828+j2.828} using INVERSE RADIX 2 DIT FFT algorithm.

   Twiddle factors- 2 marks
   FFT Signal flowgraph – 8 marks

Problems :-

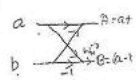
Calculate the IDFT of $X(k) = \{0, 2.8284 - j2.8284, 0, 0, 0, 0, 0, 2.828 + j2.8284\}$ using Inverse Radix-2 DIT-FFT algorithm

Sol. $N = 8$,

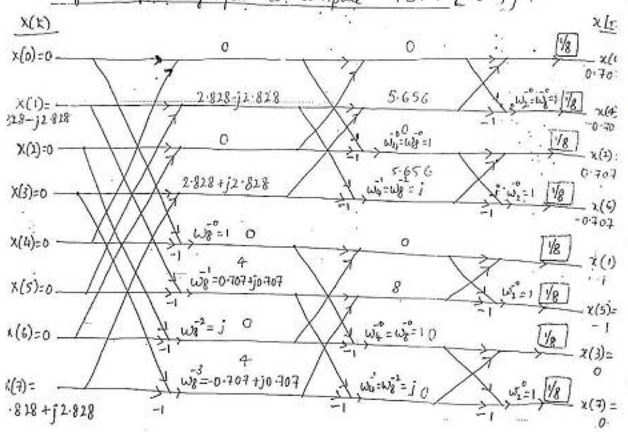
To compute 4 twiddle factors :- (W_N^{-r})

$W_8^0 = 1$, $W_8^{-1} = 0.707 + j0.707$

$W_8^{-2} = e^{\frac{j2\pi}{8}\cdot 2} = +j$, $W_8^{-3} = -0.707 + j0.707$



Signal flow graph to compute IDFT [x(n)] :-



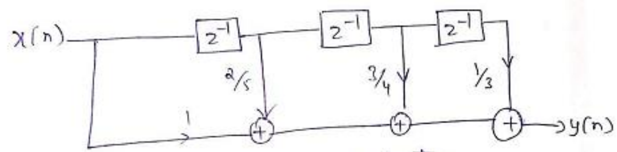
∴ $x(n) = \{0.707, 1, 0.707, 0, -0.707, -1, -0.707, 0\}$

7. An FIR filter is given by $y(n)=x(n) + 2/5x(n-1) + 3/4x(n-2) + 1/3x(n-2)$. Draw the direct and linear form realization.

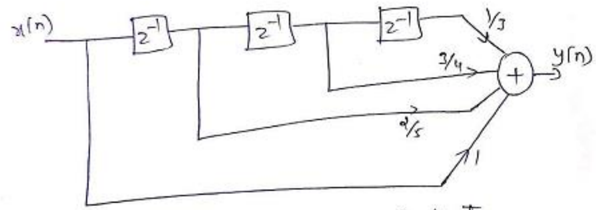
Direct form 1 realization – 5 marks

Linear phase form realization – 5marks

8. $y(n) = x(n) + \frac{2}{5}x(n-1) + \frac{3}{4}x(n-2) + \frac{1}{3}x(n-3)$



Direct Form I Realization



Linear Phase Form Realization