CMR INSTITUTE OF TECHNOLOGY, BENGALURU

(Approved by AICTE, New Delhi, Accredited by NAAC with A+ and Affiliated to VTU)

Department of Electrical and Electronics Engineering

15EE661 – Artificial Neural Network and Fuzzzy Logic (VI Semester - Open Elective)

Academic year 2018-19

Question and Solution - IAT-II

		Marks	OBE CO	OBE RBT
1(a)	Distinguish between feedforward and recurrent neural network.	[05]	CO2	L2
1(b)	What do you mean by Crisp logic, predicate logic and fuzzy logic?	[05]	CO4	L2
2	Compare single layered and multi-layer perceptron. Also discuss about on what aspects multi-layer perceptrons are advantages over single layer perceptron.	[10]	CO2	L2
3	Write short notes on learning rules of neural network. Explain the back propagation algorithm with relevant sketches and flowchart.	[10]	CO2	L3
4	Fuzzy sets A and B are defined in the interval, X = [0, 1, … 10] of real numbers by the membership functions $\mu_A(X)=x/(x+2)$, $\mu_B(X)=2^{-x}$. Obtain the output of the following fuzzy set operations. i) $A \cup B$ ii) $A \cap B$ iii) $\overline{A}$ iv) $\overline{B}$	[10]	CO4	L3
5	Illustrate the different types of membership function used in fuzzy expert system with suitable diagram.	[10]	CO4	L3
6	What do you mean by λ-cut for fuzzy sets? Analyze how the λ-cut relation for λ = 0.2, 0.4, 0.7 and 0.9 varies for the given fuzzy relation, R. $R = \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1 & 0.8 & 0.6 & 0.4 \end{bmatrix}$	[10]	CO4	L4
7	Explain in detail about Fuzzy composition and decomposition. Find the relation R(X,Z) for the fuzzy matrices using max-min composition and max-product composition. $R(X,Y) = \begin{bmatrix} 0.6 & 0.4 & 1 \\ 0 & 0 & 1 \\ 0.4 & 0 & 0.9 \end{bmatrix}$ $R(Y,Z) = \begin{bmatrix} 0.8 & 0.5 \\ 0 & 1 \\ 0 & 0.8 \end{bmatrix}$	[10]	CO4	L3

1(a)	Distinguish between feedforward and recurrent neural network.
Ans	

Feedforward NN	Recurrent NN
Feedforward Neural Network (FFNN) is the most basic artificial neural network that is used for regular regression & classification problems.	Recurrent Neural Network (RNN) is the state of the art algorithm for sequential data and is used for voice search etc.
They do not have any internal memory.	They have internal memory.
In a FFNN the information only moves in one direction from the I/P layer through the hidden layers to the O/P.	In a RNN the information cycles through a loop. when it makes a decision it takes into consideration the current and previous I/P

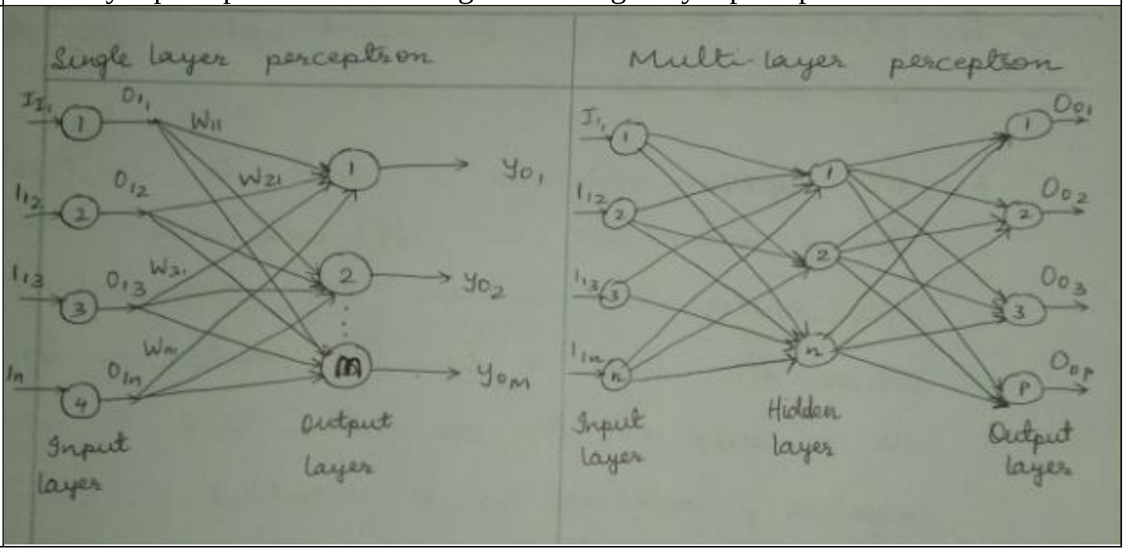
1(b)	What do you mean by Crisp logic, predicate logic and fuzzy logic?
Ans	

⟹ Crisp logic: is the same as boolean logic (either 0 or 1).
Either a statement is true(1) or it is not (0).
Membership in a set is all or nothing.

Ex: A jelly bean belongs to the class of food known as candy whereas a mashed potato does not belong to that class.

∴ Candy : Jelly bean (1)
Mashed potato (0)

Predicate logic: is a statement that contains variables (predicate variables), and they may be true or false depending on the values of these variables.

	Ex: P(x) = "X2 is greater than X" is a predicate. It contains one predicate variable x and the statement is either true or false depending on the values of X2 & X Fuzzy logic: is a form of many-valued logic in which the truth values of variables may be any real no. between 0 & 1 inclusive. It is employed to handle the concept of partial truth, where the truth value may range between completely true & completely false. Membership in a set is a degree. Ex: Consider the set of "hot" days in Coimbatore in 2006. Was July 17 "hot"? — "very hot" — "sort of hot" — "not hot" } degree of membership.
2	Compare single layered and multi-layer perceptron. Also discuss about on what aspects multi-layer perceptrons are advantages over single layer perceptron.
Ans	

Input $I_1 = \begin{Bmatrix} I_{1_1} \\ I_{1_2} \\ I_{1_n} \end{Bmatrix}_{n\times1}$

Output $O_O = \begin{Bmatrix} O_{O_1} \\ O_{O_2} \\ O_{O_m} \end{Bmatrix}_{m\times1}$

Assume linear transfer function for input layer and unipolar sigmoidal function for output layer

$\{O_I\}_{n\times1} = \{I_I\}_{m\times1}$

$I_{O_j} = W_{1j}I_{1_j} + W_{2j}I_{2I} + \; W_{nj}I_{In}$

Input-output mapping of hidden layer and multilayer perception is given by

$O = N_3\left[N_2\left[N_1[I]\right]\right]$

<u>Advantages of Multi-layer network than Single layer Network:</u>

1. Adaptive learning: An ability to learn how to do tasks based on the data given for training or initial experience.
2. One of the preferred techniques for gesture recognition.
3. MLP/Neural networks do not make any assumption regarding the underlying probability density functions or other probabilistic information about the pattern classes under consideration in comparison to other probability based models.
4. They yield the required decision function directly via training.
5. A two layer backpropagation network with sufficient hidden nodes has been proven to be a universal approximator.

3	Write short notes on learning rules of neural network. Explain the back propagation algorithm with relevant sketches and flowchart.
Ans	Learning rule (or) learning process is a method (or) a mathematical logic. It improves the Artificial Neural Networks performance and applies this rule over the network. Thus learning rules updates the weights and bias levels of a network when a network simulates in a specific data environment. Applying learning rule is an iterative process. It helps a neural network to learn from the existing conditions and improve its performance.

Different learning rules in the neural network.

*** Hebbian learning rules :-**

It identifies, how to modify the weights of nodes of a netw-
-ork. $\therefore \; W_{ij} = X_i * X_j$

*** Perceptron learning rule :-**

Network starts its learning by assigning a random value to each weight.

$$\therefore \sum_i \sum_j (E_{ij} - O_{ij})^2$$

*** Delta learning rule :-**

Modification in sympatric weight of a node is equal to the multiplication of error and the input.

$$\therefore \Delta W = \eta (t - y) x_i$$

*** Correlation learning rule :-**

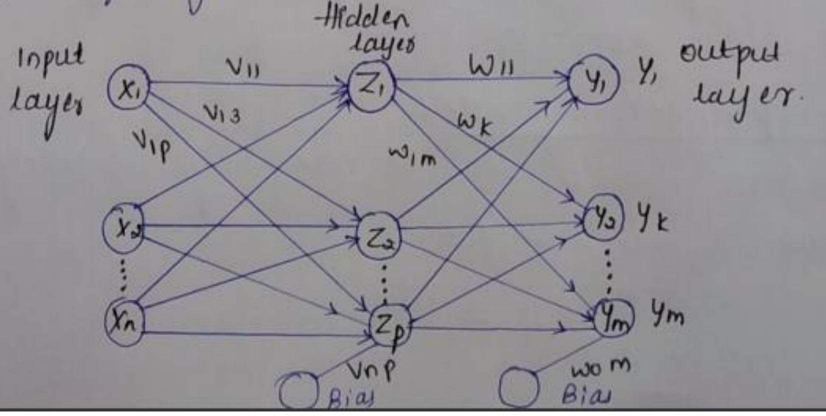
The correlation rule is the supervised learning.

$$\therefore \Delta W_{ij} = \eta x_i d_j$$

*** Outstar learning rule :-**

we can use it when it assumes that nodes(or) neurons in a network arranged in a layer.

$$W_{jk} = \begin{cases} \eta (y_k - W_{jk}) & \text{if node } j \text{ wins the competition} \\ 0 & \text{if node } j \text{ losses the competition} \end{cases}$$

	## Back propagation Algorithm Used for error detection and correction in neural network. This algorithm consists of 4 phases (i) Initialization of weight (ii) Feed forward (iii) Back-propagation of error (iv) updation of weight & bias 

(i) Initialization of weight

three steps

Step (1) Initialize weight to some random value

Step (2) while stopping condition is false, do step 3 to 10.

Step (3) For every training pair do step 4 to 9.

(ii) Feed forward

Calculate the input of the hidden layer and the output of the hidden layer and the input of the output layer and output of the output layer.

Step (4) propagate input to upper layer

Step (5)

$$Z_{inj} = V_{hj} + \sum_{i=1}^{n} x_i v_{ij} \qquad V_{hj} \rightarrow bias$$

(i/p of the j^{th} neuron of hidden layer) $= V_{hj} + x_1 v_{1j} + x_2 v_{2j} + x_3 v_{3j} + \cdots$

$$Z_j = f(Z_{-inj})$$

↓ activation fun.

Step (6)

$$y_{ink} = w_{0k} + \sum_{i=1}^{p} z_j w_{jk}$$

(i/p of the k^{th} neuron of the o/p layer) $= w_{0k} + Z_1 w_{1k} + Z_2 w_{2k} + \cdots$

$$y_k = f(y_{-ink})$$

(iii) Back propagation of error

Step (7)

$$\delta_k = (t_k - y_k) f'(y_{-ink})$$

error for y_k; t_k → desired o/p; y_k → calculated o/p

Step ⑧

$$\delta\text{-}in_j = \sum_{k=1}^{m} \delta_j w_{jk}$$

$$= \delta_j w_{j1} + \delta_j w_{j2} + \dots = \delta_j w_{j1} + \delta_j w_{j2} + \dots$$

$$\Delta_j = \delta in_j \, f'(z_{inj})$$

error o/p from the hidden layer

⑨ Updation of weight & bias

After finding the errors we will update the weights

$$\Delta w_{jk} = \alpha \, \delta_k \, Z_i$$

↳ learning rate

$$\Delta w_{0k} = \alpha \delta_k$$

$$\Delta V_{ij} = \alpha \, \Delta_j \, x_i$$

$$\Delta V_{hj} = \alpha \Delta_j = \alpha \, \Delta_j$$

Step ⑨

new weights $w_{jk}(new) = w_{jk}(old) + \Delta w_{jk}$

$w_{0k}(new) = w_{0k}(old) + \Delta w_{0k}$

III^ly

$V_{ij}(new) = V_{ij}(old) + \Delta V_{ij}$

$V_{hj}(new) = V_{hj}(old) + \Delta V_{hj}$

Step ⑩ Stopping algorithm

(i) No of iteration

(ii) Minimum of errors.

4 Fuzzy sets A and B are defined in the interval, X = [0, 1, … 10] of real numbers by the membership functions $\mu_A(X)=x/(x+2)$, $\mu_B(X)=2^{-x}$. Obtain the output of the following fuzzy set operations.

i) $A \cup B$ ii) $A \cap B$ iii) $\overline{A}$ iv) $\overline{B}$

<table>
<tr><td>Ans</td><td>
solⁿ:- $X = [0, 1, 2, ----, 10]$
$\mu_A(x) = \frac{x}{x+2}$
$A = \{0, 0.33, 0.5, 0.6, 0.66, 0.714, 0.75, 0.77, 0.8, 0.82, 0.83\}$
$\mu_B(x) = 2^{-x}$
$B = \{1, 0.5, 0.25, 0.125, 0.0625, 0.031, 0.016, 0.007, 0.003, 0.001, 0.0009\}$
(i) $A \cup B = \{1, 0.5, 0.5, 0.6, 0.66, 0.714, 0.77, 0.8, 0.82, 0.83\}$
(ii) $A \cap B = \{0, 0.33, 0.25, 0.125, 0.0625, 0.03, 0.01, 0.007, 0.003, 0.001, 0.0009\}$
(iii) $\bar{A} = \{1, 0.6, 0.5, 0.4, 0.33, 0.28, 0.25, 0.22, 0.199, 0.18, 0.167\}$.
(iv) $\bar{B} = \{0, 0.5, 0.75, 0.875, 0.93, 0.96, 0.98, 0.992, 0.996, 0.998, 0.999\}$
</td></tr>
<tr><td>5</td><td>Illustrate the different types of membership function used in fuzzy expert system with suitable diagram.</td></tr>
<tr><td>Ans</td><td>
Different types of Membership functions:

Triangular waveform
Trapezoidal waveform
Gaussian waveform
Bell-shaped waveform
Sigmoidal waveform and
S-curve waveform

<u>Selection of Membership function:</u>

For those systems that need significant dynamic variation in a short period of time,
a triangular or trapezoidal waveform should be utilized.
For those system that need very high control accuracy,
a Gaussian or S-curve waveform should be selected.

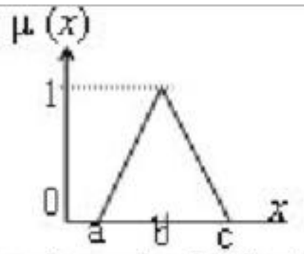
Various forms of MF:

Membership functions can be symmetrical or asymmetrical.

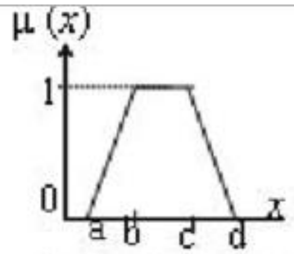
</td></tr>
</table>

- They are typically defined on one-dimensional universes, or on multidimensional universes.
- For example, the membership functions shown here are one-dimensional curves.
- In two dimensions these curves become surfaces and for three or more dimensions these surfaces become hyper surfaces.

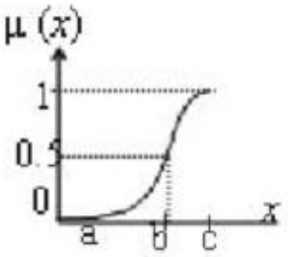
Types of Membership functions:



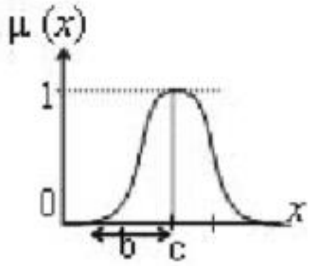
(a) Triangular membership function.



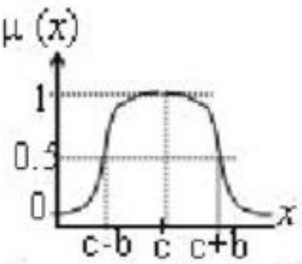
(b) Trapezoidal membership functior



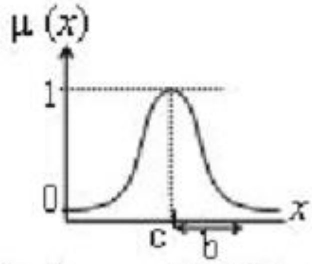
(c) S-shape membership function.



(d) π-shape membership function.



(e) Beta shape membership function



(f) Gaussian shape membership func

6 What do you mean by λ-cut for fuzzy sets? Analyze how the λ-cut relation for λ = 0.2, 0.4, 0.7 and 0.9 varies for the given fuzzy relation, R.

$$R = \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1 & 0.8 & 0.6 & 0.4 \end{bmatrix}$$

Ans **Alpha Cut – Definition:**

The Alpha cut of a fuzzy set is the (crisp) set of all elements that have a membership value greater than or equal to λ. Alpha cuts allows us to describe a fuzzy set as a composition of crisp sets. The concept is very important as it is used to extrapolate fuzzy functions from crisp ones.

$$R = \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1.0 & 0.8 & 0.6 & 0.4 \end{bmatrix}$$

$$R_\lambda = \{ x \mid \mu_{\underset{\sim}{R}}(x) \geq \lambda \} \quad x \in [0,1]$$

$$R_{0.2} = \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1.0 & 0.8 & 0.6 & 0.4 \end{bmatrix}$$

$$R_{0.4} = \begin{bmatrix} 0 & 0.5 & 0.7 & 1 & 0.9 \\ 0 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1.0 & 0.8 & 0.6 & 0.4 \end{bmatrix}$$

	$R_{0.7} = \begin{bmatrix} 0 & 0 & 0.7 & 1 & 0.9 \\ 0 & 0 & 0.7 & 1 & 0.8 \\ 0 & 0 & 0.8 & 0.9 & 0 \\ 0.9 & 1.0 & 0.8 & 0 & 0 \end{bmatrix}$ $R_{0.9} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0.9 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.9 & 0 \\ 0.9 & 1.0 & 0 & 0 & 0 \end{bmatrix}$
7	Explain in detail about Fuzzy composition and decomposition. Find the relation R(X,Z) for the fuzzy matrices using max-min composition and max-product composition. $R(X,Y) = \begin{bmatrix} 0.6 & 0.4 & 1 \\ 0 & 0 & 1 \\ 0.4 & 0 & 0.9 \end{bmatrix}$ $R(Y,Z) = \begin{bmatrix} 0.8 & 0.5 \\ 0 & 1 \\ 0 & 0.8 \end{bmatrix}$
Ans	**<u>Fuzzy composition:</u>**

	Composition of fuzzy relations Two fuzzy relation R and S are defined on sets A, B and C. That is, $R \subseteq A \times B$, $S \subseteq B \times C$. The composition $S \cdot R = SR$ of two relations R and S is expressed by the relation from A to C. For $(x, y) \in A \times B$, $(y, z) \in B \times C$, $\mu_{S \cdot R}(x, z) = \max_y [\min(\mu_R(x,y), \mu_S(y,z))]$ $= \vee_y [\mu_R(x,y) \wedge \mu_S(y,z)]$ $M_{S \cdot R} = M_R \cdot M_S$. **Fuzzy decomposition:** Decomposition of fuzzy relations Fuzzy relation can be said to be composed of several R_α's as following. $R = \bigcup_\alpha \alpha R_\alpha$ when α is a value in the level set. R_α is a α-cut relation. αR_α is a fuzzy relation. The membership function of αR_α is defined as, $\mu_{\alpha R_\alpha}(x,y) = \alpha \cdot \mu_{R_\alpha}(x,y)$, for $(x,y) \in A \times B$ Thus we can decompose a fuzzy relation R into several αR_α. **Max-min Composition:**

Using max-min composition

$$R(x,y) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.4 & 1 \\ 0 & 0 & 1 \\ 0.4 & 0 & 0.9 \end{bmatrix}_{3 \times 3} \end{array} \qquad S(y,z) = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0.8 & 0.5 \\ 0 & 1 \\ 0 & 0.8 \end{bmatrix}_{3 \times 2} \end{array}$$

$$R(x,z) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} - & - \\ - & - \\ - & - \end{bmatrix}_{3 \times 2} \end{array}$$

$$\mu_R(x_1, z_1) = \max\left[\min(x_1y_1, y_1z_1), \min(x_1y_2, y_2z_1), \min(x_1y_3, y_3z_1)\right]$$

$$\mu_R(x_1, z_1) = \max\left[\min(0.6, 0.8), \min(0.4, 0), \min(1, 0)\right]$$

$$= \max\left[0.6, 0, 0\right]$$

$$= 0.6$$

Similarly we find x_1z_2, x_2z_1, x_2z_2, x_3z_1 & x_3z_2.

$$R(x,z) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_2 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.8 \\ 0 & 0.8 \\ 0.4 & 0.8 \end{bmatrix} \end{array}$$

Max-Product composition:

	Using max-product composition, $\mu_R(x_1, z_1) = \max[x_1y_1 \times y_1z_1, x_1y_2 \times y_2z_1, x_1y_3 \times y_3z_1]$ $\mu_R(x_1, z_1) = \max[0.6 \times 0.8, 0.4 \times 0, 1 \times 0]$ $= \max[0.48, 0, 0]$ $= 0.48$ Similarly, $\mu_R(x_1, z_2) = \max(0.6 \times 0.5, 0.4 \times 1, 1 \times 0.8]$ $= \max[0.30, 0.4, 0.8]$ $= 0.8$ $\mu_R(x_2, z_1) = \max[0 \times 0.8, 0 \times 0, 1 \times 0]$ $= \max[0, 0, 0]$ $= 0$ $\mu_R(x_2, z_2) = \max[0 \times 0.5, 0 \times 1, 1 \times 0.8]$ $= \max[0, 0, 0.8]$ $= 0.8$

$$\mu_R(x_3, z_1) = \max[0.4 \times 0.8,\ 0 \times 0,\ 0.9, 0]$$

$$= \max[0.32,\ 0,\ 0]$$

$$= 0.32$$

$$\mu_R(x_3, z_2) = \max[0.4 \times 0.5,\ 0 \times 1,\ 0.9 \times 0.8]$$

$$= \max[0.20,\ 0,\ 0.72]$$

$$= 0.72.$$

$$R(x, z) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0.48 & 0.8 \\ 0 & 0.8 \\ 0.32 & 0.72 \end{bmatrix} \end{array}$$
