IAT-2 PHYSICS SCHEME

1.**EXPRESSION FOR FERMI ENERGY** **[7]**

From Fermi –Dirac theory

$$n = \int_0^{E_F} g(E).f(E).dE = \int_0^{E_F} \frac{4\pi(2m)^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}} dE.1$$

$$= \frac{4\pi(2m)^{3/2}}{h^3} \frac{E_F^{\frac{3}{2}}}{\frac{3}{2}}$$

$$E_F^{\frac{3}{2}} = \frac{h^3 3n}{8\pi(2m)^{\frac{3}{2}}}$$

$$E_F = \frac{h^2}{8m}\left[\frac{3n}{\pi}\right]^{\frac{2}{3}}$$

5.b. **[3]**

f (E) = $\frac{1}{e^{(\frac{E-E_F}{kT})}+1} = \frac{1}{e^{\frac{-0.2X1.6X10^{-19}}{1.38X10^{-23}X300}}+1} = 0.99$

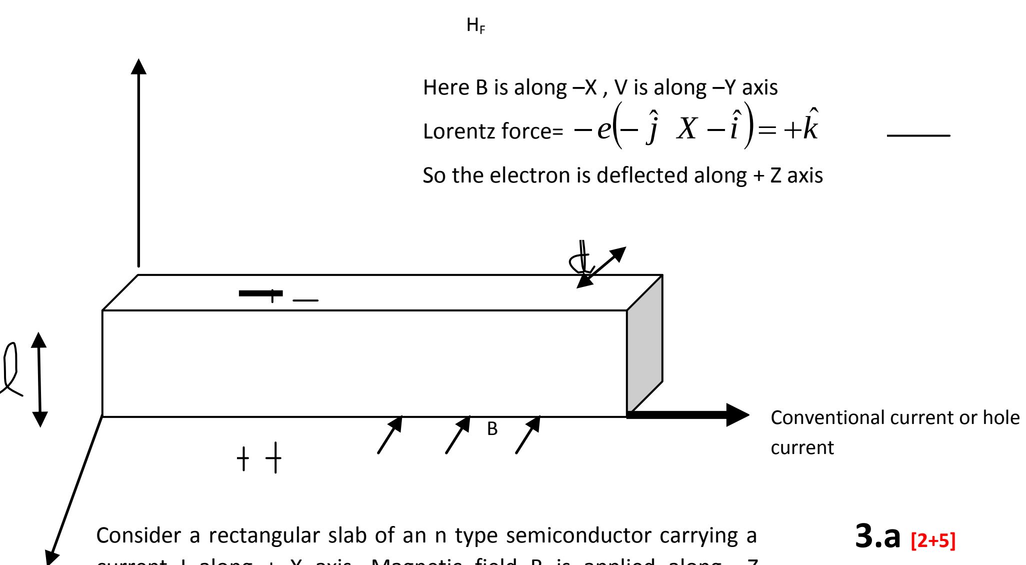
2.a **[5+2]**

Hall effect: When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.

Here B is along –X , V is along –Y axis

Lorentz force= $-e\left(-\hat{j}\ \ X - \hat{i}\right) = +\hat{k}$

So the electron is deflected along + Z axis



Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. The develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up.

Expression for Hall Coefficient:

At equilibrium, Lorentz force is equal to force due to applied electric field

Bev =- e E_H

Hall Field E_H = Bv

$$Current\ density\ \ J = -n_e ev$$

$$v = \frac{J}{n_e e}$$

$$E_H = B\frac{-J}{n_e e}$$

Hence $\frac{E_H}{JB} = -\frac{1}{n_e e} = R_H$

$$V_H = E_H.l = -R_H JBl$$

2.b. **[3]**

To show that energy levels below Fermi energy are completely occupied:

For E **<** E_F, at T = 0,

f (E) = $\frac{1}{e^{(\frac{E-E_F}{kT})}+1} = 1$

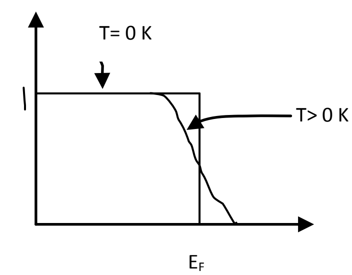
To show that energy levels above Fermi energy are empty:

For E > E_F , at T=0

f (E) = $\frac{1}{e^{(\frac{E-E_F}{kT})}+1} = 0$

At ordinary temperatures, for E = E_F,

f(E) = $\frac{\ }{2}$



3.a **[2+5]**

INTERNAL FIELDS IN A DIELECTRIC:

It is the resultant of the applied field and the field produced due to all the dipoles.

FOR 1-D $E_i = E_a + \frac{1.2\mu}{\pi\varepsilon d^3}$ $\quad \because \mu = \alpha E_a$

In three dimensional case , $(1/d^3)$ could be replaced by N, the number of atoms per unit volume and (1.2/Π) by a constant γ which depends on the crystal structure.

Hence $E_i = E_a + \frac{\gamma N\mu}{\varepsilon}$ =

$$E_a + \left[\frac{\gamma}{\varepsilon_0}\right]P \qquad \because N\mu = P$$

CLAUSIUS – MOSOTTI RELATION:

This expression relates dielectric constant of an insulator (ε) to the polarization of individual atoms (α) comprising it.

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

where N is the number of atoms per unit volume
α is the polrisability of the atom
ε_r is the relative permittivity of the medium
ε_o is the permittivity of free space.

Proof:

If there are N atoms per unit volume, the electric dipole moment per unit volume – known as polarization is given by

$$P = N\alpha E_i$$

By the definition of polarization P, it can be shown that

$$P = \varepsilon_0 E_a(\varepsilon_r - 1) = N\alpha E_i$$

$$\varepsilon_0\varepsilon_r E_a - \varepsilon_0 E_a = N\alpha E_i$$

$$\varepsilon_r = 1 + \frac{N\alpha E_i}{\varepsilon_0 E_a} \qquad \text{.......................(1)}$$

The internal field at an atom in a cubic structure (γ =1/3) is of the form

$$E_i = E_a + \frac{p}{3\varepsilon_0} = E_a + \frac{N\alpha E_i}{3\varepsilon_0}$$

$$\frac{E_i}{E_a} = \frac{1}{\left[1 - \left(\frac{N\alpha}{3\varepsilon_0}\right)\right]}$$

Substituting for $\frac{E_i}{E_a}$ in equation (1)

$$\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}\left[\frac{1}{\left(1 - \frac{N\alpha}{3\varepsilon_0}\right)}\right] = \frac{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right] + \frac{N\alpha\varepsilon_0}{\varepsilon_0}}{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right]}$$

$$= \frac{1 + \frac{2}{3}\left(\frac{N\alpha}{\varepsilon_0}\right)}{1 - \frac{1}{3}\left[\frac{N\alpha}{\varepsilon_0}\right]}$$

$$\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right] = \frac{\frac{1 + (2/3)\frac{N\varepsilon}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} - 1}{\frac{1 - (2/3)\frac{N\alpha}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} + 2} = \frac{N\alpha}{3\varepsilon_0}$$

3.b. [3]

$\alpha_e = \varepsilon_0(\varepsilon_r - 1)/N = 2.62\text{x}10^{-40}$

$\mu = \alpha_e \text{ x } E = 7.86 \text{ x}10^{-36}$ Cm

4.a. [7]

Expression for Fermi Level in Intrinsic Semiconductor

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}} =$$

$$2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f + E_v + E_c}{kT}$$

$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

4.b. [3]

$$\sigma = \frac{1}{\rho} = ne(\mu_e + \mu_h)$$

$$\rho = 2.15\Omega m$$

5.A [6]

Conductivity of Intrinsic semiconductors:

Current density J = n e V_d

For a semiconductor, J = n_e e V_d (e) + n_h e V_d (h) ……………(1)

But drift velocity V_d = µE=µ.J/σ

Using (1), σ = n_e e μ_e + n_h e μ_h

In an intrinsic semiconductor, number of holes is equal to number of electrons.

, $\sigma_{int} = n_e e[\mu_e + \mu_{hole}]$

n_e is the electron concentartion

n_p is the hole concentration

μ_e is the mobility of electrons

μ_h is the mobility of holes

5.b. [4]

$$V_H = E_H.l = -R_H JBl$$

$$= -\frac{1}{ne}.\frac{I}{A}.B.l = 0.44X10^{-3}V$$

6.a [2+3]

Hooke's law: Strain is proportional to stress at smaller magnitudes. As the stress is increased to large magnitudes strain increases more rapidly and the linear relationship between stress and strain ceases to hold. This is referred as elastic limit (A).

Effect of stress: **ELASTIC FATIGUE**

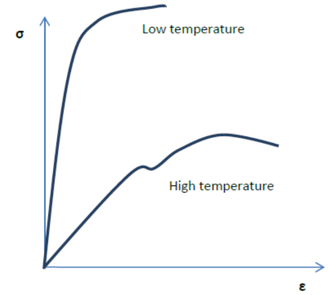
Elastic properties of a body repeatedly subjected to stress show random variation.

Ex: Piston and connecting rods in a locomotive are subjected to repeated tensions and compressions during each cycle. Their elastic properties randomly fluctuate. It may break under a stress less than elastic limit.

2. Annealing : Annealing operation involves heating and gradual cooling. The crystal grains form a uniform orientation forming larger domains.This causes decrease in elastic properties. Operations like hammering, rolling break up the crystal grains resulting in increase of elastic properties.

3. Temperature : Inter molecular forces decreases with rise in temperature. Hence the elasticity decreases with rise in temperature. (But the elasticity of invar steel (alloy) does not change with change of temperature).Carbon filament which is highly elastic at ordinary temperature, becomes plastic when heated.

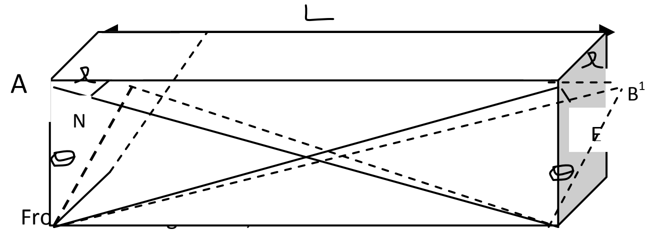
Stress



4. Impurities : Presence of impurities alters elasticity. It can increase or decrease depending on the nature of impurities. Carbon is added in minute quantities to molten Iron to increase its elastic property.

6.B [5]

A shear of ϴ is equivalent to an elongation strain ϴ/2 and compression strain ϴ/2 at right angles to each other.



…e EB is the perpendicular from B to DB^1. …$B = DE$

$$\cos 45 = \frac{EB^1}{BB^1} \Rightarrow EB^1 = \frac{l}{\sqrt{2}}$$

$$DE = \sqrt{2}L$$

Extension strain along DE = $\frac{EB^1}{DE} = \frac{l}{2L} = \frac{\theta}{2}$ where ϴ is the shearing strain

From the triangle AA^1N, $AA^1N = 45^0$

$$\cos 45 = \frac{AN}{AA^1} \Rightarrow AN = \frac{l}{\sqrt{2}}$$

$$CN = \sqrt{2}L$$

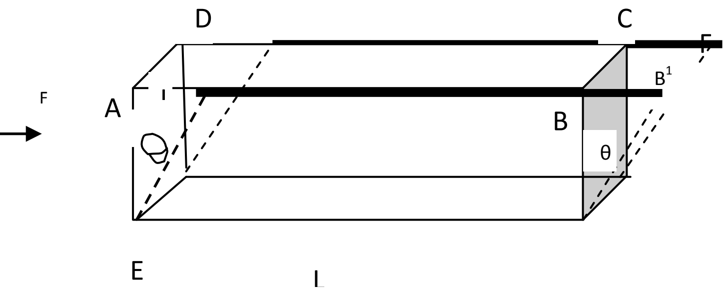
Compression strain along AC = $\frac{AN}{CN} = \frac{l}{2L} = \frac{\theta}{2}$

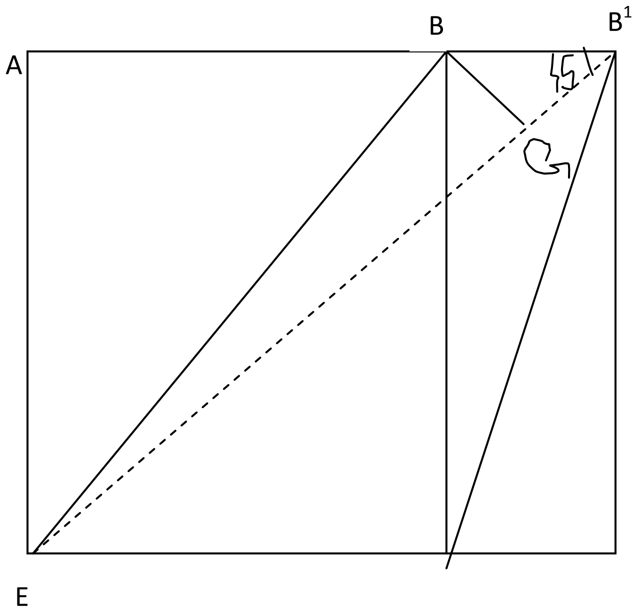
7.A [2+5]

Linear strain (α) - It is the increase per unit length per unit stress along the force.

Lateral strain (β) - It is the lateral contraction per unit length per unit stress perpendicular to force

RIGIDITY MODULUS





Let the face ABCD of a cube of side L be sheared by a Force F through an angle θ.

Shearing stress = $\frac{F}{L^2} = T$

Shearing Strain $= \frac{l}{L} = \theta$

Rigidity Modulus = $\frac{T}{\theta}$

Shearing stress along AB is equivalent to sum of expansive stress along EB and compressive stress along AF. Let α be the longitudinal expansive strain per unit Stress per unit length and β be the lateral compressive strain per unit stress per unit length respectively. BG is perpendicular to EB^1

$\therefore EB = EG$

Elongation along EB is GB^1= $EB.\alpha.T.$

Compression along AF = $AF.T.\beta$

Net extension GB^1= $L.\sqrt{2}.T(\alpha+\beta))$

Also , from right angled triangle BB^1G,

Elongation GB^1= $l.\cos 45 = \frac{l}{\sqrt{2}}$

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$$L.\sqrt{2}.T(\alpha+\beta)) = \frac{l}{\sqrt{2}}$$

$$\frac{T}{\frac{l}{L}} = \frac{1}{2(\alpha+\beta)} \qquad Also \qquad Y = \frac{T}{\frac{L.\alpha.T}{L}} = \frac{1}{\alpha}$$

$$n = \frac{1}{2(\alpha+\beta)}$$

$$n = \frac{1}{2(\alpha+\beta)}$$

$$n = \frac{1}{2\alpha(1+\sigma)}$$

7.b [3]

POISSON RATIO

Within the elastic limit, the lateral strain is proportional to longitudinal strain and the ratio between them is a constant for a material known as Poisson ratio.

$$\sigma = \frac{\beta}{\alpha}$$

$$K = \frac{Y}{3(1-2\sigma)} \qquad\qquad n = \frac{Y}{2(1+\sigma)}$$

LIMITS OF σ

$$3k(1-2\sigma) = 2n(1+\sigma)$$

1. If σ be a positive quantity, (1-2σ) should be positive

   2σ<1

   σ < 0.5

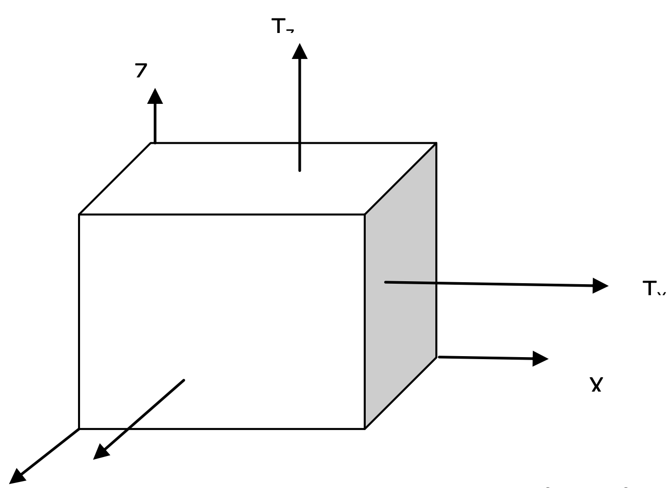
   When σ =0.5, the material is said to be incompressible

2. If σ be a negative quantity, (1 + σ) should be positive

   σ>-1

8.A [5+2]

RELATION BETWEEN BULK MODULUS (K) - α –β



Let stresses T_X , T_Y and T_Z act perpendicular to faces of a unit cube as shown in the figure .Let α be the increase per unit length per unit stress (linear strain) along the force, β be the lateral contraction (lateral strain) per unit length per unit stress perpendicular to force.

Elongation produced along X axis = $T_X .\alpha.1$

Contraction produced along X axis $= (T_y.\beta.1 + T_Z.\beta.1)$

Change in Length of AB $= T_x\alpha - T_y\beta - T_Z\beta$

Change in Length of BG $= T_y\alpha - T_x\beta - T_z\beta$

Change in Length of BC $= T_z\alpha - T_y\beta - T_x\beta$

Change in Volume of cube = $(T_x\alpha - T_y\beta - T_Z\beta)$ X $(T_y\alpha - T_x\beta - T_z\beta)$ X $(T_z\alpha - T_y\beta - T_x\beta)$

$$= T_X\alpha + T_Y\alpha - T_Z\beta - T_X\beta - T_y\beta + T_Z\alpha - T_x\beta - T_Y\beta - T_Z\beta$$

$$= (\alpha - 2\beta)(T_X + T_Y + T_Z)$$

neglecting terms containing $\alpha.\beta, \alpha^2, \beta^2$

$$= (\alpha - 2\beta)(3T) \text{ if } T_X = T_Y = T_Z$$

Bulk Modulus = $\dfrac{T}{3T(\alpha - 2\beta)} = \dfrac{1}{3(\alpha - 2\beta)}$

$$K = \frac{1}{3(\alpha - 2\beta)}$$

$$K = \frac{1}{3\alpha(1 - 2\sigma)}$$

$$K = \frac{Y}{3(1 - 2\sigma)}$$

RELATION BETWEEN ELASTIC CONSTANTS

$$\alpha - 2\beta = \frac{1}{3K} \text{......(1)}$$

$$\alpha + \beta = \frac{1}{2n} \text{..........(2)}$$

$$(2) - (1)$$

$$3\beta = \frac{1}{2n} - \frac{1}{3K}$$

$$\beta = \frac{3K - 2n}{18nK}$$

$$2.(2) + (1)$$

$$\alpha = \frac{3K + n}{9Kn}$$

$$Y = \frac{1}{\alpha} = \frac{\sigma}{\beta}$$

$$\frac{9}{Y} = \frac{3}{n} + \frac{1}{K}$$

8.b. [3]

Bulk **Modulus of Elasticity**

$$k = \frac{Bulk\ \ stress}{volumes\ train}$$

$$= \frac{T}{\frac{\Delta V}{V}}$$

$\Delta V = 102.5$ m^3

Final volume = V-ΔV=(10^{-5} – 102) m^3