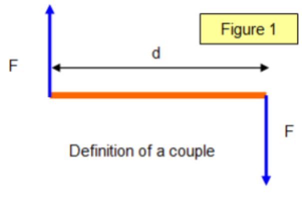
SOLUTION– JUNE 2019

Sub:	Elements of Civil Engineering and Mechanics	Sub Code:	18CIV24	Branch:	ALL

1A Explain couple and explain its characteristics. **[04]**

Two parallel forces equal in magnitude and opposite in direction and seperated by a definite distance are said form a couple.fig.1 shows the representation

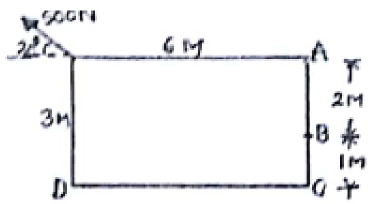


<u>Characteristics</u>

- The sum of forces forming a couple in any direction is zero, which means the translator effect of the couple is zero. The rotational effect of couple on the body is zero.
- The rotational effect of a couple about any point is a constant and it is equal to the product of the magnitude of the forces and perpendicular distance between the two forces.
- The effect of couple is unchanged if couple is rotated, shifted and replaced by another pair of forces whose rotational effects are the same.

1 b Find the moment of force at A B C D as shown in the figure. **[06]**

Fig.Q1(c)



M_A = 500sin30 X 6 = 1500Nm
M_B = 500sin30 X 6 + 500cos30 X 2 = 2366.02Nm
M_C = 500sin30 X 6 + 500cos30 X 3 =2799.03Nm
M_D = - 500cos30 X 3 = 1299.03Nm

2 a State basic idealization in mechanics **[08]**

Particle

An object that has no size but has a mass concentrated at a point, is called a particle. In mathematical sense a particle is a body whose dimensions approach zero so that it may be analyzed as a point mass.

Rigid Body

A body is said to be rigid when the relative movements between its parts are negligible. Actually, every body must deform to a certain degree under the action of forces, but in many cases the deformation is negligible and may not be considered in the analysis. This rigid body concept leads to simplified computations. Refer Fig. 1.2 (a).

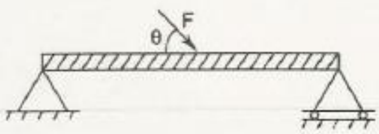


Fig. 1.2 (a) Rigid body

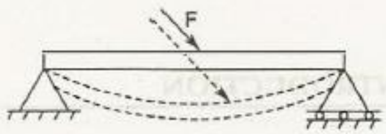


Fig. 1.2 (b) Deformable body

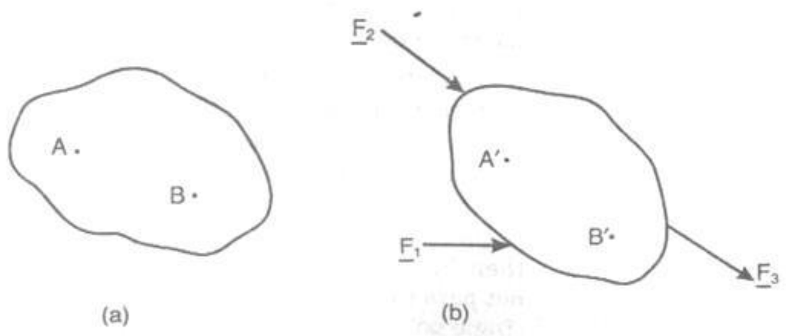
Continuum

A body consists of several particles. It is a well known fact that each particle can be sub-divided into molecules, atoms and electrons. It is not feasible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter which will not separate even when various forces considered are acting simultaneously. In other words, we say the body is treated as a continuum.

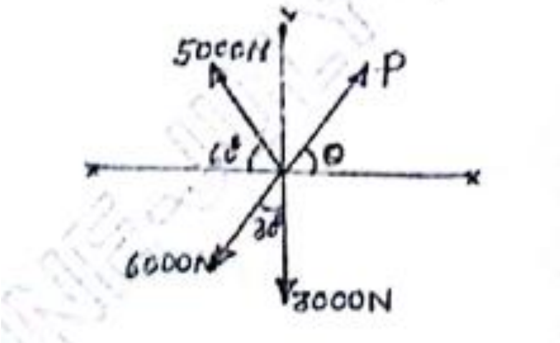
Rigid Body

As already stated, in Civil Engineering, we treat a body as rigid, when the relative position of any two particles in the body do not change even after the application of a system of forces. For examples, let the body shown in figure (a) move to a position as shown in figure (b) when the system of forces F_2 and F_3 are applied. If the body is treated as a rigid body, the relative position of A to B is the same as A' and B', i.e.,

AB = A'B'



2b Four forces are acting on a gusset plate of a bridge truss as shown in the figure. Determine the force P and angle θ to maintain equilibrium of joint. [10]



$\sum F_X$ = -5000 cos 60-6000sin30+Pcosθ = 0

Pcosθ = 5500

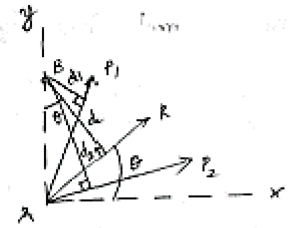
$\sum F_Y$ = 5000sin60 -3000-6000cos30+Psinθ = 0

Psinθ = 3866.02

θ = 34.99^o
P = 6713.44kN

3 a.	State and prove Varignon's theorem of moment.	[10]

Proof



$R \times d_2 = R\sin\theta \times 0 + R\cos\theta \times AB$

$R \times d_2 = R_x \cdot AB$

$P_1 \cdot d_1 = P_{1x} \, AB$

$P_2 \, d_2 = P_{2x} \, AB$

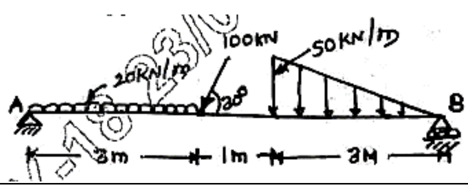
$P_1 \cdot d_1 + P_2 \, d_2 = AB\,(\underbrace{P_{1x} + P_{2x}})$

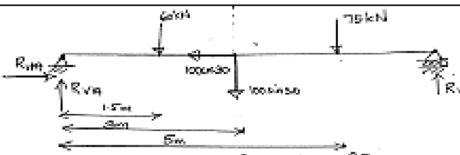
$= AB\,(R_x)$

$= Rd$

$\rightarrow P_1 d_1 + P_2 d_2 = Rd$

3b.	Determine the support reactions.	[10]





$\Sigma F_x = 0 \Rightarrow R_{HA} = 100\cos 30 = \underline{86.6\text{ kN}}$

$\Sigma F_y = 0 \Rightarrow R_{VA} - 60 - 100\sin 30 - 75 + R_{VB} = 0$

$R_{VA} + R_{VB} = 185\text{ kN}$

$\Sigma M_A = 0 \Rightarrow (60 \times 1.5) + (100\sin 30 \times 3) + (75 \times 5) (-R_{VB} \times 7) = 0$

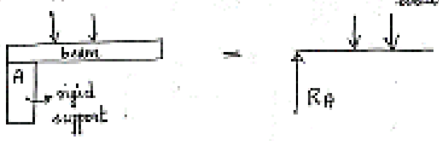
$\Rightarrow R_{VB} = 87.85\text{ kN}$

$R_{VA} = 185 - R_{VB}$
$= 185 - 87.85$
$= \underline{\underline{97.14\text{ kN}}}$

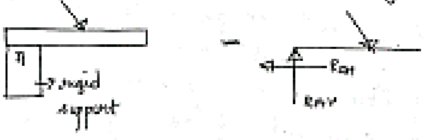
4 a. Explain with neat sketches: **[10]**

(i) Types of loads
(ii) Types of support
(iii) Types of beams

1. Simple support



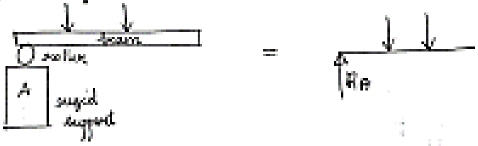
Here the beam is simply placed on a rigid support. Here the reaction will be perpendicular to the rigid support opposite to the loading direction.

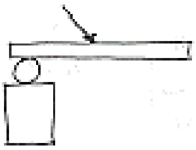


This R_{AH} is actually due to friction at point of contact b/w beam and the rigid support. Since this point of contact is very small & inclined loads come on the beam we will opt for other supports. (hinged support).

2. Roller Support

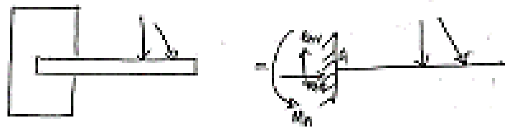
The beam is placed on a rigid support with a roller in between to remove the friction at point of contact between the beam and the support





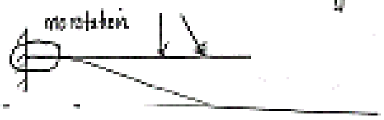
= unstable
as no horizontal
reaction can be developed at A
→ the beam slips of the support

Fixed Support

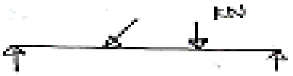


This support restricts the beam from translation in vertical and horizontal direction as well as from rotation.

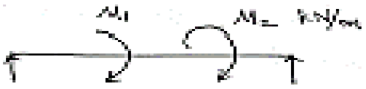
eg: When reinforcements in beam are taken into columns which makes the support fixed.



1) Point loads or Concentrated loads.



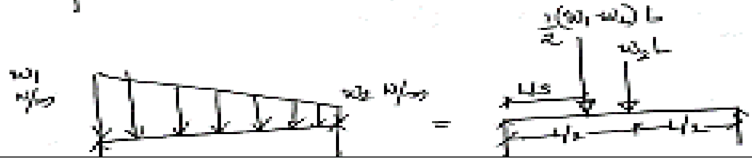
) Couple moments



) UDL - Uniformly Distributed Load

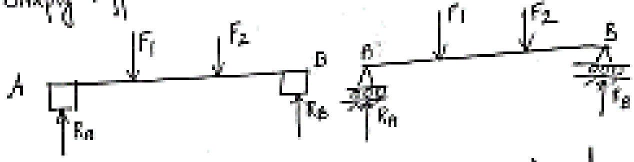
eg: weight of beam always acts like a UDL.

Uniformly Varying load (UVL)



Types of Beams

① Simply supported beams and Beams on roller support



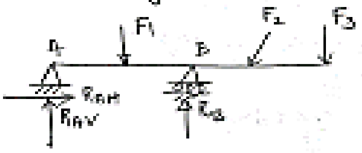
2 eqns 2 unknowns — The support reactions R_A and R_B must be found by $\boxed{\Sigma F_y = 0 \text{ and } \Sigma M = 0}$. If inclined loads come on the beam then the beam will become unstable.

② Beams with one end hinged and the other roller.

3 eqns 3 unknowns

The reactions R_{AH}, R_{AV}, R_B can be found by $\boxed{\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M = 0}$. The problem will simplify to the first case if inclined loads are not present.

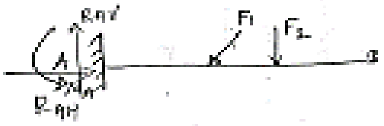
③ Overhanging beams



Cannot be stable if both supports are rollers.

R_{AH}, R_{AV}, R_B can be solved by $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0$
3 equations and 3 unknowns.

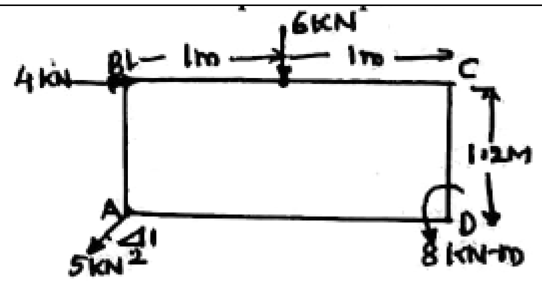
④ Cantilever Beams.



R_{AV}, R_{AH}, M_A can be solved by $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0$
3 equations and 3 unknowns

In all the above case the number of unknown is equal to the number of equations. Hence all the above type of beams are statically determinate

4 b	Determine the resultant with respect to D.	[10]



$\Sigma F_x = 40 - 50\cos 26.56 = -4.72\,kN$

$\Sigma F_y = -6 - 50\sin 26.56 = -28.36\,kN$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$= \sqrt{-4.72^2 + (28.36)^2}$

$= \underline{28.75\,kN}$

$\theta = \tan^{-1}\dfrac{\Sigma F_y}{\Sigma F_x} = \tan^{-1}\dfrac{-28.36}{-4.72}$

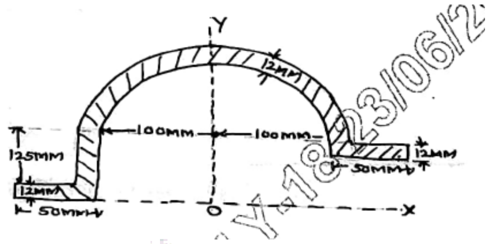
$= \underline{80.55^\circ}$

$\Sigma M_O = 40\times1.2 - 6\times1 + (50\times\cos 26.56\times 2)$

$- 50\times\sin 26.56\times 2 - 8 = -10.71\,kNm$

$d = \dfrac{|\Sigma M_O|}{|R|} = \dfrac{10.71}{28.75} = \underline{0.37\,m}$

5 a. Determine the centroid with respect to X and Y axis. [10]



Component	Area mm^2	x (mm)	y (mm)
1	$50\times12 = 600$	$-(100+\frac{50}{2}) = -125$	$\frac{12}{2} = 6$
2.	$125\times12 = 1500$	$-(100+12/2) = -106$	$12+125/2 = 74.5$
3.	$50\times12 = 600$	$100+50/2 = 125$	$125+12/2 = 131$
4 5	$-\frac{\pi\times(100)^2}{2} = 15707.9$	0	$125+12+\frac{4\times100}{3\pi} = 179.44$
5 4	$+\frac{\pi\times(112)^2}{2} = +19704.07$	0	$125+12+\frac{4\times112}{3\pi} = 184.553$

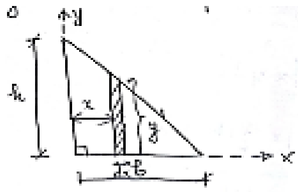
$\bar{X} = \dfrac{(600\times-125)+(1500\times-106)+(600\times125)-(15707.9\times0)+(19704.07\times0)}{6696.17}$

$= \underline{-23.75\,mm}$

$\bar{Y} = \dfrac{(600\times6)+(1500\times74.5)+(600\times131)+-(15707.9\times179.44)+(19704.07\times184.553)}{6696.17}$

$= \underline{151.03\,mm}$

6a. Find the centroid of a triangle by method of integration.



: similar triangles

$\frac{y'}{(b-x)} = \frac{h}{b}$

$y' = \frac{h}{b}(b-x)$

Area of the shaded element

$dA = y'dx$

$A = \int dA = \int_0^b y'dx$

$= \int_0^b \frac{h}{b}(b-x)\, dx$

$= \frac{h}{b}\left(bx - x^2/2\right)_0^b$

$= \frac{h}{b}\left(b^2 - b^2/2\right)$

$= \frac{h}{b} \times \frac{b^2}{2} = \frac{bh}{2}$

$\bar{x} = \frac{\int x\, y\, dx}{A}$

$= \frac{\int_0^b x\, \frac{h}{b}(b-x)\,.\,dx}{\frac{1}{2}bh}$

$= \frac{\frac{h}{b}\int_0^b bx - x^2\, dx}{\frac{1}{2}bh}$

$= \frac{\frac{h}{b}\left(b\frac{x^2}{2} - \frac{x^3}{3}\right)_0^b}{\frac{1}{2}bh}$

$= \frac{h}{b} \times \frac{b^3}{2} - \frac{b^3}{3} \times \frac{2}{bh}$

$= \frac{\cancel{h}}{\cancel{b}} \times \frac{b^3}{6} \times \frac{2}{\cancel{bh}}$

$\bar{X} = \frac{b}{3}$

distance of centroid of subdivision from x-axis

$\bar{y} = \frac{\int y\, dA}{A}$

$= \frac{1}{A}\int_0^b \frac{h}{b}(b-x) \times \frac{h}{b}(b-x)\, dx \times \frac{1}{2}$ centroidal dist $= y = y/2$

$= \frac{1}{A}\, \frac{h^2}{b^2}\int_0^b \frac{(b^2 + x^2 - 2bx)}{2}\, dx$

$= \frac{1}{2A}\, \frac{h^2}{b^2}\left(b^2x + \frac{x^3}{3} - bx^2\right)_0^b = \frac{h^2}{2b^2A}\left(\cancel{b^3} + \frac{b^3}{3} - \cancel{b^3}\right)$ =h/3