CBCS SCHEME

USN

18MCA23

Second Semester MCA Degree Examination, June/July 2019
Discrete Mathematical Structures and Statistics

Time: 3 hrs. Max. Marks: 100

Note: *Answer FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Prove the following conditional is a tautology.
$[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \leftrightarrow [(p \to q) \wedge (q \to r) \wedge (r \to p)]$ **(07 Marks)**

b. Given the following proposition, write
i) Direct proof ii) Indirect-proof
"If n-is an odd integer, then (n+11) – is an even integer." **(07 Marks)**

c. Using the laws of logic prove the following conditional expression:
$[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$ **(06 Marks)**

OR

2 a. Prove the following argument is valid :

$$\begin{array}{l} p \to q \\ r \to s \\ \underline{p \vee s} \\ \therefore q \vee s \end{array}$$

(07 Marks)

b. Negate and simplify the following:
i) $\exists x, [p(x) \vee g(x)]$ ii) $[\exists x, [p(x) \vee q(x)]] \to r(x)$ **(07 Marks)**

c. Summarize the laws of logic. **(06 Marks)**

Module-2

3 a. Determine sets A and B, given that:
$A - B = \{1, 2, 4\}$, $B - A = \{7, 8)$ and $A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$. **(07 Marks)**

b. For any three sets A, B, C prove that
i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(07 Marks)**

c. State and prove the addition theory in probability. **(06 Marks)**

OR

4 a. A problem is given to four students A, B, C, D whose chances of solving it are 1/2, 1/3, 1/4, 1/5 respectively. Find the probability that the problem is solved. **(07 Marks)**

b. The probabilities that three persons x, y, z hit a target in one attempt are 1/6, 1/4 and 1/3 respectively. If each of these shoots once at a target-find:
i) The probability that the target is hit
ii) The probability that the target is hit by exactly one person. **(07 Marks)**

c. Prove the Demorgan laws, for any two sets:
i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Find the number of permutations of the letters of the word 'INSTITUTION'
i) How many of these begin with I?
ii) How many of these begin with I and end with N?
iii) In how many the 3 T's are together? (07 Marks)

b. Prove the following by using Mathematical induction for every positive integer n:

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{1}{6}\, n(n+1)(2n+1)$$
(07 Marks)

c. Given the recurrence relation $a_n = a_{n-1} + 4$ with $a_1 = 2$ obtain an explicit formula for the given sequence. (06 Marks)

OR

6 a. Find the coefficient of x^4 – in the expansion of $\left(2x^2 - \frac{3}{x}\right)^8$. (07 Marks)

b. A man has 7-relatives, 4-of them are ladies and 3-gentlemen. His wife has also 7-relatives 3-of them are ladies and 4-gentlemen. In how many ways can they invite a dinner party of 3-ladies and 3-gentlemen so that there are 3 – of the man's relative and 3 – of the wife's relatives? (07 Marks)

c. The Fibonacci numbers are defined by $F_0 = 1$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Evaluate F_2 to F_{10}. (06 Marks)

Module-4

7 a. The probability distribution of a finite random variable – X – is given by

X :	-2	-1	0	1	2	3
P(X) :	0.1	K	0.2	2K	0.3	K

Find: i) The value of K ii) Mean iii) Variance and standard deviation. (07 Marks)

b. In a certain town the duration of the shower is exponentially distributed with a mean 5-min. What is the probability that a shower will last for
i) 10-min or more ii) less than 10-min iii) Between 10 and 12 min. (07 Marks)

c. The weekly wages of workers in a company are normally distributed with mean of Rs.700 and standard deviation of Rs.50. Find the probability that the weekly wage of a randomly chosen worker is
i) Between Rs.650/- and Rs.750
ii) More than Rs.750/- (06 Marks)

OR

8 a. Obtain the mean and standard deviation of the Poisson distribution. (07 Marks)

b. The probability density function of a variate X – is given by the following table:

X :	0	1	2	3	4	5	6
P(X):	K	3K	5K	7K	9K	11K	13K

Find: i) The value of K ii) $P(X < 4)$, $P(X \geq 5)$. (07 Marks)

c. The number of telephone lines busy at an instant of time is a binomial variate with $P = 0.2$. If at an instant 10 lines are choosen at random what is the probability that
i) 5-lines are busy ii) At most 2-lines are busy. (06 Marks)

Module-5

9 a. By the method of least squares, find the straight line that fits the following data: ($y = ax + b$)

x:	1	2	3	4	5
y:	14	27	40	55	68

(07 Marks)

b. Find the correlation coefficient for the two groups,

x :	92	89	87	86	83	77	71	63	53	50
y :	86	83	91	77	68	85	52	82	37	57

(07 Marks)

c. Define the terms
i) Coefficient of correlation
ii) Regression
iii) Principle of least squares. (06 Marks)

OR

10 a. Find the correlation coefficient 'r' and the equations of the lines of regression for the following values of x and y

x :	1	2	3	4	5
y :	2	5	3	8	7

(10 Marks)

b. Fit a curve of the form $y = ae^{bx}$, to the following data:

x:	5	15	20	30	35	40
y:	10	14	25	40	50	62

(10 Marks)

* * * * *

1. a) $\left[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)\right] \leftrightarrow \left[(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)\right]$

LHS

$(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)$

$\equiv (p \rightarrow q) \wedge (q \rightarrow p) \wedge (q \rightarrow r) \wedge (r \rightarrow q) \wedge (r \rightarrow p) \wedge (p \rightarrow r)$

$\equiv \{(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)\} \wedge \{(q \rightarrow p) \wedge (r \rightarrow q) \wedge (p \rightarrow r)\}$

$\equiv \{(p \rightarrow r) \wedge (r \rightarrow p)\} \wedge \{(q \rightarrow p) \wedge (p \rightarrow r) \wedge (r \rightarrow q)\}$

syllogism

$\equiv (p \leftrightarrow p) \wedge (q \rightarrow r) \wedge (r \leftrightarrow q)$

$\equiv (p \leftrightarrow p) \wedge (q \leftrightarrow q)$

$\equiv T_0 \wedge T_0 \equiv T_0$

RHS

$(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)$

$\equiv (p \rightarrow r) \wedge (r \rightarrow p)$

$\equiv p \leftrightarrow p$

$\equiv T_0$

b. p : n is an odd integer

q : $n+1$ is an even integer

$p \rightarrow q$

Direct Proof Hypothesis: n is an odd integer

$n = 2k+1$

Analysis: $n+1 = 2k+2$
$= 2(k+1)$ even

Conclusion $n+1$ is an even integer

Indirect Proof:

Hypotheses: Let n be not an odd integer

$n = 2k$

Analysis : $n+1 = 2k+1$ not a multiple of 2

$n+1$ is odd integer

Conclusion: If n is not an odd integer

then $n+1$ is an odd integer.

c. $[(p \vee q) \wedge (p \vee \sim q)] \vee q$ disterubutive law

$\equiv [p \vee (q \wedge \sim q)] \vee q$

$\equiv (p \vee F_0) \vee q$

$\equiv p \vee q$

2 a) $(p \to q) \wedge (r \to s) \wedge (p \vee s)$

$\equiv p \to q \wedge (\sim r \vee s) \wedge (p \vee s)$

$\equiv (p \to q) \wedge (\sim r \wedge p) \vee s$ Disterubutive

$\equiv (p \to q) \wedge (p \wedge \sim r) \vee s$ conjuactive simplification

$\equiv (p \to q) \wedge p \vee s$

$\equiv p \wedge (p \to q) \vee s$

$\equiv q \vee s$

b) $\exists x \, [p(x) \vee q(x)]$

$\sim [\exists x, p(x) \vee q(x)] \equiv \forall x, \sim (p(x) \vee q(x))$

$\equiv \forall x, \sim p(x) \wedge \sim q(x)$

c. Idempotent $p \vee p \equiv p$, $p \wedge p \equiv p$

Associative $(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Commutative $p \vee q \equiv q \vee p$ $\quad p \wedge q \equiv q \wedge p$

Distributive $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Identity $p \vee F \equiv p$ $\quad p \wedge T \equiv p$

$p \vee T \equiv T$ $\quad p \wedge F \equiv F$

$p \vee \sim p \equiv T$ $\quad p \wedge \sim p \equiv F$

Absorption $p \vee (p \wedge q) \equiv p$ $\quad p \wedge (p \vee q) \equiv p$

De Morgans $\sim (p \wedge q) \equiv \sim p \vee \sim q$

$\sim (p \vee q) \equiv \sim p \wedge \sim q$

Involution $\sim (\sim p) \equiv p$

3a) $A - B = \{1, 2, 4\}$

$B - A = \{7, 8\}$

$A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$

$A \cap B = \{5, 9\}$

$A = \{1, 2, 4, 5, 9\}$

$B = \{4, 5, 7, 8\}$

b) Let A, B, C be any 3 sets

Let $x \in A \cap (B \cup C)$

Let $x \in A$ & $x \in B \cup C$

$\Rightarrow x \in A$ & $x \in B$ or/and C

$\Rightarrow x \in A$ & $x \in B$ or $x \in A$ and $x \in C$

$\Rightarrow x \in A \cap B$ or $x \in A \cap C$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Let $x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow x \in A \cap B$ or $x \in A \cap C$

$\Rightarrow x \in A$ & $x \in B$ or $x \in A$ & $x \in C$

$\Rightarrow x \in A$ & $(x \in B$ or $x \in C)$

$\Rightarrow x \in A \cap (B \cup C)$

$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ (2)

(1) & (2) $\Rightarrow$

~~Let $x \in A$~~

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Let $x \in A \cup (B \cap C)$

$x \in A$ or $x \in B \cap C$

$x \in A$ or $x \in B$ & $x \in C$

$x \in A$ or $x \in B$ & $x \in A$ or $x \in C$

$x \in A \cup B$ & $x \in A \cup C$

$x \in (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ (3)

Let $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow x \in A \cup B$ & $x \in A \cup C$

$\Rightarrow (x \in A$ or $x \in B)$ & $(x \in A$ or $x \in C)$

$\Rightarrow x \in A$ or $(x \in B$ & $x \in C)$

$\Rightarrow x \in A \cup (B \cap C)$

$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ (4)

c. For any 2 sets A, B

a) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ b) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Proof Let $x \in \overline{A \cup B}$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$

$\Rightarrow x \in \overline{A} \cap \overline{B}$

$\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ (1)

Let $x \in \overline{A} \cap \overline{B}$

$x \in \overline{A}$ & $x \in \overline{B}$

$x \notin A$ & $x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in \overline{A \cup B}$.

$\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$ (2)

(1) & (2) $\Rightarrow \overline{A} \cap \overline{B} = \overline{A \cup B}$

b) Let $x \in \overline{A \cap B}$

$x \notin A \cap B \Rightarrow x \notin A$ & $x \notin B$

$\Rightarrow x \in \overline{A}$ & $x \in \overline{B}$

$\Rightarrow x \in \overline{A} \cup \overline{B}$

$\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$ (3)

Let $x \in \overline{A} \cup \overline{B}$

$x \in \overline{A}$ or $x \in \overline{B}$

$x \notin A$ or $x \notin B$

$x \notin A \cap B$

$x \in \overline{A \cap B}$

$\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$ (4)

(3) + (4) $\Rightarrow$ $\cancel{\overline{A \cup B} = \overline{A} \cap \overline{B}}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$

4 a) Let A, B, C, D be 4 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.

$P(A) = \frac{1}{2}$ $\quad P(B) = \frac{1}{3}$ $\quad P(C) = \frac{1}{4}$

$P(D) = \frac{1}{5}$

P(problem solved) = 1 − P(problem not solved)

$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(\bar{C}) = 1 - P(C) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(\bar{D}) = 1 - P(D) = 1 - \frac{1}{5} = \frac{4}{5}$

$\bar{A}, \bar{B}, \bar{C}, \bar{D}$ are for not solving the problem

P(problem not solved)

$= P(\bar{A})\, P(\bar{B})\, P(\bar{C})\, P(\bar{D})$

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} = \frac{1}{5}$

P(problem solved) = 1 − P (problem not solved)

$= 1 - \frac{1}{5} = \frac{4}{5}$

③ + ④

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

c. For any two sets A, B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

where $A \cap B \neq \phi$

For any 3 sets A, B, C

$$P(A \cup B \cup C) = P(A \cup D)$$

where $D = B \cup C$

$$= P(A) + P(D) - P(A \cap D)$$

$$= P(A) + P(B \cup C) - P(A \cap (B \cup C))$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P((A \cap B) \cup (A \cap C))$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - \left(P(A \cap B) + P(A \cap C) - P(A \cap B \cap A \cap C)\right)$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

$$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap C) - P(A \cap B) + P(A \cap B \cap C)$$

5c $a_n = a_{n-1} + 4, \quad a_1 = 2$

$\underline{n=2}$ $a_2 = a_1 + 4 = 2 + 4$

$\underline{n=3}$ $a_3 = a_2 + 4 = 2 + 4 + 4$

$\underline{n=4}$ $a_4 = a_3 + 4 = 2 + 4 + 4 + 4$

$\underline{n=5}$ $a_5 = a_4 + 4 = 2 + 4 + 4 + 4 + 4$

$\cdots$

$$a_n = 2 + (\underbrace{4 + 4 + \cdots + 4}_{n-1})$$

$$= 2 + 4(n-1)$$

$$= 4n - 2$$

Explicit form $\{a_n\} = \{4n - 2\}$

a) INSTITUTION – 11 letters

No. of arrangements $= \dfrac{11!}{3!\,2!\,3!\,1!\,1!}$

3 I's, 2 N's, 3 T's, 1 S, 1 O

$$= \frac{1 \times \cancel{2} \times \cancel{3} \times \cancel{4} \times 5 \times \cancel{6} \times 7 \times 8 \times 9 \times 10 \times 11}{(\cancel{6})(\cancel{2})(\cancel{6})}$$

$$= (10)(7)(72)(110)$$

$$= 554400$$

5.b) Let $S(n)$ be the stt

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1)$$

$S(1)$: $1^2 = \frac{1}{6}\, 1(1+1)(2+1)$

$$1^2 = 1^2$$

Let $S(k)$ be true

To S.T $S(k+1)$ is true

$S(k)$: $1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{1}{6} k(k+1)(2k+1)$

Consider $\underbrace{1^2 + 2^2 + 3^2 + \dots + k^2}$ $+ (k+1)^2$

$$= \frac{1}{6} k(k+1)(2k+1) + (k+1)^2$$

$$= (k+1)\left\{\frac{1}{6} k(2k+1) + (k+1)\right\}$$

$$= (k+1)\left\{\frac{2k^2+k}{6} + k+1\right\}$$

$$= (k+1)\,\frac{2k^2+7k+6}{6}$$

$$= \frac{1}{6}(k+1)(2k^2+4k+3k+6)$$

$$= \frac{1}{6}(k+1)\left[2k(k+2)+3(k+2)\right]$$

$$= \frac{1}{6}(k+1)(2k+3)(k+2)$$

$$= \frac{1}{6}(k+1)$$

$\therefore$ $S(k+1)$ is true

By Mathematical Induction

$S(n)$ is true $\forall n \in Z^+$

6 a) $\left(2x^2 - \frac{3}{x}\right)^8$

$$= \sum_{r=0}^{8} 8C_r (2x^2)^r \left(-\frac{3}{x}\right)^{8-r}$$

$$= \sum_{r=0}^{8} 8C_r \, 2^r (-3)^{8-r} x^{2r} \cdot \frac{1}{x^{8-r}}$$

$$= \sum_{r=0}^{8} 8C_r \, 2^r (-3)^{8-r} x^{3r-8}$$

To find the co-eff of x^4

$3r - 8 = 4 \quad 3r = 12 \quad r = 4$

Ans $8C_4 \, 2^4 (-3)^{8-4} = \frac{8!}{4!\,4!} (16)(81)$

$$= \frac{\cancel{1 \times 2 \times 3 \times 4} \times 5 \times \overset{2}{\cancel{6}} \times 7 \times \cancel{8}}{(\cancel{24})(\cancel{24})} (16)(81)$$

$$= (70)(16)(81) = 90,720$$

6c) $F_0 = 0 \quad F_1 = 1 \quad F_n = F_{n-1} + F_{n-2}, \; n \geq 2$

$\{F_n\} = \{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots\}$

$F_2 = 1 \quad F_3 = 2 \quad F_4 = 3 \quad F_5 = 5$

$F_6 = 8 \quad F_7 = 13 \quad F_8 = 21 \quad F_9 = 34 \quad F_{10} = 55$

6 b Find the no. of committees of 5 selected from 7 men and 5 women if the committee has at least 1 man and 1 woman.

There are 12 people - 7 men and 5 women.

5 people can be selected from 12 people in $12C_5$ ways.

There are $7C_5$ committees consisting of 5 men and $1 = 5C_5$ committee consisting of 5 women

The no. of committees consisting atleast 1 man and 1 woman is

$$12C_5 - 7C_5 - 1$$

$$= \frac{12!}{7!.5!} - \frac{7!}{5!.2!} - 1$$

$$= 792 - 21 - 1 = 770$$

7.

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	k

We K.T $\sum P(x) = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$4k + 0.6 = 1 \quad 4k = 0.4 \quad \boxed{k = 0.1}$

Mean $\mu = \sum x P(x)$

$= -2(0.1) - 1(k) + 0(0.2) + 1(2k) + 2(0.3) + 3k$

$= -0.2 - k + 2k + 0.6 + 3k$

$= 0.4 + 4k = 0.4 + 4(0.1) = 0.8$

Variance $= \sigma^2 = E(X^2) - \mu^2$

$= \sum x^2 P(x) - \mu^2$

$= \left[(-2)^2(0.1) + (-1)^2 k + 0^2(0.2) + 1^2(2k) + 2^2(0.3) + 3^2(k)\right] - (0.8)^2$

$= \left[4(0.1) + 0.1 + 2(0.1) + (1.2) + 9(0.1)\right] - (0.8)^2$

$\sigma^2 = 2.16$

S.D $\sigma = \sqrt{2.16} = 1.4697$

ii) begin with I and end with N.

I NSTITUTIO N

I and N are fixed

$$\frac{9!}{3!\,1!\,2!\,1!\,2!} =$$

$$\frac{1 \times \cancel{2} \times 3 \times \cancel{4} \times 5 \times 6 \times 7 \times 8 \times 9}{(\cancel{6})(2)(\cancel{2})}$$

$$= (210)(72) = 15120$$

No. of arrangements $= 15120$

iii) T T T I N S O U
3 3 2 1 1 1

Treat all of T's as one letter

There are 9 letters.

No. of permutations $\dfrac{9!}{3!\,2!\,1!\,1!\,1!}$

$$\frac{1 \times 2 \times \cancel{3} \times \cancel{4} \times 5 \times 6 \times 7 \times 8 \times 9}{(\cancel{6})(\cancel{2})}$$

$$= (10)(42)(72) = 30,240$$

8 a)　$P(m, x) = \dfrac{e^{-m} m^x}{x!}$　$x = 0, 1, 2, 3 \ldots$

Mean μ　$E(x) = \sum x P(m, x)$

$= 0 P(m, 0) + 1 P(m, 1) + 2 P(m, 2) + 3 P(m, 3) + \cdots$

$= m e^{-m} + \dfrac{e^{-m} m^2}{2!}(2) + \dfrac{e^{-m} m^3}{3!}(3) + \dfrac{e^{-m} m^4}{4!}(4) + \cdots$

$\cancel{= m e^{-m} \left\{ 1 + \frac{m e^{-m}}{1!} + m \right.}$

$\cancel{= m e^{-m} \left\{ 1 + m e^{-m} + \frac{m^2 e^{-m}}{1} \right.}$

$= m e^{-m} \left\{ 1 + \dfrac{m}{1!} + \dfrac{m^2}{2!} + \dfrac{m^3}{3!} + \cdots \right\}$

$= m e^{-m} e^{m} = m$

Variance $\sigma^2 = \sum (x - \mu)^2 P(\mu, x)$

$= \sum (x^2 - 2\mu x + \mu^2) P(\mu, x)$

$= \sum x^2 P(\mu, x) - 2\mu \sum x P(\mu, x) + \mu^2 P(\mu, x)$

$$= \sum (x^2 - x + x)\, P(\mu, x)$$
$$- 2\mu \sum x\, P(\mu, x) + \mu^2 P(\mu, x)$$

$$= \sum (x^2 - x)\, P(\mu, x) + (1 - 2\mu) \sum x P(\mu, x)$$
$$+ \mu^2 P(\mu, x)$$

$$0 + 0 + 2(1) P(\mu, 2) + 3(2) P(\mu, 3)$$
$$+ 4(3) P(\mu, 4) + \dots + (1 - 2\mu)\mu$$
$$+ \mu^2$$

$$= 2 \frac{\mu^2}{2!} e^{-\mu} + (3)(2) \frac{\mu^3 e^{-\mu}}{3!} + (4)(3) \frac{\mu^3 e^{-\mu}}{4!}$$
$$+ \dots + \mu - 2\mu^2 + \mu^2$$

$$\mu^2 e^{-\mu} \left\{ 1 + \frac{\mu}{1!} + \frac{\mu^2}{2!} + \dots \right\} + \mu - \mu^2$$

$$\mu^2 e^{-\mu} e^{\mu} + \mu - \mu^2 = \mu^2 + \mu - \mu^2$$
$$= \mu$$

$$\text{Var } \sigma^2 = \mu$$
$$\text{S.D } \sigma = \sqrt{\mu}$$

8 b)

x	0	1	2	3	4	5	6
$P(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

We k.T $\sum P(x) = 1$

$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$49k = 1$ $\quad \boxed{k = \frac{1}{49}}$

$P(x < 4) = k + 3k + 5k + 7k = 16k = \frac{16}{49}$

$P(x \geq 5) = 11k + 13k = 24k = \frac{24}{49}$

8 c) $b(n, p, x) = nC_x \, p^x q^{n-x}$

$n = 10 \quad p = 0.2 = \frac{1}{5}$

$P(10, \frac{1}{2}, x) = 10C_x (0.2)^x (0.8)^{10-x}$

a) P(no line is busy) $= P(x = 0)$

$= 10C_0 (0.2)^0 (0.8)^{10}$

$= 0.1074$

b) P(5 lines are busy) $= P(x = 5)$

$= 10C_5 (0.2)^5 (0.8)^5$

$= \frac{(10)\cancel{(9)}\cancel{(8)}(7)(6)\cancel{5!}}{\cancel{5!}\,(1\cancel{20})} (0.2)^5 (0.8)^5$

$= 252 \, (0.00032)$

$= 0.2642$

c) P(at least one line busy)

$= P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0) = 1 - 0.1074$

$= 0.8926$

d) P(at most 2 lines are busy)

$= P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2)$

$= 0.1074 + 10C_1 (0.2)(0.8)^9 + 10C_2 (0.2)^2 (0.8)^8$

$= 0.1074 + 0.2684 + 0.302$

$= 0.6778$

e) P(all lines are busy)

$P(x = 10) = 10C_{10} \left(\frac{1}{5}\right)^{10} \left(\frac{4}{5}\right)^0$

$= 1.024 \times 10^{-7}$

7c) $\mu = 700 \quad \sigma = 50$ x is the weekly wage

SNV $Z = \frac{x-\mu}{\sigma} = \frac{x-700}{50}$

at $x = 650$, $Z = \frac{650-700}{50} = -1$

at $x = 750$ $Z = \frac{750-700}{50} = 1$

$P(650 < x < 750) = P(-1 < Z < 1)$

$= 2P(0 < Z < 1)$

$= 2(0.3413) = 0.6826$

$P(x > 750) = P(Z > 1) = P(Z \geq 0) - P(0 < Z < 1)$

$= 0.5 - A(1)$

$= 0.5 - 0.3413 = 0.1587$

b) $P(\mu, x) = \frac{e^{-\mu}\mu^{x}}{x!}$

~~μ~~ Mean $\cancel{\mu}$ = 5 min

$\frac{1}{\mu} = 5 \Rightarrow \mu = \frac{1}{5}$

$P(\frac{1}{5}, x) = \frac{e^{-1/5}(1/5)^{x}}{x!}$

a) $P(x \geq 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5} dx$

$= \frac{1}{5} \left(\frac{e^{-x/5}}{-1/5} \right)_{10}^{\infty}$

$= -\left(e^{-\infty} - e^{-2} \right) = e^{-2}$

b) $P(x < 10) = \int_{0}^{10} \frac{1}{5} e^{-x/5} dx$

$= \frac{1}{5} \left(\frac{e^{-x/5}}{-1/5} \right)_{0}^{10} = -\left(e^{-2} - e^{0} \right)$

$= 1 - e^{-2}$

c) $P(10 < x < 12) = \int_{10}^{12} \frac{1}{5} e^{-x/5} dx$

$= 1/5 \left(\frac{e^{-x/5}}{-1/5} \right)_{10}^{12}$

$= -\left(e^{-12/5} - e^{-10/5} \right)$

$= e^{-2} - e^{-12/5}$

2

10 a)

x	y	$X = x-\bar{x}$	$Y = y-\bar{y}$	X^2	Y^2
1	2	-2	-3	4	9
2	5	-1	0	1	0
3	3	0	-2	0	4
4	8	1	3	1	9
5	7	2	2	4	4

$\bar{x} = \frac{15}{5} = 3 \quad \bar{y} = \frac{25}{5} = 5$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}}$$

$$\Sigma XY = 6+0+0+3+4 = 13$$

$$r = \frac{13}{\sqrt{10}\sqrt{26}} = \frac{13}{\sqrt{260}}$$

$$= \frac{13}{\sqrt{13\times5\times4}} = \frac{\sqrt{13}}{2\sqrt{5}}$$

$$= \frac{3.606}{2\sqrt{5}}$$

$$= 0.806$$

x	y	$Y = \log_e y$	xY	x^2
5	10	1.609	8.045	25
15	14	2.7081	40.622	225
20	25	2.996	59.92	400
30	40	3.401	102.03	900
35	50	3.555	124.425	1225
40	62	3.689	147.56	1600
145	201	17.958	482.607	4375

$$\sum Y = nA + b\sum x$$

$$\sum xY = A\sum x + b\sum x^2$$

$$6A + 145b = 17.958 \quad (1)$$

$$145A + 4375b = 482.607 \quad (2)$$

$$145 \times 6A + (145 \times 145)b = 17.958 \times 145 \quad (3)$$

$$(145 \times 6)A + (4375 \times 6)b = 482.607 \times 6 \quad (4)$$

$$(3) - (4) \quad (21025 - 26250)b = 2603.91 - 2895.642$$

$$-5225b = -291.73$$

$b = 0.0558$

$6A = 17.958 - 145b$

$17.958 - (145 \times 0.0558)$

$= 9.867$

$A = \dfrac{9.867}{6} = 1.6445$

$a = e^{A} = e^{1.645} = 5.181$

$y = ae^{bx} = 5.181e^{9.867x}$

9

b)

$r = \dfrac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x \sigma_y}$

x	y	$z = x - y$	x^2	y^2	z^2
92	86	6	8464	7396	36
89	83	6	7921	6889	36
87	91	−4	7569	8281	16
86	77	9	7396	5929	81
83	68	15	6889	4624	225
77	85	−8	5929	7225	64
71	52	19	5041	2704	361
63	82	−19	3969	6724	361
53	37	16	2809	1369	256
50	57	−7	2500	3249	49
751	718	33	58487	54390	1485

$\bar{x} = \frac{751}{10} = 75.1 \quad \bar{y} = \frac{718}{10} = 71.8$

$\bar{z} = \frac{33}{10} = 3.3$

$\sigma_x^2 = \frac{\Sigma x^2}{n} - \bar{x}^2 = \frac{58487}{10} - (75.1)^2$
$= 208.69$

$\sigma_y^2 = \frac{\Sigma y^2}{n} - \bar{y}^2 = \frac{54390}{10} - (71.8)^2 = 283.76$

$$\sigma_z^2 = \frac{\Sigma z^2}{n} - \bar{z}^2 = \frac{1485}{10} - (3.3)^2$$
$$= 137.61$$

$$r = \frac{208.69 + 283.76 - 137.61}{2\sqrt{208.69}\sqrt{283.76}} = 0.73$$

9 a

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	55	220	16
5	68	340	25

$$\bar{x} = \frac{15}{5} = 3 \qquad \bar{y} = \frac{81+55+68}{5} = \frac{136+68}{5}$$
$$= \frac{204}{5} = 40.8$$
$$\sim 41$$

$$\Sigma x = 15 \qquad \Sigma y = 204 \qquad \Sigma xy = \frac{680+68}{5}$$
$$= \frac{748}{5} = 149.6$$

$$\Sigma x^2 = 55$$

$\Sigma y = a \Sigma x + nb$

$\Sigma xy = a \Sigma x^2 + b \Sigma x$

$204 = 15a + 5b$ (1)

$149.6 = 55a + 15b$ (2)

(1) $\times 3$ − (2)

$$\begin{aligned} 45a + 15b &= 612 \\ 55a + 15b &= 149.6 \\ \hline -10a &= 462 \\ a &= -46.2 \end{aligned}$$

$5b = 204 - 15(a)$

$= 204 - 15(-46.2) = 897$

$b = \frac{897}{5} = 179.4$

$a = -46.2, \quad b = 179.4$

<u>Ans</u> $y = ax + b = -46.2x + 179.4$

c. Co-variance of two independent magnitudes is known as correlation.
The numerical measure of correlation between 2 variables x & y is called as Pearson's Co-eff of correlation

c. The least squares principle states that the SRF should be constructed so that the sum of the squared distance between the observed values of the dependent variable and the values estimated from SRF is minimised.

6.b Man has 7 relatives - 4 L and 3 G
Wife has 7 relatives - 3 L and 4 G

No. of ways they invite a dinner party of 3 L and 3 G

$$= ({}^4C_3 \times {}^4C_3) + ({}^3C_3 \times {}^3C_3)$$

$$= (4 \times 4) + (1 \times 1) = 17$$

L for ladies G for Gentlemen