USN 1CR14EC434

MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2019
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the ratio in which the point P(5, 4, –6) divides the line joining the points Q(3, 2,–4) and R(9, 8,–10). **(06 Marks)**

b. Find the angles between any two diagonals of a cube. **(07 Marks)**

c. Find the projection of AB on the line CD, where A = (1, 2, 3), B = (1, 1, 1), C = (0, 0, 1) and D = (2, 3, 0). **(07 Marks)**

2 a. Show that the points (2, 2, 0), (4, 5, 1), (3, 9, 4) and (0, –1,–1) are coplanar. Find the equation of the plane containing them. **(06 Marks)**

b. Find the equation of the plane through the intersection of the planes $2x + 3y - z = 5$ and $x - 2y - 3z + 8 = 0$ and perpendicular to the plane $x + y - z = 2$. **(07 Marks)**

c. Find the shortest distance between the lines :

$$\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1} \text{ and } \frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}.$$ **(07 Marks)**

3 a. Show that the position vectors of the vertices of a triangle $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ from a right – angled triangle. **(06 Marks)**

(07 Marks)

b. If $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$ then find the angle between $(2\vec{a}+\vec{b})$ and $(\vec{a}+2\vec{b})$.

c. Find the sine of the angle between $\vec{a} = 2\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}+2\hat{k}$. **(07 Marks)**

4 a. Find the constant 'a' so that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are coplanar. **(06 Marks)**

b. Find the angle between the tangents to the curve $x = t^2, y = t^3, z = t^4$ at $t = 2$ and $t = 3$. **(07 Marks)**

c. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $\hat{i}+\hat{j}+2\hat{k}$. **(07 Marks)**

5 a. Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$. **(06 Marks)**

b. Find the constants a, b, c such that the vector :

$\vec{F} = (x+y+az)\hat{i}+(x+cy+2z)\hat{k}+(bx+2y-z)\hat{j}$ is irrotational. **(07 Marks)**

c. Prove that $\nabla^2(\log r) = \frac{1}{r^2}$ where $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ and $r = |\vec{r}|$. **(07 Marks)**

6 a. Find the laplace transform of $5\sin 2t + 3\cos 4t$. (05 Marks)

b. Find Lapace transform of $e^{-3t}\cos 4t$. (05 Marks)

c. Find $\alpha\left\{\frac{1-\cos t}{t}\right\}$. (05 Marks)

d. Find $\alpha\{f(t)\}$ where $f(t) = \begin{cases} 3t, & 0 < t < 2 \\ 6, & 2 < t < 4 \end{cases}$

Given that f(t) is the paiodic function with the period 4. (05 Marks)

7 a. Find the inverse Laplace transform of $\frac{5S+1}{S^2+16}$. (06 Marks)

b. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)(s+3)}$. (07 Marks)

c. Find the inverse Laplace transform of $\log(1-a/s)$. (07 Marks)

8 a. Solve $y'' - 3y' + 2y = 12e^{-t}, y(0) = 2, y'(0) = 6$ using Laplace transform method. (10 Marks)

b. Solve the simultaneous equation using Laplace transforms $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$, given that $x = 1, y = 0$ when $t = 0$. (10 Marks)

* * * * *

1 a) $P(5,4,-6)$ divides line joining the points $(3,2,-4)$ and $(9,8,-10)$

$C(5,4,-6)$ ~~Let m_1 and~~

$A(3,2,-4)$ $B(9,8,-10)$

Let $m_1 : m_2$ denote the ratio in which the point C divides the line AB. Then using the section formula for each co-ordinate which is given by

$$(x,y,z) = \left(\frac{x_2 m_1 + x_1 m_2}{m_1 + m_2}, \frac{y_2 m_1 + y_1 m_2}{m_1 + m_2}, \frac{z_2 m_1 + z_1 m_2}{m_1 + m_2} \right)$$

$$= \left(\frac{9m_1 + 3m_2}{m_1 + m_2}, \frac{8m_1 + 2m_2}{m_1 + m_2}, \frac{-10m_1 - 4m_2}{m_1 + m_2} \right)$$

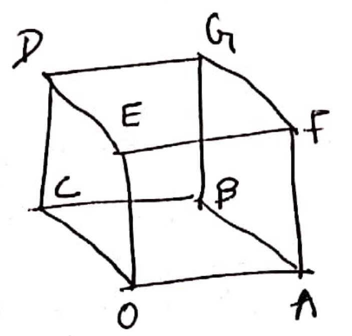
$\therefore$ equating one of them we get

$$5 = \frac{9m_1 + 3m_2}{m_1 + m_2} \Rightarrow 5m_1 + 5m_2 = 9m_1 + 3m_2$$

$$4m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \boxed{m_1 : m_2 = 1 : 2}$$

b)



Let OABCDEFG be a cube with vertices as below.

$O(0,0,0)$, $A(a,0,0)$, $B(a,a,0)$
$C(0,a,0)$ $D(0,a,a)$ $E(0,0,a)$

$F(a,0,a)$ $G(a,a,a)$

There are four diagonals.

Let us consider one of them say OG & AD

$$\vec{OG} = (a-0)\hat{i} + (a-0)\hat{j} + (a-0)\hat{k}$$

$$= a\hat{i} + a\hat{j} + a\hat{k}.$$

$$\vec{AD} = (0-a)\hat{i} + (a-0)\hat{j} + (a-0)\hat{k}$$

$$= -a\hat{i} + a\hat{j} + a\hat{k}.$$

$$\vec{OG}\cdot\vec{AD} = -a^2 + a^2 + a^2 = a^2.$$

angle between any two vectors is given by

$$\cos^{-1}\left(\frac{\vec{OG}\cdot\vec{AD}}{|\vec{OG}||\vec{AD}|}\right) = \cos^{-1}\left(\frac{a^2}{\sqrt{3}a\sqrt{3}a}\right) = \cos^{-1}\left(\frac{a^2}{3a^2}\right)$$

$$= \cos^{-1}\left(\frac{1}{3}\right)$$

c) $A(1,2,3)$, $B(1,1,1)$, $C(0,0,1)$ & $D(2,3,0)$

$$\vec{AB} = \vec{OB} - \vec{OA} = 0i - 1j - 2\hat{k}$$

$$\vec{CD} = \vec{OD} - \vec{OC} = 2\hat{i} + 3\hat{j} - \hat{k}.$$

Projection of AB on CD : $\dfrac{\vec{AB}\cdot\vec{CD}}{|\vec{CD}|} = \dfrac{-3+2}{\sqrt{4^2+3^2+1^2}}$

$$= \frac{-1}{\sqrt{26}} //$$

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2) a) $A(2,2,0)$, $B(4,5,1)$, $C(3,9,4)$, $D(0,-1,-1)$

$\vec{AB} = (2,3,1)$ $\vec{AC} = (1,7,4)$ $\vec{AD} = (-2,-3,-1)$

$\vec{AB}\cdot(\vec{AC}\times\vec{AD})$

$$\vec{AC}\times\vec{AD} = \begin{vmatrix} i & j & k \\ 1 & 7 & 4 \\ -2 & -3 & -1 \end{vmatrix} = i(5) - j(7) + k(11)$$

$$= 5i - 7j + 11\hat{k}.$$

$$\vec{AB}\cdot(\vec{AC}\times\vec{AD}) = (2i+3j+k)(5i-7j+11k)$$

$$= 10 - 21 + 11 = 21 - 21 = \underline{\underline{0}}$$

normal vector $\vec{n} = 5\hat{i} - 7\hat{j} + 11\hat{k}$

The equation of the plane is given by

$\vec{n}\cdot(\vec{r} - \vec{r}_0) = 0$) $\vec{r} = xi + yj + z\hat{k}$.

$$(5i - 7j + 11k)\left((xi + yj + z\hat{k}) - (2i + 2j + 0\hat{k})\right) = 0$$

$$(5i - 7j + 11k)\left((x-2)i + (y-2)j + z\hat{k}\right) = 0$$

$$5(x-2) - 7(y-2) + 11z = 0$$

$$5x - 10 - 7y + 14 + 11z = 0$$

$$\boxed{5x - 7y + 11z = -4}$$

b)

$2x + 3y - z = 5$ & $x + y - z = 2$

$x - 2y - 3z = -8$

~~we for~~ The equation of the ~~line~~ Plane passing through the intersection of 2 plane is given by

$(2x+3y-z-5)+\lambda(x-2y-3z+8)=0$

$(2+\lambda)x+(3-2\lambda)y-(1+3\lambda)z+(8\lambda-5)=0$

Also, the plane is perpendicular to the plane $x+y-z=2$

So the normal vector $\vec{N}$ is ~~perpen~~ to the plane is perpendicular to the normal vector of $x+y-z=2$

$\vec{N}=(2+\lambda)i+(3-2\lambda)j-(1+3\lambda)\hat{k}$

Direction ratios: $(2+\lambda),\ (3-2\lambda),\ -(1+3\lambda)$

$\vec{n}=1i+1j-z\hat{k}$

Direction ratios $1, 1, -1$

$\vec{N}\perp\vec{n}$

$\Rightarrow \vec{N}\cdot\vec{n}=0 \Rightarrow 2+\lambda+3-2\lambda+1+3\lambda=0$

$2\lambda+6=0 \Rightarrow \lambda=3$

$(2+3)x+(3-6)y-(1+9)z+(24-5)=0$

$\boxed{5x-3y-10z=-19}$

c) The shortest distance between two lines is given by

$$\frac{\left|\begin{matrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{matrix}\right|}{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}$$

(3)

$\frac{x-6}{3} = \frac{y-7}{-1}, \frac{z-4}{1}$ and

$\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$

$a_1 = 3, \quad b_1 = -1, \quad c_1 = 1 \;|\; a_2 = -3, \; b_2 = 2, \; c_2 = 4$

$x_1 = 6, \quad y_1 = 7, \quad z_1 = 4 \;|\; x_2 = 0, \; y_2 = -9, \; z_2 = 2$

$$\begin{vmatrix} -6 & -16 & -2 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = -6(-6) + 16(12+3) - 2(3)$$

$$36 + 240 - 6 = 270$$

$$\sqrt{(2\cdot 3 - 3)^2 + (-4-2)^2 + (-3-12)^2} = \sqrt{9 + 36 + 225} = \sqrt{270}$$

$\therefore \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$

$$\begin{array}{r|l} 3 & 270 \\ 3 & 90 \\ 3 & 30 \\ 2 & 10 \\ 5 & 5 \end{array}$$

5(a) $\phi = x^3+y^3+z^3-3xyz$, $\nabla\phi = \frac{\partial\phi}{\partial x}i+\frac{\partial\phi}{\partial y}j+\frac{\partial\phi}{\partial z}k$

$\vec{F} = \text{grad}\,\phi = (3x^2-3yz)i+(3y^2-3xz)j+(3z^2-3xy)k$

$\text{div}\,\vec{F} = 6(x+y+z)$

$$\text{curl}\,\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \end{vmatrix}$$

$\text{curl}\,\vec{F} = i\{-3x+3x\} - j\{-3y+3y\} + k(-3z+3z) = 0$

(b) $\vec{F}$ is irrotational if $\text{curl}\,\vec{F} = 0$

$$\nabla\times\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y+az & bx+2y-z & x+cy+2z \end{vmatrix} = 0$$

$\Rightarrow i(c+1) - j(1-a) + k(b-1) = 0$

$\Rightarrow c+1=0 \quad 1-a=0 \quad b-1=0$

$c=-1 \quad a=1 \quad b=1$

(c) $\vec{r} = xi+yj+zk$, $r = |\vec{r}|$

$r = \sqrt{x^2+y^2+z^2}$

$$\nabla^2(\log r) = \frac{\partial^2}{\partial x^2}(\log r) + \frac{\partial^2}{\partial y^2}(\log r) + \frac{\partial^2}{\partial z^2}(\log r)$$

$$= \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2+z^2}\right) + \frac{\partial}{\partial z}\left(\frac{z}{x^2+y^2+z^2}\right)$$

$$= \frac{x^2+y^2+z^2(1)-x(2x)}{(x^2+y^2+z^2)^2} + \frac{x^2+y^2+z^2-2y^2}{(x^2+y^2+z^2)^2} + \frac{x^2+y^2+z^2-2z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{y^2+z^2-\cancel{x^2}+\cancel{x^2}+\cancel{z^2}-\cancel{y^2}+x^2+\cancel{y^2}-\cancel{z^2}}{(x^2+y^2+z^2)^2} = \frac{(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}$$

$$= \frac{1}{x^2+y^2+z^2} = \frac{1}{r^2}$$

$\therefore \nabla^2(\log r) = \frac{1}{r^2}$

CMR

3 (a) A B C be the vertices of a triangle

$\vec{OA} = 2i - j + k \quad \vec{OB} = i - 3j - 5k \quad \vec{OC} = 3i - 4j - 4k$

$\vec{AB} = -i - 2j - 6k = \vec{OB} - \vec{OA}$

$\vec{BC} = \vec{OC} - \vec{OB} = 2i - j + k$

$\vec{CA} = \vec{OA} - \vec{OC} = -i + 3j + 5k$

$AB = |\vec{AB}| = \sqrt{41} \quad BC = |\vec{BC}| = \sqrt{6} \quad CA = |\vec{CA}| = \sqrt{35}$

$BC^2 + CA^2 = 6 + 35 = 41 = AB^2$

$\therefore$ ABC is a right angled triangle.

(b) $2\vec{a} + \vec{b} = 5i + 3j - 4k$

$\vec{a} + 2\vec{b} = 7i + k$

$\therefore$ Angle between $(2\vec{a} + \vec{b})$ and $(\vec{a} + 2\vec{b})$ is given by

$$\cos\theta = \frac{(2\vec{a} + \vec{b}) \cdot (\vec{a} + 2\vec{b})}{|2\vec{a} + \vec{b}|\,|\vec{a} + 2\vec{b}|} = \frac{35 - 4}{\sqrt{25+9+16}\sqrt{49+1}} = \frac{31}{50}$$

$$\theta = \cos^{-1}\left(\frac{31}{50}\right)$$

(c) $$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix} = i(-4+2) - j(4-1) + k(-4+2)$$

$\vec{a} \times \vec{b} = -2i - 3j - 2k \quad \therefore |\vec{a} \times \vec{b}| = \sqrt{4+9+4} = \sqrt{17}$

$|\vec{a}| = \sqrt{4+4+1} = \sqrt{9} = 3 \quad |\vec{b}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$$\sin\theta = \frac{\sqrt{17}}{3 \times 3} = \frac{\sqrt{17}}{9}.$$

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CMR

4(a) Let $\vec{A} = 2i - j + k$ $\quad \vec{B} = i + 2j - 3k$ $\quad \vec{C} = 3i + aj + 5k$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & a & 5 \end{vmatrix} = 0$$

$$2(10 + 3a) + 1(5 + 9) + 1(a - 6) = 0$$

$$20 + 6a + 14 + a - 6 = 0$$

$$7a + 28 = 0 \Rightarrow 7a = -28 \Rightarrow \boxed{a = -4}$$

(b) $\vec{r} = xi + yj + zk \Rightarrow \vec{r} = t^2 i + t^3 j + t^4 k$

$$\vec{T} = \frac{d\vec{r}}{dt} = 2ti + 3t^2 j + 4t^3 k$$

Let $\vec{a} = [\vec{T}]_{t=2} = 4i + 12j + 32k = 4[i + 3j + 8k]$

$|\vec{a}| = 4\sqrt{74}$ and $\vec{b} = [\vec{T}]_{at\ t=3} = 3[2i + 9j + 36k]$

$|\vec{b}| = 3\sqrt{1381}$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{4(i + 3j + 8k) \cdot 3(2i + 9j + 31k)}{4\sqrt{74} \quad 3\sqrt{1381}} = \frac{2 + 27 + 248}{\sqrt{74}\ \sqrt{1381}}$$

$$\cos\theta = 0.8665$$

$$\theta = \cos^{-1}(0.8665) = 30^\circ$$

(c) Let $\phi = x^2 y z^3$

Directional derivative of ϕ along $\vec{a}$ is $\nabla\phi \cdot \vec{a}$

$$\nabla\phi = 2xyz^3 i + x^2 z^3 j + 3x^2 y z^2 k, \quad [\nabla\phi]_{(1,1,1)} = 2i + j + 3k$$

Let $\vec{a} = i + j + 2k \Rightarrow |\vec{a}| = \sqrt{6}$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{i + j + 2k}{\sqrt{6}} \quad \therefore \nabla\phi \cdot \hat{a} = (2i + j + 3k) \cdot \frac{(i + j + 2k)}{\sqrt{6}}$$

$$\nabla\phi \cdot \hat{a} = \frac{2 + 1 + 6}{\sqrt{6}}$$

$$\therefore \boxed{\nabla\phi \cdot \hat{a} = \frac{9}{\sqrt{6}}}$$

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6 (a) Let $f(t) = 5\sin 2t + 3\cos 4t$

$\therefore L[f(t)] = L[5\sin 2t + 3\cos 4t]$

$= 5L[\sin 2t] + 3L[\cos 4t]$ (by linear property)

$= 5 \cdot \frac{2}{s^2+2^2} + 3 \cdot \frac{s}{s^2+4^2}$

$$\boxed{\therefore L[f(t)] = \frac{10}{s^2+4} + \frac{3s}{s^2+16}}$$

(b) $L[\cos 4t] = \frac{s}{s^2+16}$

$\therefore L[e^{-3t}\cos 4t] = \frac{s+3}{(s+3)^2+16}$ (by first shifting theorem)

$= \frac{s+3}{s^2+6s+9+16}$

$$\boxed{\therefore L[e^{-3t}\cos 4t] = \frac{s+3}{s^2+6s+25}}$$

(c) $L[1-\cos t] = \frac{1}{s} - \frac{s}{s^2+1}$

$\therefore L\left(\frac{1-\cos t}{t}\right) = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+1}\right) ds$

$= \left[\log s - \frac{1}{2}\cdot\log(s^2+1)\right]_s^\infty$

$= \left[\log\left(\frac{s}{(s^2+1)^{1/2}}\right)\right]_s^\infty$

$= \lim_{s\to\infty} \log\left(\frac{s}{(s^2+1)^{1/2}}\right) - \log\left(\frac{s}{(s^2+1)^{1/2}}\right)$

$= 0 - \log\left(\frac{s}{(s^2+1)^{1/2}}\right)$

$$\boxed{\therefore L\left(\frac{1-\cos t}{t}\right) = -\log\left(\frac{s}{(s^2+1)^{1/2}}\right)}$$

6 d We have

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$

$$= \frac{1}{1-e^{-4s}} \int_0^4 e^{-st} f(t)\,dt$$

(given $f(t)$ is periodic function with the period 4.)

$$= \frac{1}{1-e^{-4s}} \left[\int_0^2 e^{-st} \cdot 3t\,dt + \int_2^4 e^{-st} \cdot 6\,dt \right]$$

$$= \frac{1}{1-e^{-4s}} \left(\left[3t \cdot \frac{e^{-st}}{-s} - (3)\frac{e^{-st}}{(-s)^2} \right]_0^2 + 6\left[\frac{e^{-st}}{-s} \right]_2^4 \right)$$

$$= \frac{1}{1-e^{-4s}} \left(\frac{-3}{s}\left(2e^{-2s} - 0\right) - \frac{3}{s^2}\left(e^{-2s} - 1\right) - \frac{6}{s}\left(e^{-4s} - e^{-2s}\right) \right)$$

$$= \frac{1}{1-e^{-4s}} \left[-\frac{6}{s} e^{-2s} - \frac{3}{s^2}\left(e^{-2s} - 1\right) - \frac{6}{s}\left(e^{-4s} - e^{-2s}\right) \right]$$

$$= \frac{\cancel{-6se^{-2s}} - 3e^{-2s} + 3 - 6se^{-4s} \cancel{+6se^{-2s}}}{s^2(1-e^{-4s})}$$

$$= \frac{3 - 3e^{-2s} - 6se^{-4s}}{s^2(1-e^{-4s})}$$

$$\boxed{\therefore\ L[f(t)] = \frac{3 - 3e^{-2s} - 6se^{-4s}}{s^2(1-e^{-4s})}}$$

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7 (a) $L^{-1}\left[\frac{5s+1}{s^2+16}\right] = L^{-1}\left[\frac{5s}{s^2+16}+\frac{1}{s^2+16}\right]$

$= 5L^{-1}\left[\frac{s}{s^2+16}\right]+L^{-1}\left[\frac{1}{s^2+16}\right] = 5\cos 4t+\frac{1}{4}\sin 4t$

$$\boxed{\therefore L^{-1}\left[\frac{5s+1}{s^2+16}\right] = 5\cos 4t+\frac{1}{4}\sin 4t}$$

(b) Let $F(s) = \frac{1}{(s+1)(s+2)(s+3)} = \frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{s+3}$

$\Rightarrow 1 = A(s+2)(s+3)+B(s+1)(s+3)+C(s+1)(s+2)$

put $s=-1 \Rightarrow 1 = 2A \Rightarrow A = 1/2$

put $s=-2 \Rightarrow 1 = -B \Rightarrow B = -1$

put $s=-3 \Rightarrow 1 = 2C \Rightarrow C = 1/2$

$\therefore F(s) = \frac{1/2}{s+1}+\frac{-1}{s+2}+\frac{1/2}{s+3}$

Taking Inverse Laplace transform, we get

$L^{-1}[F(s)] = \frac{1}{2}L^{-1}\left[\frac{1}{s+1}\right]-L^{-1}\left[\frac{1}{s+2}\right]+\frac{1}{2}L^{-1}\left[\frac{1}{s+3}\right]$

$= \frac{1}{2}e^{-t}-e^{-2t}+\frac{1}{2}e^{-3t}$

$$\boxed{\therefore L^{-1}\left[\frac{1}{(s+1)(s+2)(s+3)}\right] = \frac{1}{2}\left[e^{-t}-2e^{-2t}+e^{-3t}\right]}$$

(c) Let $F(s) = \log\left(1-\frac{a}{s}\right)$

$= \log\left(\frac{s-a}{s}\right)$

$\Rightarrow F(s) = \log(s-a)-\log(s)$

$\therefore F'(s) = \frac{1}{s-a}-\frac{1}{s}$

$L^{-1}[F'(s)] = e^{at}-1$

we have $L^{-1}[F(s)] = \frac{(-1)^1}{t^1}L^{-1}[F'(s)]$

$\therefore L^{-1}\left[\log\left(1-\frac{a}{s}\right)\right] = -\frac{1}{t}(e^{at}-1)$

$$\boxed{L^{-1}\left[\log\left(1-\frac{a}{s}\right)\right] = \frac{1-e^{at}}{t}}$$

⑧ ⓐ Given $y'' - 3y' + 2y = 12e^{-t}$

taking laplace transform, we get

$$L[y''] - 3L[y'] + 2L[y] = 12L[e^{-t}]$$

$$\Rightarrow s^2 L[y] - s\,y(0) - y'(0) - 3[s.L[y] - y(0)] + 2L[y] = 12.\frac{1}{s+1}$$

$$\Rightarrow s^2 L[y] - s(2) - 6 - 3[s\,L[y] - 2] + 2L[y] = \frac{12}{s+1}$$

$$\Rightarrow s^2 L[y] - 2s - \cancel{6} - 3s\,L[y] + \cancel{6} + 2\,L[y] = \frac{12}{s+1}$$

$$\Rightarrow (s^2 - 3s + 2)\,L[y] = \frac{12}{s+1} + 2s$$

$$\Rightarrow (s-1)(s-2)\,L[y] = \frac{2s^2 + 2s + 12}{s+1}$$

$$\therefore L[y] = \frac{2s^2 + 2s + 12}{(s+1)(s-1)(s-2)} \quad \text{——} \quad ①$$

Consider

$$\frac{2s^2 + 2s + 12}{(s+1)(s-1)(s+2)} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s-2}$$

$$\Rightarrow 2s^2 + 2s + 12 = A(s-1)(s-2) + B(s+1)(s-2) + C(s+1)(s-1)$$

put $s = 1 \Rightarrow 16 = -2B \Rightarrow B = -8$

put $s = 2 \Rightarrow 24 = 3C \Rightarrow C = 8$

put $s = -1 \Rightarrow 12 = 6A \Rightarrow A = 2$

$$\therefore \frac{2s^2 + 2s + 12}{(s+1)(s-1)(s+2)} = \frac{2}{s+1} - \frac{8}{s-1} + \frac{8}{s-2} \quad \text{——} \quad ②$$

substitute eq ② in ①

$$\therefore L[y] = \frac{2}{s+1} - \frac{8}{s-1} + \frac{8}{s-2}$$

$$\Rightarrow y = L^{-1}\left[\frac{2}{s+1} - \frac{8}{s-1} + \frac{8}{s-2}\right]$$

$$= 2L^{-1}\left[\frac{1}{s+1}\right] - 8L^{-1}\left[\frac{1}{s-1}\right] + 8L^{-1}\left[\frac{1}{s-2}\right] = 2e^{-t} - 8e^{t} + 8e^{2t}$$

$$\boxed{\therefore y = 2e^{-t} - 8e^{t} + 8e^{2t}}$$

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⑧ (b) Given $\frac{dx}{dt} + y = \sin t$

$$\frac{dy}{dt} + x = \cos t$$

$$\Rightarrow x' + y = \sin t$$
$$y' + x = \cos t$$

Applying Laplace Transform, we get

$$L[x'] + L[y] = L[\sin t]$$
$$L[y'] + L[x] = L[\cos t]$$

$$\Rightarrow s\,L[x] - x(0) + L[y] = \frac{1}{s^2+1}$$
$$s\,L[y] - y(0) + L[x] = \frac{s}{s^2+1}$$

$$\Rightarrow s\,L[x] - 1 + L[y] = \frac{1}{s^2+1}$$
$$s\,L[y] - 0 + L[x] = \frac{s}{s^2+1}$$

$$\therefore s\,L[x] + L[y] = \frac{1}{s^2+1} + 1 \quad \text{—— ①}$$
$$L[x] + s\,L[y] = \frac{s}{s^2+1} \quad \text{—— ②}$$

multiplying eq ① by 's' and adding eq ②, we get

$$(s^2-1)\,L[x] = s$$

$$\Rightarrow L[x] = \frac{s}{s^2-1}$$

$$\Rightarrow x = L^{-1}\left[\frac{s}{s^2-1}\right]$$

$$\therefore x = \cosh t$$

Since $y = \sin t - x'$

$$\Rightarrow y = \sin t - \sinh t.$$

$$\boxed{\begin{aligned} &\therefore x = \cosh t \\ &y = \sin t - \sinh t \end{aligned}}$$