USN | | | | | | | | | | | MATDIP301

Third Semester B.E. Degree Examination, June/July 2019
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Express square root of $1-i$ in the form of $x + iy$. **(07 Marks)**

b. Find the modulus and amplitude of the following and express each in polar form.

(i) $1-i\sqrt{3}$ (ii) $\dfrac{1-i}{1+i}$ **(07 Marks)**

c. Expand $\cos^6\theta$ in series of multiples of θ. **(06 Marks)**

2 .. Find the n^{th} derivative of $e^{ax}\cos(bx + c)$. **(06 Marks)**

b. Find the n^{th} derivative of $\dfrac{x}{(x+1)(x-2)}$. **(07 Marks)**

c. If $y = \log\left(x+\sqrt{1+x^2}\right)$, prove that $\left(1+x^2\right)y_{n+2} + (2n+1)xy_{n+1} + n^2y = 0$. **(07 Marks)**

3 Find the angle between radius vector and the tangent of the curve $r = a(1 + \cos\theta)$. **(06 Marks)**

b. Find the Taylor's series expansion of the function e^x about $x = 1$. **(07 Marks)**

c. Obtain the Maclaurin's series expansion of the function $\log_e(1 + x)$ up to third degree terms. **(07 Marks)**

4 a. If $\cos u = \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{2}\cot u$. **(06 Marks)**

b. If $x = r\cos\theta$ and $y = r\sin\theta$, prove that $JJ' = 1$. **(07 Marks)**

c. If $x^y + y^x = c$, where c is a constant, find $\dfrac{dy}{dx}$. **(07 Marks)**

5 a. Obtain the reduction formula $I_n = \int \sin^n x\, dx$, where n is a positive integer. **(06 Marks)**

b. Evaluate : $\displaystyle\int_0^1\int_0^{\sqrt{x}} xy(x+y)\,dx\,dy$ **(07 Marks)**

c. Evaluate : $\displaystyle\int_0^1\int_0^{1-z}\int_0^{1-z-y}(x+y+z)\,dx\,dy\,dz$ **(07 Marks)**

6 a. Prove the following :
$\beta(m, n) = \beta(n, m)$ **(06 Marks)**

b. Prove that $\Gamma\left(\dfrac{1}{2}\right) = \sqrt{\pi}$ **(07 Marks)**

c. Using Gamma function, evaluate the integral $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^4}}\,dx$ **(07 Marks)**

7 a. Solve : $(x+y+1)^2\dfrac{dy}{dx}=1$ **(06 Marks)**

b. Solve : $\dfrac{dy}{dx}=1+x^2+y^2+x^2y^2$. **(07 Marks)**

c. Solve : $(x^2-xy+y^2)dx-xy\,dy=0$ **(07 Marks)**

8 Solve the following second order O.D.Es.

a. $\dfrac{d^2y}{dx^2}+y=e^x$ **(06 Marks)**

b. $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y=\cos^2 x$ **(07 Marks)**

c. $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}-2y=2(1+x)$. **(07 Marks)**

* * * * *

3.c obtain the maclaurin's series Expansion of the function $\log_e(1+x)$ upto third degree terms

Matdip301
June 2019
Solution Manual

Soln $y(x) = y(0) + x y_1(0) + \frac{x^2}{2!} y_2(0) + \cdots$

$y = \log(1+x) \qquad y(0) = \log 1 = 0$

$y_1 = \frac{1}{1+x} \qquad y_1(0) = 1 \qquad y_2 = \frac{-1}{(1+x)^2} \qquad y_2(0) = -1$

$y_3 = \frac{2}{(1+x)^3} \qquad y_3(0) = 2 \qquad y_4 = \frac{-6}{(1+x)^4} \qquad y_4(0) = -6$

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} -$

2.a Find the nth derivative of $e^{ax}\cos(bx+c)$

Soln Let $y = e^{ax}\cos(bx+c)$ Then

$y_1 = e^{ax}\cos(bx+c) - b e^{ax}\sin(bx+c)$

$= e^{ax}[k(\cos\alpha)\cos(bx+c) - k(\sin\alpha)\sin(bx+c)]$

$y_1 = k e^{ax}\cos(bx+c+\alpha)$

$\therefore \quad y_2 = k[a e^{ax}\cos(bx+c+\alpha) - b e^{ax}\sin(bx+c+\alpha)]$

$y_2 = k e^{ax}[k(\cos\alpha)\cos(bx+c+\alpha) - k(\sin\alpha)\sin(bx+c+\alpha)]$

$= k^2 e^{ax}\cos(bx+c+2\alpha)$

$\therefore$

$$\boxed{y_n = k^n e^{ax}\cos(bx+c+n\alpha)}$$

(1).

a. $\sqrt{1-i} = \quad r = \sqrt{x^2+y^2} = \sqrt{1^2+(-1)^2} = \sqrt{2}$.

$\theta = \tan^{-1}\left|\left(\frac{y}{x}\right)\right| = \tan^{-1}\left|\frac{-1}{1}\right| = \tan^{-1}(+1) = \pi/4$

$Z = 1-i$ is in fourth quadrant $\therefore \theta = 2\pi - \alpha$.

$= 2\pi - \pi/4$

$= 7\pi/4$

$Z = \sqrt{2}\,(\cos 7\pi/4 + i \sin 7\pi/4)$

$z^{1/2} = (\sqrt{2})^{1/2}\left[\cos 7\pi/4 + i \sin 7\pi/4\right]^{1/2}$.

$= 2^{1/4}\left[\cos 7\pi/8 + i \sin 7\pi/8\right]^{1/2}$.

$= 2^{1/4}\left[\cos\left(2k\pi + 7\pi/4\right) + i \sin\left(2k\pi + 7\pi/4\right)\right]^{1/2}$

$k = 0, 1, 2 -$

$= 2^{1/4}\left[\cos 7\pi/8 + i \sin 7\pi/8\right] \quad k = 0$.

$= 2^{1/4}\left[\cos 15\pi/8 + i \sin \frac{15\pi}{8}\right] \quad k = 1$.

b. i) $1 - i\sqrt{3}$ ii) $\frac{1-i}{1+i}$

Modulus $r = \sqrt{x^2+y^2} = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4}$

$= 2$.

$\frac{1-i}{1+i} = \frac{1-i}{(1+i)} \times \frac{(1-i)}{(1-i)} = \frac{(1-i)^2}{1-i+i+1} = \frac{1+i^2-2i}{2}$

$$= \frac{1 \cdot -1 - 2i}{2}$$

$$= -i$$

Modulus $= \sqrt{0^2 + (-1)^2} = \sqrt{1} = 1$.

i) amplitude $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$= \tan^{-1}\left(-\frac{\sqrt{3}}{1}\right)$$

$$= -\pi/3$$

ii) amplitude $\Rightarrow \theta = \tan^{-1}\left(-\frac{1}{0}\right) = \tan^{-1}(\infty) = \pi/2$

i) $x + iy = 1 - i\sqrt{3}$

$$r = \sqrt{x^2 + y^2} = \sqrt{4} = 2.$$

$$z = re^{i\theta} = 2e^{i(-\pi/3)}$$

2) $x + iy = -i$

$$z = re^{i\theta} = e^{i\pi/2}.$$

C. $\cos^6\theta$ -

$z = \cos\theta + i\sin\theta$.

$z + \frac{1}{z} = 2\cos\theta$, $z^p + \frac{1}{z^p} = 2\cos p\theta$

$$(2\cos\theta)^6 = \left(z + \frac{1}{z}\right)^6$$

$$= z^6 + 6C_1 z^5 \cdot \frac{1}{z} + 6C_2 z^4 \cdot \frac{1}{z^2} + 6C_3 z^3 \cdot \frac{1}{z^3} +$$

$$6C_4 z^2 \cdot \frac{1}{z^4} + 6C_5 z \cdot \frac{1}{z^5} + 6C_6 \cdot \frac{1}{z^6}$$

$$= \left(z^6 + \frac{1}{z^6}\right) + 6C_1 z^4 + 6C_2 z^2 + 6C_3 + 6C_4 \cdot \frac{1}{z^2}$$

$$+ 6C_5 \frac{1}{z^4} +$$

$$= (2\cos 6\theta) + 6\left[z^4 + \frac{1}{z^4}\right] + 15\left[z^2 + \frac{1}{z^2}\right]$$

$$+ 20 \ldots$$

$$= 2\cos 6\theta + 6(2\cos 4\theta) + 15(2\cos 2\theta) + 20$$

$$= 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$$

2.
b.

$$\frac{x}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}.$$

$$x = A(x-2) + B(x+1)$$

Put $x = 2$, $2 = A(0) + B(3)$

$$\Rightarrow B = 2/3$$

Put $x = -1$, $-1 = A(-3) + B(0)$

$$\Rightarrow A = 1/3$$

$$\therefore \frac{x}{(x+1)(x-2)} = \frac{1}{3(x+1)} + \frac{2}{3(x-2)}$$

$$= \frac{1}{3} \frac{(-1)^n \cdot n!}{(x+1)^{n+1}} + \frac{2}{3} \frac{(-1)^n \cdot n!}{(x-2)^{n+1}}$$

c. $y = \log(x + \sqrt{1+x^2})$

$$y_1 = \frac{1}{x + \sqrt{1+x^2}} \left[1 + \frac{2x}{2\sqrt{1+x^2}}\right]$$

$$\Rightarrow y_1 = \frac{1}{x+\sqrt{1+x^2}}\left[\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right]$$

$$\sqrt{1+x^2}\, y_1 = 1$$

Squaring on b.s

$$(1+x^2)y_1^2 = 1$$

diff again

$$(1+x^2)\,2y_1y_2 + y_1^2 \cdot 2x = 0$$

$\div$ by $2y_1$

$$\underset{u}{(1+x^2)}\,\underset{v}{y_2} + \underset{u}{x}\,\underset{v}{y_1} = 0.$$

$$(1+x^2)y_{n+2} + 2xn\,y_{n+1} + n(n-1)y_n + xy_{n+1} + ny_n = 0$$

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2-n+n)\,y_n = 0$$

$$(1+x^2)y_{n+2} + (2n+1)\,xy_{n+1} + n^2y_n = 0.$$

3

a. $r = a(1+\cos\theta)$

$$\frac{1}{r}\frac{dr}{d\theta} = \cancel{b}\ \frac{-\sin\theta}{1+\cos\theta} \qquad \tan\phi = r\frac{d\theta}{dr}.$$

$$= \frac{-\cancel{2}\sin\theta/2\ \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}. \qquad \frac{1}{r}\frac{dr}{d\theta} = \cot\phi.$$

$$\cot\phi = -\tan(\theta/2) = \cancel{\cot}\,(\pi/2 + \theta/2)$$

$$\cancel{\cot}\ \phi = \cot(\pi/2 + \theta/2)$$

$$\phi = \cancel{\pi/2} + \theta/2 \Rightarrow \phi = \pi/2 + \theta/2$$

b. $y = e^x$

Taylor's series expansion is

$$y = y(x_0) + (x - x_0)\, y'(x_0) + \frac{(x - x_0)^2}{2!} \cdot y_0''(x_0) + \frac{(x - x_0)^3}{3!} y'''(x_0) + \cdots$$

$y(1) = e^1$

$y'(x) = e^x \qquad y'(1) = e^1$

$y''(x) = e^x \qquad y''(1) = e^1$

$$= e^1 + (x-1) \cdot e^1 + \frac{(x-1)^2}{2!} e^1 + \frac{(x-1)^3}{3!} e^1 + \cdots$$

$$= e^1 \left[1 + (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!} + \cdots \right]$$

4.

a. $\cos u = \dfrac{x+y}{\sqrt{x} + \sqrt{y}} \Rightarrow \cos u = \dfrac{x\left[1 + y/x\right]}{\sqrt{x}\left[1 + \frac{\sqrt{y}}{\sqrt{x}}\right]}$

$= \sqrt{x}\, f(y/x) \Rightarrow n = 1/2$

$\cos u$ is a homogeneous func of degree $n = 1/2$

$x u_x + y u_y = n u$. (By Euler's theorem)

$$-x \sin u \cdot \frac{\partial u}{\partial x} + y \cdot \sin y \frac{\partial u}{\partial y} = \frac{1}{2} \cdot \cos u$$

$$\Rightarrow x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \frac{\cos u}{\sin u}$$

$$\Rightarrow x u_x + y u_y = -\frac{1}{2} \cot u$$

$$\Rightarrow \left[x \frac{\partial}{\partial x}(\cos u) + y \frac{\partial}{\partial y}(\cos u) = n \cos u \right]$$

4 c) Let $f = f(x, y) = x^y + y^x$.

Then $f = c$

$$\Rightarrow \frac{df}{dx} = 0$$

$$\Rightarrow \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\partial f/\partial x}{\partial f/\partial y} \quad \text{--- (1)}$$

From $f = x^y + y^x$, $\frac{\partial f}{\partial x} = yx^{y-1} + y^x \log y$

& $\frac{\partial f}{\partial y} = x^y \log x + xy^{x-1}$

Putting these in (1)

$$\frac{dy}{dx} = -\frac{yx^{y-1} + y^x \log y}{xy^{x-1} + x^y \log x}.$$

$$= \int_0^1 \left[\frac{z}{2} - \frac{7z^3}{6}\right] dz$$

$$= \left[\frac{z^2}{4} - \frac{7z^4}{24}\right]_0^1$$

$$= \frac{1}{4} - \frac{7}{24}$$

$$= \frac{6-7}{24} = -\frac{1}{24}$$

4b) If $x = r\cos\theta$ & $y = r\sin\theta$

$r = \sqrt{x^2+y^2}$; $\theta = \tan^{-1}(y/x)$

$$J = \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r \quad \text{——— (1)}$$

$$J' = \frac{\partial(r,\theta)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{vmatrix} = \begin{vmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{vmatrix}$$

$$= \frac{x^2+y^2}{(x^2+y^2)^{3/2}} = \frac{1}{\sqrt{x^2+y^2}} = \frac{1}{r} \quad \text{——— (2)}$$

From (1) & (2).

$$J \cdot J' = r \cdot \frac{1}{r} = 1$$

$$= \int_{x=0}^{1} \left(\frac{x^3}{2} + \frac{x^{5/2}}{3} \right) dx$$

$$= \frac{x^4}{8} + \frac{x^{7/2}}{3 \cdot 7/2} \Big]_0^1$$

$$= \frac{1}{8} + \frac{2}{21}$$

$$=$$

5c) $$\int_{z=0}^{1} \int_{y=0}^{1-z} \int_{x=0}^{1-z-y} (x+y+z) \, dx \, dy \, dz$$

$$= \int_{z=0}^{1} \int_{y=0}^{1-z} \left[\frac{x^2}{2} + yx + zx \right]_0^{1-z-y} dy \, dz$$

$$= \int_{z=0}^{1} \int_{y=0}^{1-z} \left[\frac{(1-z-y)^2}{2} + y(1-z-y) + z(1-z-y) \right] dy \, dz$$

$$= \int_{z=0}^{1} \int_{y=0}^{z} \left\{ \frac{1}{2} (1+y^2+z^2-2z+2zy-2y) + y - zy - y^2 + z - z^2 - yz \right\} dy \, dz$$

$$= \int_{z=0}^{1} \left[\frac{y}{2} + \frac{y^3}{6} + \frac{z^2 y}{2} - \cancel{zy} + \cancel{\frac{zy^2}{2}} - \cancel{\frac{y^2}{2}} + \cancel{\frac{y^2}{2}} - \cancel{\frac{zy^2}{2}} - \frac{y^3}{3} + \cancel{zy} - z^2 y - \frac{zy^2}{2} \right]_0^z dz$$

$$= \int_{z=0}^{1} \left(\frac{y}{2} - \frac{y^3}{6} - \frac{z^2 y}{2} - \frac{zy^2}{2} \right)_0^z dz$$

$$= \int_0^1 \left(\frac{z}{2} - \frac{z^3}{6} - \frac{z^3}{2} - \frac{z^3}{2} \right) dz$$

5a) Let $I_n = \int \sin^n x \, dx$

$$= \int \sin^{n-1} x \sin x \, dx$$

$$= \sin^{n-1} x (-\cos x) + \int \cancel{\sin x}(n-1) \sin^{n-2} x \cdot \cos x \cos x \, dx$$

$$= -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \cdot dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x (1 - \sin^2 x) \, dx$$

$$= \sin^{n-1} x (\cos x) + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$\Rightarrow I_n = -(\sin^{n-1} x)(\cos x) + (n-1) I_{n-2} - (n-1) I_n$$

$$\Rightarrow \boxed{I_n = -\frac{(\sin^{n-1} x)(\cos x)}{n} + \frac{(n-1)}{n} I_{n-2}}$$

5b) $$\int_{x=0}^{1} \int_{y=0}^{\sqrt{x}} xy(x+y) \, dy \, dx$$

$$= \int_{x=0}^{1} \int_{y=0}^{\sqrt{x}} (x^2 y + x y^2) \, dy \, dx$$

$$= \int_{x=0}^{1} \left[\frac{x^2 y^2}{2} + \frac{x y^3}{3} \right]_0^{\sqrt{x}} dx$$

$$= \int_{x=0}^{1} \left[\frac{x^2}{2}(x - 0) + \frac{x}{3}(x\sqrt{x} - 0) \right] dx$$

$$= \int_{x=0}^{1} \left(\frac{x^3}{2} + \frac{x^{5/2}}{3} \right) dx$$

6(a) We have $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$

Put $x = 1-y$ or $1-x = y$ $\therefore dx = -dy$

When $x=0, y=1$ and when $x=1, y=0$.

$$\therefore \beta(m,n) = \int_{y=1}^{0} (1-y)^{m-1} y^{n-1} (-dy)$$

$$= \int_{y=0}^{1} y^{n-1}(1-y)^{m-1} dy = \beta(n,m)$$

6(b) Wkt $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$

$$\therefore \Gamma(1/2) = 2\int_0^\infty e^{-x^2} dx = 2\int_0^\infty e^{-y^2} dy$$

Hence $\{\Gamma(1/2)\}^2 = 4\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy = 4 \cdot \frac{\pi}{4} = \pi$

Thus $\Gamma(1/2) = \sqrt{\pi}$.

6(c) Let $I = \int_0^1 \frac{1}{\sqrt{1-x^4}} dx$

Put $x^4 = \sin^2\theta$ or $x = \sin^{1/2}\theta$ $\therefore dx = \frac{1}{2}\sin^{-1/2}\theta \cos\theta\, d\theta$

θ varies from 0 to $\pi/2$.

$$\therefore I = \int_{\theta=0}^{\pi/2} \frac{1/2 \sin^{-1/2}\theta \cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}} = \frac{1}{2}\int_0^{\pi/2} \frac{\sin^{-1/2}\theta\, \cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}}$$

$$= \frac{1}{2}\cdot\frac{1}{2}\beta\left(\frac{-1/2+1}{2}, \frac{0+1}{2}\right) = \frac{1}{4}\beta\left(1/4, 1/2\right) = \frac{1}{4}\frac{\Gamma(1/4)\sqrt{\pi}}{\Gamma(3/4)}$$

7. a Solve $(x+y+1)^2 \frac{dy}{dx} = 1$

Sol: $x+y+1 = t$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{dt}{dx} - 1$$

$\therefore$ $(x+y+1)^2 \frac{dy}{dx} = 1$ — (1) Reduces to

$$\frac{dt}{dx} - 1 = \frac{1}{t^2} \Rightarrow \frac{dt}{dx} = \frac{1}{t^2} + 1 = \frac{t^2+1}{t^2}$$

$$\frac{t^2}{t^2+1} dt = dx = \frac{t^2+1-1}{t^2+1} dt = dx$$

On integration

$$\Rightarrow t - \tan^{-1}(t) = x + c$$

$$\boxed{(x+y+1) - \tan^{-1}(x+y+1) = x + c}$$

b. Solve $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

Soln: $1 + x^2 + y^2 + x^2y^2 = (1+x^2)(1+y^2)$

$\therefore$ $\frac{dy}{dx} = (1+x^2)(1+y^2)$ or $\frac{dy}{1+y^2} = (1+x^2)\,dx$

On integration

$$\tan^{-1} y = x + \frac{x^3}{3} + c$$

c. Solve $(x^2-xy+y^2)\,dx - xy\,dy=0$

Soln

$$\frac{dy}{dx} = \frac{x^2-xy+y^2}{xy} \quad \text{--- (1)} \quad H.E$$

$\therefore$ put $y = vx$.

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^2-x^2v+x^2v^2}{x^2v}$$

$$v + x\frac{dv}{dx} = \frac{1-v+v^2}{v} \Rightarrow x\frac{dv}{dx} = \frac{1-v}{v}$$

$$\Rightarrow \frac{v}{1-v}\,dv = \frac{dx}{x}$$

$$= \frac{v}{-(v-1)}\,dv = \frac{dx}{x}$$

$$= \frac{v-1+1}{-(v-1)}\,dv = \frac{dx}{x} \quad = \quad -v - \log(v-1) = \log x + c$$

$$\Rightarrow -(y/x) - \log(y/x - 1) = \log x + c$$

8.a solve $\frac{d^2y}{dx^2} + y = e^x$

Soln

$D^2y + y = 0$

$m^2 + 1 = 0$

$m = \pm i \Rightarrow y = c_1 \cos x + c_2 \sin x$

$P.I = \frac{e^x}{D^2+1} \Rightarrow$ Put $D = a = 1.$

$P.I = \frac{e^x}{1.}$

$\therefore$ GS $\boxed{y = c_1 \cos x + c_2 \sin x + e^x}$

b. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \cos^2 x$

Soln. $D^2y + 2Dy + y = 0$

$m^2 + 2m + 1 = 0$

$(m+1)^2 = 0 \Rightarrow m = -1, -1$

$\boxed{y = (c_1 + c_2 x)e^{-x}}$

$P.I = \frac{\cos^2 x}{f(D)} = \frac{\cos^2 x}{D^2 + 2D + 1}$

$\cos^2 x = \frac{1 + \cos 2x}{2} \quad \therefore PI = \frac{1}{2}\left[\frac{1 + \cos 2x}{D^2 + 2D + 1}\right]$

$PI = \frac{1}{2}\left[\frac{e^{0x}}{D^2+2D+1} + \frac{\cos 2x}{-2^2+2D+1}\right] = \frac{1}{2}\left(\frac{e^{0x}}{1} + \frac{\cos 2x}{2D-3}\right)$

$= \frac{1}{2}\left[1 + \frac{\cos 2x}{2D^2 - 3^2}(2D+3)\right] = \frac{1}{2}\left[\frac{2(-\sin 2x).2) + 3\cos 2x}{-8-9}\right]$

$$y = (c_1 + c_2 x) e^{-x} + \frac{1}{-34}\left[-4\sin x + 3\cos 2x\right]$$

c. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 2(1+x)$

Sol

$D^2y + Dy - 2y = 0$

$m^2 + m - 2 = 0$

$m(m+2) - 1(m+2) = 0$

$m - 1 = 0 \quad m + 2 = 0$

$m = 1 \quad m = -2$

$$\boxed{y = c_1 e^{x} + c_2 e^{-2x}}$$

P.I

$$
\begin{array}{r|l}
 & -1 + x - \tfrac{1}{2} \\
\hline
-2 + D + D^2 & 2 + 2x \\
 & -2 \\
\hline
 & 2x \\
 & 2x - 1 \\
\hline
 & 0 + 1 \\
 & +1 \\
\hline
 & 0
\end{array}
$$

$$\boxed{y = c_1 e^{x} + c_2 e^{-x} - 1 + x - \tfrac{1}{2}}$$