CBCS SCHEME

USN

15CS36

Third Semester B.E. Degree Examination, June/July 2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Simplify the switching network shown in Fig Q1(a)

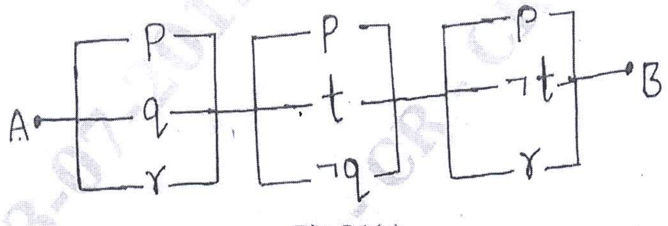


Fig Q1(a) **(08 Marks)**

b. Give a direct proof of the statement "If n is an odd integer then n^2 is also an odd integer". **(04 Marks)**

c. Let p(x), q(x) and r(x) be open statements that are defined for the given universe. Show that the argument.

$\forall x,[p(x) \to q(x)]$

$\underline{\forall x,[q(x) \to r(x)]}$

$\therefore \exists x,[p(x) \to r(x)]$ is valid **(04 Marks)**

OR

2 a. Define tautology, prove that for any proposition p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to q)$ is a tautology using truth table. **(05 Marks)**

b. Show that RVS follows logically form the premises CVD, CVD$\to \neg$H, $\neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to (RVS)$. **(04 Marks)**

c. Using rules of inference shows that the following argument is valid.

$\forall x,[p(x) \vee q(x)] \wedge \exists x, \neg p(x) \wedge$

$\forall x,[\neg q(x) \vee r(x)] \wedge \forall x,[s(x) \to \neg r(x)]$

$\therefore \exists x, \neg S(x)$ **(07 Marks)**

Module-2

3 a. Prove by mathematical induction that, for all integers $n \geq 1$, $1 + 2 + 3 + \ldots + n = \frac{1}{2}n(n+1)$. **(06 Marks)**

b. The Fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Evaluate F_2 to F_{10}. **(04 Marks)**

c. In the word S, O, C, I, O, L, O, G, I, C, A, L.

i) How many arrangements are there for all letters?

ii) In how many of these arrangements all vowels are adjacent? **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

4 a. Obtain the recursive definition for the sequence $\{a_n\}$ in each of the following cases.
(i) $a_n = 5n$ (ii) $a_n = 6^n$ (iii) $a_n = n^2$ **(06 Marks)**

b. Find the coefficient of
i) $x^9 y^3$ in the expansion fo $(2x - 3y)^{12}$
ii) x^{12} in the expansion of $x^3 (1 - 2x)^{10}$ **(04 Marks)**

c. A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols, the transmitter will also send a total of 45 blank spaces between the symbols, with atleast 3 spaces between each pair of consecutive symbols. In how many ways can the transmitter send such a message? **(06 Marks)**

Module-3

5 a. Let $f : R \rightarrow R$ be defined by
$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$ determine $f(0)$, $f(-1)$, $f^{-1}(0)$, $f^{-1}(+3)$, $f^{-1}([-5, 5])$ **(08 Marks)**

b. Define an equivalence relation. Write the partial order relation for the positive divisors of 36 and write its Hasse diagram (HASSE). **(08 Marks)**

OR

6 a. Consider the function $f : R \rightarrow R$ defined by $f(x) = 2x + 5$. Let a function $g : R \rightarrow R$ be defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that g is an inverse of f. **(03 Marks)**

b. State Pigeonhole principle. Let ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that atleast two of their points are such that the distance between them is less than ½ cm. **(05 Marks)**

c. If $A = \{1, 2, 3, 4\}$, R and S are relations on A defined by $R = \{(1, 2), (1, 3), (2, 4), (4, 4)\}$ $S = \{(1, 1), (1, 2), (1,3), (1, 4), (2, 3), (2, 4)\}$ find RoS, SoR, R^2, S^2 and write down their matrices. **(08 Marks)**

Module-4

7 a. Find the number of derangements of 1, 2, 3, 4 list all such derangements. **(04 Marks)**

b. Determine the number of integers between 1 and 300 (inclusive) which are divisible by exactly 2 of 5, 6, 8. **(06 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day? **(06 Marks)**

OR

8 a. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for 5 classes C_1, C_2, C_3, C_4, C_5 one teacher for each class T_1 and T_2 donot wish become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5. In how many ways can teachers be assigned the work (without displeasing any teacher)? **(08 Marks)**

b. Solve the recurrence relation,
$a_n = 2(a_{n-1} - a_{n-2})$, where $n \ge 2$ and $a_0 = 1$, $a_1 = 2$. **(08 Marks)**

Module-5

9 a. Prove that the undirected graph G = (V, E) has an Euler circuit if and only if G is connected and every vertex in G has even degree. **(08 Marks)**

b. Define binary rooted tree and Balanced tree. Draw all the spanning trees of the graph shown in Fig 9(b)

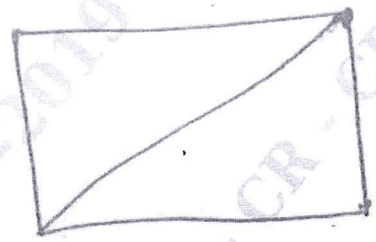


Fig Q9(b) **(08 Marks)**

OR

10 a. Define, with an example for each Regular graph, complement of a graph, Euler trail and Euler circuit and complete graph. **(10 Marks)**

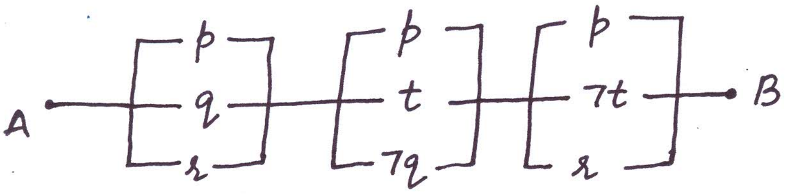
b. Apply Merge sort to the list
6, 2, 7, 3, 4, 9, 5, 1, 8 **(06 Marks)**

* * * * *

Solution : DMS (15CS36), June / July 2019

MODULE-1

1(a)



Given network is given by

$u \equiv (p \vee q \vee r) \wedge (p \vee t \vee \neg q) \wedge (p \vee \neg t \vee r)$

$\equiv p \vee \{(q \vee r) \wedge (t \vee \neg q)\} \wedge \{p \vee (\neg t \vee r)\}$ (associative & distributive)

$\equiv p \vee [(q \vee r) \wedge (t \vee \neg q) \wedge (\neg t \vee r)]$ (Distributive)

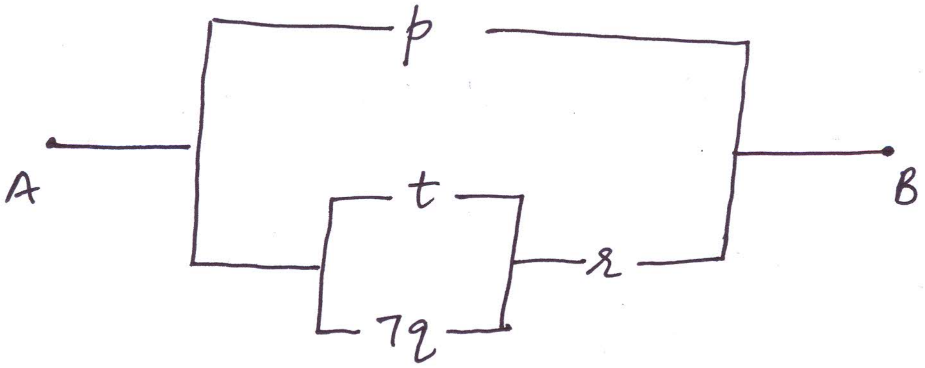
$\equiv p \vee [\{r \vee (q \wedge \neg t)\} \wedge (t \vee \neg q)]$ (commu. & distri.)

$\equiv p \vee [\{r \wedge (t \vee \neg q)\} \vee \{(q \wedge \neg t) \wedge (t \vee \neg q)\}]$ (Distri.)

$\equiv p \vee [\{r \wedge (t \vee \neg q)\} \vee \{(q \wedge \neg t) \wedge (\neg (q \wedge \neg t))\}]$ (DeMorgan's)

$\equiv p \vee [\{r \wedge (t \vee \neg q)\} \vee F_0]$ (complement law)

$\equiv p \vee [r \wedge (t \vee \neg q)]$ (by identity)



1(b) Here

p: n is an odd integer

q: n^2 is also an odd integer

Given form $p \to q$

<u>Direct proof</u>: Let p be true

$\therefore \quad n = 2K+1$

$\Rightarrow \quad n^2 = (2K+1)^2 = 4K^2+1+4K = 2(2K^2+2K)+1$

$= 2m+1$

which is an odd no.

$\therefore$ q is true

Hence $p \to q$ is true

1(c)

$$\begin{array}{ll} & \forall x, [p(x) \to q(x)] \\ & \forall x, [q(x) \to r(x)] \\ \hline \Rightarrow & \forall x, [p(x) \to r(x)] \quad \text{(Rule of Syllogism)} \\ \hline \Rightarrow & p(a) \to r(a) \quad \text{Rule of universal specification} \\ \hline \Rightarrow & \exists x, p(x) \to r(x) \end{array}$$

$\therefore$ Given argument is valid.

2(a) Given statement

$[(p \to q) \wedge (q \to r)] \to (p \to q)$

The following is the truth table:

p	q	r	① $p \to q$	② $q \to r$	③ ① $\wedge$ ②	③ $\to$ ①	
0	0	0	1	1	1	1	
0	0	1	1	1	1	1	
0	1	0	1	0	0	1	All
0	1	1	1	1	1	1	entries
1	0	0	0	1	0	1	are 1.
1	0	1	0	1	0	1	
1	1	0	1	0	0	1	
1	1	1	1	1	1	1	

Since all entries are 1 $\therefore$ given statement is tautology

2(b)

$C \vee D$

$C \vee D \to \neg H$

$\neg H \to (A \wedge \neg B)$

$(A \wedge \neg B) \to (R \vee S)$

$\neg H$ Modus Pones in (i) & (ii)

$\neg H \to (A \wedge \neg B)$

$(A \wedge \neg B) \to (R \vee S)$

$\neg H$

$\neg H \to (R \vee S)$ Syllogism in (ii) & (iii)

$\therefore R \vee S$ Modus Pones

The argument is valid.

2(c) $\forall x, (p(x) \vee q(x)) \wedge \exists x, \neg p(x) \wedge$
$\forall x, [\neg q(x) \vee r(x)] \wedge \forall x, [s(x) \to \neg r(x)]$

Incomplete info.
(Can't be solved)

MODULE – 2

3(a) Let $S(n): 1+2+3+\dots+n = \frac{1}{2}n(n+1)$

Basis step – For ~~nt~~ $n=1$

$LHS = 1$ & $R.H.S. = \frac{1}{2}\times 1\times 2 = 1$

$\therefore$ $S(n)$ is true for $n=1$

Let $S(n)$ is true for $n=k$

i.e. $1+2+3+\dots+k = \frac{1}{2}k(k+1)$ —— (i)

Inductive step:- we've to prove that $S(n)$ is true for $n=k+1$

i.e. $1+2+3+\dots+k+(k+1) = \frac{1}{2}(k+1)(k+2)$

LHS. $1+2+3+\dots+k+(k+1)$

$= \frac{1}{2}k(k+1)+(k+1)$ {using (i)}

$= (k+1)\left\{\frac{1}{2}k+1\right\}$

$= \frac{1}{2}(k+1)(k+2)$ RHS

$\therefore$ $S(n)$ is true for $n=k+1$

Hence $S(n)$ is true for $\forall n \geq 1$.

3(b) Given $F_0=0$, $F_1=1$, $F_n = F_{n-1}+F_{n-2}$, $\forall n\geq 2$

Using above recurrence relation

$F_2 = F_0+F_1 = 1$

$F_3 = F_2+F_1 = 1+1 = 2$

$F_4 = F_3 + F_2 = 2+1 = 3$; $F_8 = F_7 + F_6 = 13+8 = 21$

$F_5 = F_4 + F_3 = 3+2 = 5$; $F_9 = F_8 + F_7 = 21+13 = 34$

$F_6 = F_5 + F_4 = 5+3 = 8$; $F_{10} = F_9 + F_8 = 34+21 = 54$

$F_7 = F_6 + F_5 = 8+5 = 13$

$\boxed{3(c)}$ Given word SOCIOLOGICAL

(i) The total no. of arrangements – (3 Os, 2Ls, 2I, 2Cs, 1S, 1 G, 1A)

$$= \frac{12!}{3!2!2!2!1!1!1!}$$

(ii) If vowels are adjacent

$\underbrace{OOOIIA}$ SLLCCG

consider this one as a single group

$\therefore$ Total no. of arrangements

$$= \frac{7!}{2!2!1!1!} \times \frac{6!}{3!2!1!}$$

$\boxed{4(a)}$ (i) $a_n = 5n$

$a_1 = 5,\ a_2 = 10,\ a_3 = 15,\ a_4 = 20 \ldots\ldots$

Recursive def.

$a_1 = 5$ & $a_n = a_{n-1} + 5$, for $n \geq 2$

(ii) $a_n = 6^n$

$a_1 = 6,\ a_2 = 6^2,\ a_3 = 6^3, \ldots\ldots$

Rec. def

$a_1 = 6,\quad a_n = 6a_{n-1}\ ,\ n \geq 2$

(iii) $a_n = n^2$

$a_1 = 1,\ a_2 = 4,\ a_3 = 9,\ a_4 = 16, \ldots\ldots$

we've $a_n = n^2 \Rightarrow a_{n-1} = (n-1)^2$

$\therefore\ a_n - a_{n-1} = n^2 - (n-1)^2 = n^2 - (n^2+1-2n)$

$a_n - a_{n-1} = 2n+1$

$\Rightarrow a_n = a_{n-1} + (2n+1),\quad a_1 = 1,\quad n \geq 2$

$\boxed{4(b)}$ (i) coefficient of x^9y^3 in expansion of $(2x-3y)^{12}$

The bino. theorem

$$(2x-3y)^{12} = \sum_{0}^{12} \binom{12}{r} (2x)^r (-3y)^{12-r}$$

$$= \sum_{0}^{12} \binom{12}{r} (2)^r (-3)^{12-r} x^r y^{12-r}$$

The coeff of x^9y^3, corresponds to $r = 9$

$$\therefore \text{Coeff} = \binom{12}{9} (2)^9 (-3)^3 = -(2)^{10}(3)^3 \times 11 \times 10$$

(ii) By Bino. theorem

$$(1-2x)^{10} = \sum_{0}^{10} \binom{10}{r} (1)^{10-r} (-2x)^r$$

$$\therefore x^3(1-2x)^{10} = \sum_{0}^{10} \binom{10}{r} (-2)^r x^{r+3}$$

The coeff of x^{12}, corresponds to $r = 9$

$$\therefore \text{Coeff} = \binom{10}{9} (-2)^9 = -5120$$

$\boxed{4(c)}$ 12 symbols can be arranged in 12! ways.

For each arrangement, there are 11 positions b/w 12 symbols. Since there must be at least three spaces b/w successive symbols, 33 of the 45 will be used up. The remaining 12 spaces can be accommodated in 11 positions in $C(11+12-1,12)$ $= C(22,12)$ ways.

$\therefore$ By product rule, the required no. is

$$= 12! \times C(22,12) = \frac{22!}{10!} = 3.097445 \times 10^{14}$$

MODULE – 3

$\boxed{5(a)}$

Given $f(x) = \begin{cases} 3x-5, & x > 0 \\ -3x+1, & x \leq 0 \end{cases}$

- $f(0) = -3 \times 0 + 1 = 1$
- $f(-1) = -3 \times (-1) + 1 = 4$
- $f^{-1}(0) = \{x \in R \mid f(x) = 0\}$

$\Rightarrow$ $3x-5=0$ & $-3x+1=0$

$x = 5/3 > 0$ $\qquad$ $x = 1/3 \not\leq 0$

$\therefore$ $f^{-1}(0) = \{5/3\}$

- $f^{-1}(3) = \{x \in R \mid f(x) = 3\}$

$3x-5=3$, $-3x+1=3$

$x = 8/3 > 0$, $-3x = +2 \Rightarrow x = \frac{-2}{3} < 0$

$\therefore f^{-1}(3) = \left\{ \frac{-2}{3}, \frac{8}{3} \right\}$

- $f^{-1}([-5,5]) = \{ x \in R \mid -5 \leq f(x) \leq 5 \}$

$\Rightarrow \quad -5 \leq 3x - 5 \leq 5 \qquad \& \qquad -5 \leq -3x + 1 \leq 5$

$0 \leq 3x \leq 10 \qquad\qquad -6 \leq -3x \leq 4$

$-2 \leq -x \leq \frac{4}{3}$

$\Rightarrow \quad 0 \leq x \leq \frac{10}{3}$

$-\frac{4}{3} \leq x \leq 2$

$\Rightarrow x \in \left(0, \frac{10}{3}\right] \qquad \Rightarrow \left[-\frac{4}{3}, 0\right] \quad (\because x \leq 0)$

$\because x > 0$

$\therefore f^{-1}([-5,5]) = \left[-\frac{4}{3}, 0\right] \cup \left(0, \frac{10}{3}\right]$

$= \left[-\frac{4}{3}, \frac{10}{3}\right]$

$\boxed{5(b)}$ Equivalence Relation – A relation R on A is equivalence if it satisfies the following properties –

(i) Reflexive – $\forall a \in A$, we must have $(a,a) \in R$

(ii) Symmetric – ~~for~~ $a, b \in A$

if $(a,b) \in R$ then $(b,a) \in R$

(iii) Transitive – For $a, b, c \in A$

if (a,b) & $(b,c) \in R$, then $(a,c) \in R$

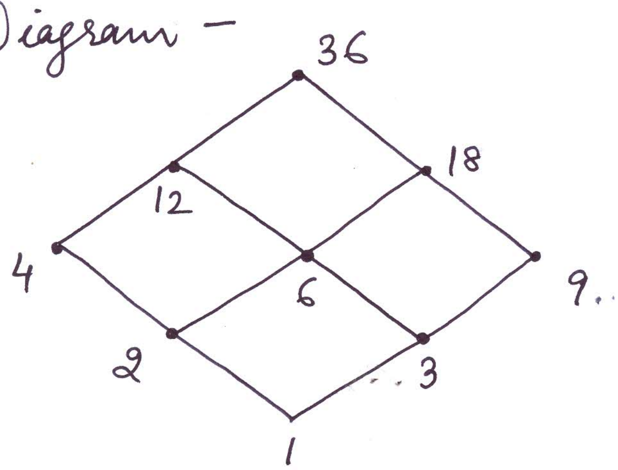
$D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

The relation R is –

aRb iff a divides b

$\therefore R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,9), (1,12), (1,18)$
$(1,36), (2,2), (2,4), (2,6), (2,12), (2,18), (2,36)$
$(3,3), (3,6), (3,9), (3,12), (3,18), (3,36), (4,4), (4,12)$
$(4,36), (6,6), (6,12), (6,18), (6,36), (9,9), (9,18), (9,36)$
$(12,12), (12,36), (18,18), (18,36), (36,36)\}$

Hasse Diagram –



$\boxed{6(a)}$ Given $f: R \to R$, $f(x) = 2x+5$

& $g: R \to R$, $g(x) = \frac{1}{2}(x-5)$

$g \circ f(x) = g\{f(x)\} = g(2x+5)$

$= \frac{1}{2}(2x+5-5) = x = I_R(x)$

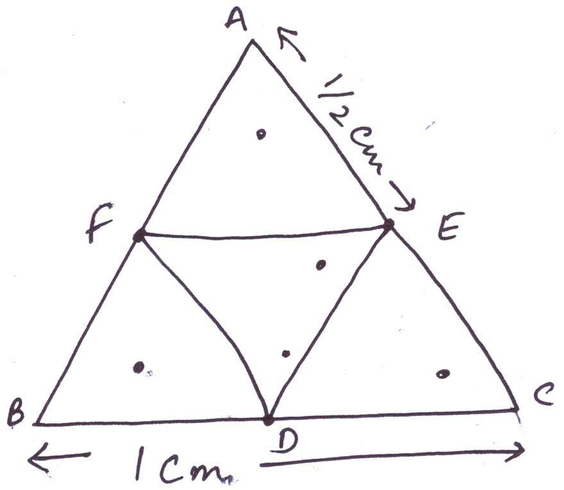
& $f \circ g(x) = f\{g(x)\} = f\{\frac{1}{2}(x-5)\}$

$= \left\{\left(2 \times \frac{1}{2}(x-5)\right) + 5\right\} = x = I_R(x)$

$\therefore$ g is inverse of f.

Even f is inverse of g.

6(b) Consider the ΔDEF formed by the mid points of the sides BC, AC and AB of the ΔABC.



This ΔABC is partitioned into 4 equilateral subtriangles, each of which has sides equal to $\frac{1}{2}$ cm.

Treating each of these four portions as ~~the~~ a pigeonhole and five points chosen insides the Δ as pigeons.

Using pigeonhole principle, we see at least two pigeons contain two or more points.

$\therefore$ The distance between such pts is less than $\frac{1}{2}$ cm.

6(c) Given $A = \{1, 2, 3, 4\}$

$R = \{(1,2), (1,3), (2,4), (4,4)\}$

$S = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4)\}$

$R \circ S = \{(1,3) (1,4)\}$

$S \circ R = \{(1,2) (1,3) (1,4), (2,4)\}$

$R^2 = R \circ R = \{(1,4) (2,4), (4,4)\}$

$S^2 = S \circ S = \{(1,1) (1,2), (1,3), (1,4), (1,3), (1,4)\}$

$$M_{ROS} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} , \quad M_{SOR} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_{R^2} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} , \quad M_{S^2} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

MODULE-4

7(a) derangements of 1, 2, 3, 4

$$d_4 = 4! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right\}$$

$$= 24 \times \left\{ 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} \right\} = 9$$

The derangements are :

2143, 2341, 2413, 3142, 3412, 3421,
4123, 4312, 4321

(b) $S = \{1, 2, 3, \ldots 300\} \Rightarrow |S| = 300$

Let A_1, A_2, A_3 be subsets of S whose elements are divisible by 5, 6, 8 respectively

$\therefore \ |A_1| = \left\lfloor \frac{300}{5} \right\rfloor = 60 \quad , \quad |A_1 \cap A_2| = \left\lfloor \frac{300}{30} \right\rfloor = 30$

$|A_2| = \left\lfloor \frac{300}{6} \right\rfloor = 50 \quad , \quad |A_1 \cap A_3| = \left\lfloor \frac{300}{40} \right\rfloor = 7$

$|A_3| = \left\lfloor \frac{300}{8} \right\rfloor = 37 \quad , \quad |A_2 \cap A_3| = \left\lfloor \frac{300}{24} \right\rfloor = 12$

$$|A_1 \cap A_2 \cap A_3| = \left\lfloor \frac{300}{120} \right\rfloor = 2$$

Now $S_0 = |S| = 300$

$$S_1 = \sum |A_i| = 60 + 50 + 37 = 147$$

$$S_2 = \sum |A_i \cap A_j| = 10 + 7 + 12 = 29$$

$$S_3 = |A_1 \cap A_2 \cap A_3| = 2$$

$\therefore$ The nos. which are divisible by exactly two of 5, 6, 8 is

$$E_2 = S_2 - \binom{3}{1} S_3 = 29 - \binom{3}{1} 2 = 29 - 6 = 23$$

$\boxed{7(C)}$ In the begining the no. of virus affected files is $1000 = a_0$

Let a_n be the no. of virus affected files after $2n$ hours. Then the no. increases by $a_n \times 250\%$ in next two hours. Thus after $2n+2$ hours, the no. is

$$a_{n+1} = a_n + \left(a_n \times \frac{250}{100}\right) = (3.5) a_n$$

which is homo. recurrence relation, and the solution is given by

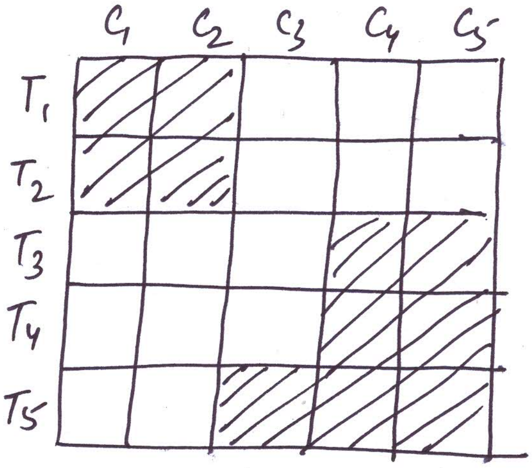
$$a_n = (3.5)^n \times a_0 = (3.5)^n \times 1000$$

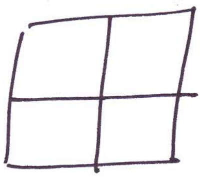
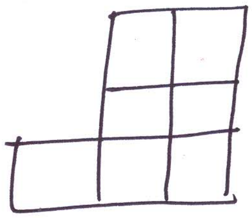
The no. of virus affected files after one day i.e. $n = 12$

$$a_{12} = 1000 \times (3.5)^{12} = 3379220508$$

8(a)

The board C is divided into two disjoint sub-boards. say D_1 & D_2



D_1 :  & D_2: 

$r(D_1, x) = 1+4x+2x^2$ & $r(D_2, x) = 1+7x+10x^2+2x^3$

$\therefore r(C,x) = r(D_1,x) \times r(D_2,x)$

$= 1+11x+40x^2+56x^3+28x^4+4x^5$

Here $r_1 = 11$, $r_2 = 40$, $r_3 = 56$, $r_4 = 28$, $r_5 = 4$

Consequently

$S_0 = 5! = 120$

$S_1 = (5-1)! \times r_1 = 264$

$S_2 = (5-2)! \times r_2 = 240$

$S_3 = (5-3)! \times r_3 = 112$

$S_4 = (5-4)! \times r_4 = 28$

$S_5 = (5-5)! \times r_5 = 4$

$\therefore$ The no. of ways in which the work can be done in

$S_0 - S_1 + S_2 - S_3 + S_4 - S_5 = 8$

8(b) The characteristic eqⁿ is $k^2 - 2k + 2 = 0$

roots are $= \dfrac{2 \pm \sqrt{4-8}}{2} = 1 \pm i$

$\therefore$ The general solution for a_n is

$$a_n = r^n (A \cos n\theta + B \sin n\theta)$$

where $r = \sqrt{1+1} = \sqrt{2}$

& $\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{n\pi}{4}$

$$\therefore \quad a_n = (\sqrt{2})^n \left(A \cos \frac{n\pi}{4} + B \sin \frac{n\pi}{4}\right)$$

Use $a_0 = 1$ & $a_1 = 2$

$\Rightarrow \quad A = 1$

& $2 = \sqrt{2}\left(A \cos \frac{\pi}{4} + B \sin \frac{\pi}{4}\right) = A + B$

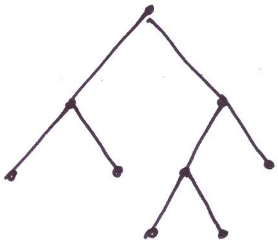
$\therefore \quad A = 1$ & $B = 1$

Therefore $a_n = (\sqrt{2})^n \left(\cos \frac{n\pi}{4} + \sin \frac{n\pi}{4}\right)$

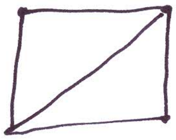
MODULE -5

9(a) Binary rooted tree – A tree is rooted if it has unique vertex, called the roots, whose in-degree is 0, and the in degree of all other vertices of T are 1, and for being binary, every vertex is of out degree two.

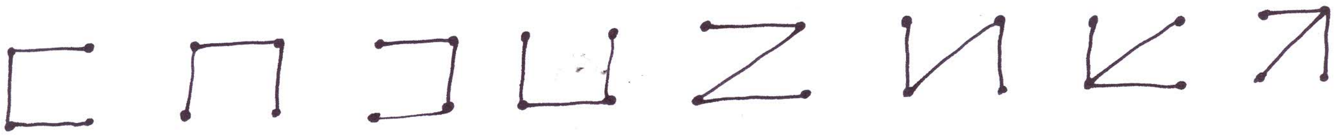
Balanced tree – If T is a rooted tree and h is the largest level number achieved by a leaf of T, then T is said to have height h. A rooted tree of height h, is said to balanced if the level no. of every leaf is h or h−1.



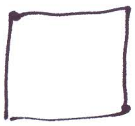
Given graph



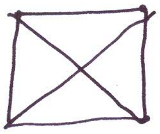
The following are the spaning trees



$\boxed{10(a)}$ **Regular graph** – A graph in which all vertices are of same degree k is called a regular graph of degree k, or k-regular graph.

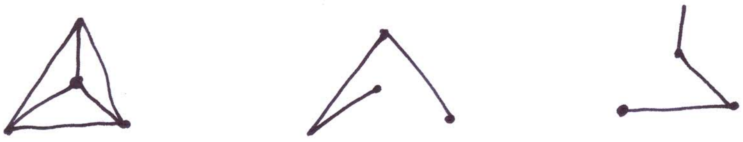


2-regular



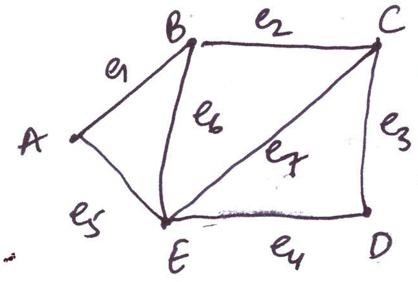
3-regular

Complement of a graph – If G is a simple graph of order n, then the complement of G in K_n is called complement of G, and is denoted by $\overline{G}$.



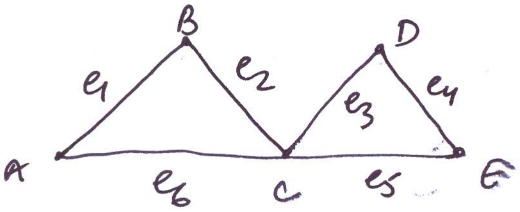
K_4 G $\overline{G}$

Euler trail – If there is a trail in G that contains all the edges of G, then that trail is called Euler trail in G.



$B e_1 A e_5 E e_6 B e_2 C e_7 E e_4 D e_3 C$

Euler Circuit – If there is a circuit in a connected graph G that contains all the edges of G, then that circuit is called an Euler circuit.



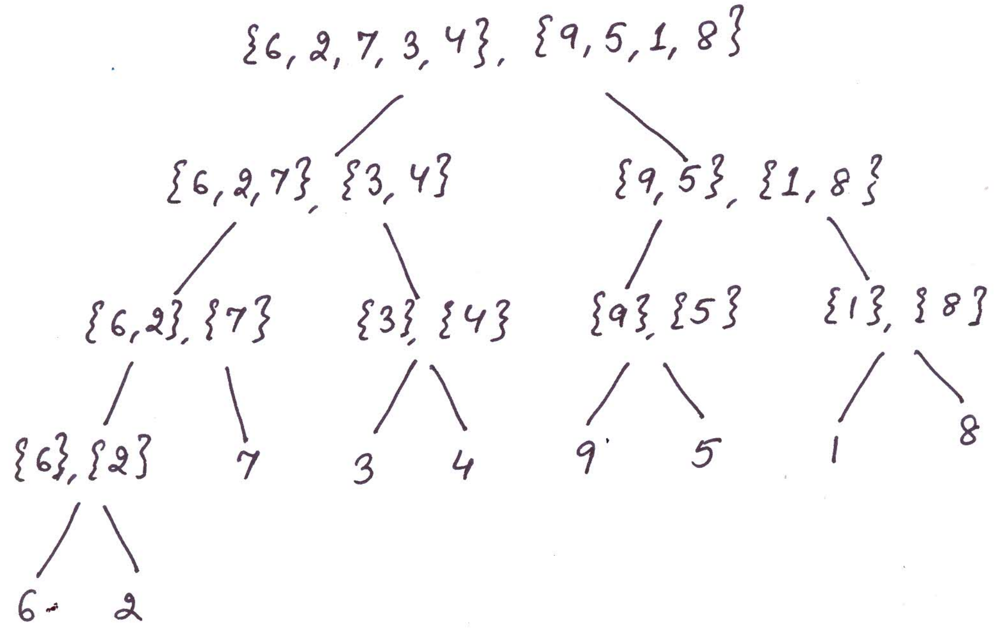
$A e_1 B e_2 C e_3 D e_4 E e_5 C e_6 A$

Complete graph – A simple graph of order ≥ 2 in which there is an edge between every pair of vertices is called a complete graph

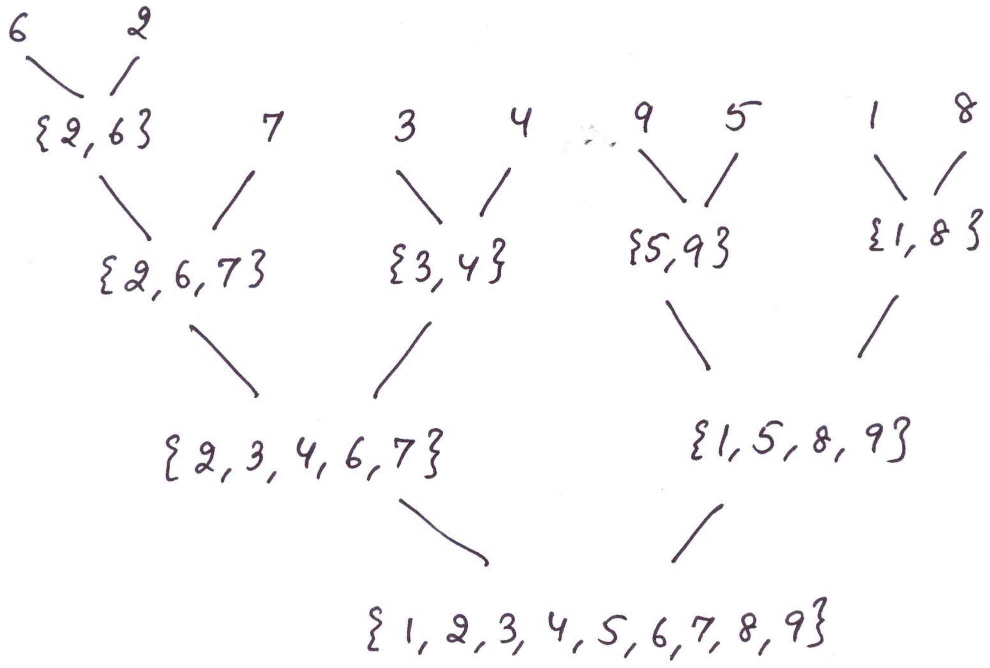
K_2 K_3

$\boxed{10(b)}$ Merge sort to the list

6, 2, 7, 3, 4, 9, 5, 1, 8



Now we merge the sublists in non-decreasing order



$\boxed{9(a)}$ First, let G has an Euler circuit. While tracing this circuit we observe that every time the circuit meets a vertex v it goes through two edges incident on v. This is true for all vertices that belong to the circuit. Since the circuit contains all edges, it meets all the vertices at least once. $\therefore$ the degree of each vertex is multiple of two i.e. even.

Conversely, suppose that all the vertices of G are of even degree. Now, we construct a circuit starting at an arbitrary vertex v and going through the edges of G s.t. no edge is traced more than once. Since every vertex is of even degree, we can depart from every vertex we enter, and the tracing can't stop at any vertex other than v. In this way, we obtain a circuit q having v as the initial & final vertex. If this circuit contains all the edges in G, then the circuit is an Euler circuit. If not, let us consider the subgraph H obtained by removing from G all edges that belong to q. The degree of vertices in this subgraph are also even. Since G is connected, q and H must have at least one vertex v' in common. Start from v', we can make a circuit q' in H. The two q & q' together constitute a circuit which starts and end at v & has more edges than q. If this circuit contains all the edges, then we get Euler circuit. Otherwise we repeat the process until we get a circuit which covers all the edges. In this way, we get Euler circuit in G.