SYSTEM MODELING AND SIMULATION(15CS834)

SOLUTIONS : VTU JUNE 2019

1.a) Simulation is the imitation of the real world or system over time

Figure 1.3. Steps in a simulation study.

Four phases according to Figure 1.3

- **First phase : a period of discovery or orientation (step 1, step2)**
- **Second phase : a model building and data collection (step 3, step 4, step 5, step 6, step 7)**
- **Third phase : running the model (step 8, step 9, step 10)**
- **Fourth phase : an implementation (step 11, step 12)**

1.b)

Inter-Arrival	Probability	Cumulative	Random N _o
time		Probability	Assessment
	0.25	0.25	$1 - 25$
2	0.40	0.65	26-65
3	0.20	0.85	66-85
$\overline{4}$	0.15	1.00	86-00

• For finding the following times -1 Mark

Avg IAT $= 24/9$ Avg ST of able = $118/10$

2.a. Components of Discrete event simulation

- 1. System: A collection of entities (e.g., people and machines) that together over time to accomplish one or more goals.
- 2. Model: An abstract representation of a system, usually containing structural, logical, or mathematical relationships which describe a system in terms of state, entities and their attributes, sets, processes, events, activities, and delays.
- 3. System state: A collection of variables that contain all the information necessary to describe the system at any time.
- 4. Entity: Any object or component in the system which requires explicit representation in the model (e.g., a server, a customer, a machine).
- 5. Attributes: The properties of a given entity (e.g., the priority of a v customer, the routing of a job through a job shop).
- 6. List: A collection of (permanently or temporarily) associated entities ordered in some logical fashion (such as all customers currently in a waiting line, ordered by first come, first served, or by priority).
- 7. Event: An instantaneous occurrence that changes the state of a system as an arrival of a new customer).
- 8. Event notice: A record of an event to occur at the current or some future time, along with any associated data necessary to execute the event; at a minimum, the record includes the event type and the event time.
- 9. Event list: A list of event notices for future events, ordered by time of occurrence; also known as the future event list (FEL).
- 10. Activity: A duration of time of specified length (e.g., a service time or arrival time), which is known when it begins (although it may be defined in terms of a statistical distribution).
- 11. Delay: A duration of time of unspecified indefinite length, which is not known until it ends (e.g., a customer's delay in a last-in, first-out waiting line which, when it begins, depends on future arrivals).
- 12. Clock: A variable representing simulated time.

C2-AT:4,DT-8 C3-AT:9,DT-13 C4-AT-11,DT-19 C5-AT:19,DT-21 C6-AT:22,DT-29

3.a) Explain the following distributions.

i) Binomial distribution ii) Uniform distribution

i)Binomial Distribution

The no of successes in n Bernoulli trials is said to follow binomial distribution.

 $P(x) = (n) p^x q^{n-x}$ for $x=0,1,2,...,n$ Mean: $E(x) = np$ Variance : $V(x) = npq$

ii)Uniform Distribution

In probability theory and statistics, the **continuous uniform distribution** or **rectangular distribution** is a family of symmetric probability distributions such that for each member of the family, all intervals of the same length on the distribution's support are equally probable. The support is defined by the two parameters, a and b, which are its minimum and maximum values.

> Pdf : $f(x) = 1/b-a$, $a < x < b$ Cdf : $F(x) = x-a/b-a$, $a < x < b$ Mean: $E(x) = a+b/2$ Variance: $v(x) = (b-a)^2/12$

4.a) List and explain the characteristics of queuing system. Briefly explain queuing notations.

characterfitties of sweeting system the key elements of querry Mitem are the cuttuments and servers. outtomed! - refers to people, m/c, twold, mechanics, parsents airplanes, amail atcservents :- matest to receptionlits, repair person, epu, any resource that provides requested service the Atlanding table and tend examples of questing system system cuptomed server Herport Auplanes Runway
Hospital patients Nuttes, doctor
Reception desk people Receptions A **Conference of the Con-**The ream customed means anything that arrives at a facility and requires service. -> the characteristics of queving system are - catting population - system confeacity - Arrival process - Queve behaviour and outeve discipline - Jervice time and service mechanism. the calling population the population of potential automets is referenced as calling population -> the calling population can be elther finite (or) infinite -> the main difference the finite and infinite is based on how arrived note & defined. 4) Quere behaviour and Rueve discipline surve behavious refers to the action of culturies while in queve worthing for leavice to begin. queve behaviour are -That's - leave when they see most the line is too long. -7 Renege - leave ofted being in the line when may see that the line is moving too slowly. -> Jockey - move from one line to another & they wink they have chooser a slowline. - sueve ducipline refers to how the who ment are Revered in the querre $HFFBO, HFO,$ $-$ SIRO - service in random 5) service times and service mechanism resince times are denoted by susais, ... they may be constant (or) of random durintion. the exponential, weiboll, gamma, lognormal distributions are used to model the lervice times. A queving system consists of no of service centers and interfloringcesting queues both let vice center contities of no of letrens a working in parallel. pasallel letrice mechanisms are einer ingle lerver(c=1), multiple server (d) unlimited servers

-> In infinite population the arrival vate is not affected by the 10 of customent who have left the calling population and solining the quelling system.

 $Ex := H0+E$

-> In finite population the arrival rate depends on the number of cultoment being lerved and waiting.

Ex: - Bo In a book hospital where nexe are prior appointments the evitoment are patients here and the assival rate of patients depends on finite arrivals. 2. System asparenty :-

system capacity is defined as the maximum no of customents allowed in susten (er) in watting queue.

EX :- An automatic car wash might have form for only 10 cars to walt in line to enter me mechanism. An arriving customed who finds the system full does not ented but returns immediately to the calling population.

- some systems such as concent ticket lates for students may be confidered to have unlimited capacity

3. The arrival process ;-

Arrival process describe how customer arrival, how the arrival are distributed in time and whether there is finite population model (or) infinite population model. - Arrivals may occur at scheduled times (or) raudom times. -> the most important model for random annivals is the poisson arrival process.

La If An represents iAT bin customed not and customed n men for pottson arrival process An is exponentially distributed with near 1/2 time unity.

4) Queve behaviour and Queve discipline

sueve behaviours refers to the action of culturies while in queve wouting for existen to begin. Bueve behaviour are -TBalk - leave when may see mot the line is too long. -> Renege - leave ofted being in the line when they see that the line is moving too slowly. ->Jockey - move from one line to another it they think they have chooser a slow line. - Queve ducipline refers to how the customent are Revered in the querie $-FFO, HFO,$ -sieo - service in vandom 5) service times and service mechanism resince times are denoted by surfaces.... they may be constant (or) of random duration. the exponential, weibuil, samma, lognormal distributions are used to model me lerrice times.

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Breving notation
       Kendall proposed a guerral notation as follows.
    A|B|C|N|RA represent Inter arrival time distribution
    B represents service time diARM
    c represents no of parallel servers
    N represents lister copacity
     K represents size of colling population
- common symbols for a and B includes m (exponential Gr)
modrov), D (constant con) deterministic); Ex (exlang of order K)
GlastHaly, general)
     ex- m/m/1/20/20
              Ly A fingle server system mat has unlunited
 queve capacity and an infinite population of potential
 arrivals
   -ywhen N and K are infinite then they may be
stopped trom the notation
     mmlilools is shortened to mmli
```
5.a) **Sol:** The sequence of Xi and subsequent Ri values is computed as follows:

 $X0 = 27$

- \bullet $\text{XI} = (a.X0+c) \text{ mod } m = (17.27+43) \text{ mod } 100 = 502 \text{ mod } 100 = 2$
- $R1 = X1/m = 2/100 = 0.02$
- \bullet $X2 = (17 \bullet 2 + 43) \text{ mod } 100 = 77 \text{ mod } 100 = 77$
- \bullet R2=77/100=0.77
- \bullet $X3 = (17\cdot 77 + 43) \text{ mod } 100 = 1352 \text{ mod } 100 = 52$
- $R3 = 52 / 100 = 0.52$

Finding the Period :

- 1. For m a power of 2, say m = 2b and c $\ 0$, the longest possible period is P = m = 2b, which is achieved provided that c is relatively prime to m (that is, the greatest common factor of c and m i s l), and $=a = 1+4k$, where k is an integer.
- 2. For m a power of 2, say m = 2b and $c = 0$, the longest possible period is $P = m/4 = 2b-2$, which is achieved provided that the seed X0 is odd and the multiplier, a is given by $a=3+8K$, for some k=0,1..
- 3. For m a prime number and $c=0$, the longest possible period is P=m-1, which is achieved provided that the multiplier , a, has the property that the smallest integer k such that a k - 1 is divisible by m is $k=$ m-1.

5.b.)

- **Rank** the data from smallest to largest. Finding the R_i o i.e. R_i 0.11 0.54 0.68 0.73 0.98
- Finding the D^+ and D^- values
	- o i.e. $D^+=i/N-R_i = 0.09$

o $D^{\dagger} = R_i - (i-1)/N = 0.34$

- \blacktriangleright Finding D value
	- o i.e. $D = max(D^+, D^-) = 0.34$
- \blacktriangleright Justification of the data accept or reject
	- o i.e. The tabular value D $_{n}= 0.565$. Since D < D i.e 0.34 < 0.565 the sequence of numbers given are accepted.

6.a)

exponential diffusion
\n
$$
+ compute m = adf + af m = destred random valueable
$$
\n
$$
adp: fQ) = 1 - e^{4x}
$$
\n
$$
= e^{4x} = R
$$
\n
$$
1 - e^{4x} = R
$$
\n
$$
1
$$

6.b.) i.e. 1. Set n=0,p=1.

- 2. R1= 0.4357 , P= $1*0.4357 = 0.4357$
- 3. Since P=0.4357 $< e^{-0.2}$ = 0.8187, accept N=0
	- Steps 1-3.(R1=0.4146 leads to N=0),accept N=0
		- o $1.$ Set n=0, $P=1$
		- o 2. R1=0.8353,P=1*0.8353=0.8353
		- o 3.Since $0.8353 > 0.8187$, reject n=0 and return to step 2 with n=1
		- o Step 2. R2= 0.9952 , P=R1R2= $0.8353*0.9952 = 0.8313$
		- o 3. Since $0.8313 > 0.8187$, reject n=1 and return to step 2 with n=2
		- o Step 2. R3=0.8004, P=R1R2R3 = $0.8313 * 0.8004 = 0.6654$
		- o 3. Since $0.6654 < 0.8187$ accept N=2.
		- o The three Poisson Variates are 0.4357, 0.4146, 0.6654

7.a)

 1011 isseful model of input dasa. Mollect dota from the real system of interest. L this requires a substantial time and release su some eales it is not possible to and release. In some way is limited (8) collect aare for a does not exist. when the process does not example operat opinion -when data ale not overturned on the used.
and knowledge of the process must be used. and knowledge of the process must accepted me input precess ut process
-when done are available als step begins with the development of facqueusy distances with the both data.
3) chester parameters mar determine a specific with the bie data. s) choose parameters family withouse of the cheoler duteibution and the 4) evaluate the crisis conducts of fit. A palameters for you and comparable stream relts ale southers of are rests x we choose distant ale sociations of the mese rests their me is not easined from these and chooses on depersent family of distributions (a) Explain david reliection in detail. Data collection :pata collection it very important with hard to achieve there are a approaches 1. classical approach

7.b). Explain Chi–square goodness of fit test. Apply it to Poisson assumption with =3.64.Data size=100 and observed frequency Oi=12,10,19,17,10,8,7,5,5,3,3,1.Consider tabular value as 11.1.

Sol: For Poisson distribution $P(x) = e^{-x} / x!$ for $x=0,1,2,...$

Compute $P(0), P(1), P(2), \ldots, P(11)$ as follows

 $P(0)=e^{-3.64}(3.64)^0/0! = 0.026$

 $P(1) = e^{-3.64}(3.64)^{1}/1! = 0.096$

 $P(2) = e^{-3.64}(3.64)^2 / 2! = 0.174$

 $P(3) = e^{-3.64}(3.64)^3 / 3! = 0.211$

……till P(11) as follows

b) it engineering dara it

- after a product b) process has performance natings provided by the manufacturer.

EX :- mean time to failure of a disk delve is 10000 hrs a laser printed can produce spages/min etc.

- company rules night specify rime (or) production standards.

- Expert option :-

- Talk to people who are experienced with the process. they can provide optimistic, personistic and most likely times. - they might also able to say whether me process is

meally conflant (et) wighly valiable and mey can define me source of valiability.

-physical or contractional limitations:

- most real processes nove physical limits on performance Exi-computer data enthy cannot be foster man a pesson cour type.

- Because of company polletes mere could be upper limits en how long a procell may take and donot ignore it.

- the nature of the process;

when data are not available then uniform, thongular, and bera distributions are often used as the models. -uniform division - poor choice bes upper and lower bounds ale likely of central values in real process. - Thiangular distr^{on} - in addition to upper and lower bounds a most likely value it given then we thengular distRN. - Beta differ - dentity function not to be plotted.

a contract the advance of the contract of

8.a)

Types of Anulations with respect to output analysis.

there are a types of simulations 1. Teeninaring simulations of the plants from 2. steady Hate Minulations

A reininating Analation is one mation hing for some dulation of time Te where & is me specified event mat thops me structures ex :- A retail shop closes every evening from which it opens from gain to 5 pm.

LHESE E = other + hours of the has been used

2) A company which sells on product would like to decide how many items to have in inventory dusing phonong for low memby. **LOIDAUS CLASS**

 L Hele $E = 100$ months

3) A communication system, confirst of several components plus several backup components, as in fig.

Here ϵ = consider the system over a period of time Te until system fails.

 $A = \{A \text{ fails } I(d) \text{ } B \text{ fails } (d) \text{ } B \text{ and } C \text{ fails } T\}$

- In mit case we cannot peedlet E in advance

to g we do not know when the component fails.

curpot avaly is too teeninating sinulations

A terminating simulation runs over a simulated time interval (O, TE)

- A common goal if to estimate

 $\theta = \mathcal{L}\left[\frac{1}{N}\sum_{i=1}^{N} Y_i\right]$ for discrete output $\varphi = \mathbb{E}\left[\frac{1}{T_{\mathcal{E}}}\int\limits_{0}^{T_{\mathcal{E}}}\gamma(\epsilon)\,d\epsilon\right]$ for continuous of φ y(E),

(3)

- In general independent replications are used. each fun using a different random no stream and independently choosen inford conditions

statistical background i-

the most confusing aspect among simulation op analy is is distinguishing within-replication data from across-replication data.

- For ex simulation of a manufactualing system

- Two performance measures of mat system -cycle time for pasts (time from relasse into the factory until completion) - work in process (wip) me rotal no of part in the factory at any time.

I let the me under the for the in part produced in the 1^p replication **MEGE**

- Across-replication data are formed by Jummeliging within-replication data &

I - sample mean of the mi cycle times from *in* replication

 s_i^2 - sample validuce of the same data.

and $H_i = \pm \alpha/2$, $m_i - 1$ $\frac{si}{\sqrt{n_i}}$ is a considence interval

half-width based on mig data set.

 $44 - 11$

winn and across-rep data for cycle-Ane

- From the across-replication data we compute the overall statistics me any of the daily cycle time **es** compare posse profitamente i Muleus es overages

the lawple valiance of the daily cycle timeaverages $s^2 = \frac{1}{R-1} \sum_{i=1}^{R} (\overline{Y_i} - \overline{Y})^2$

the confidence interval have width $H = \frac{5}{R}$ sold $1 - \frac{5}{R}$

will a short is premability off

of - burgle variance of me look does

The quantity s/p If the Mandard error

고도 교육 도용되도록

winin-replication

work to process is a continuous time ofp. devoted by tilt). The stopping time for in replication Te, could be a raudom valiable

within-Rep and Across replication wip data

- the winin-replication sample mean and valiance are defined as

$$
\overleftarrow{\gamma_i} = \frac{1}{\tau \epsilon_i} \int_0^{\tau_{\epsilon_i}} \gamma_i(\epsilon) d\epsilon
$$
\n
$$
S_i^2 = \frac{1}{\tau_{\epsilon_i}} \int_0^{\tau_{\epsilon_i}} (\gamma_i(\epsilon) - \overline{\gamma_i})^2 d\epsilon
$$
\n
$$
H_i^2 = \frac{Z_{\alpha}}{2} \sqrt{\frac{S_i^2}{\sqrt{\tau_{\epsilon_i}}}}
$$

confidence intervals with specified precision

the note length H of a loo(1-d) 1. Confidence interval for a mean θ based on the t differention. Is given by

$$
H = \frac{t_{\alpha/2}}{T}, \frac{R-1}{R}, \frac{S-1}{R}, \frac{S}{S^2} \text{ if } \text{Re} \text{ sample} \text{ variance}
$$

RV me no of replications

- luppose mat an error exitesion & is specified with probability 1-d a sufficiently large eample size should parishy $P(|\overline{q}-\theta|<\epsilon) \geq 1-\epsilon$

- Asume mat au initial sample size of Ro a canton a proba va replications has been observed

 $10.a)$

The first step in model building consists of observing the real system and the interactions among The first step in model building consists of observing the real system and the interactions among
their various components and of collecting data on their behavior. But observation alone seldom yields sufficient understanding of system behavior. Persons familiar with the system, or any subsystem, should be questioned to take advantage of their special knowledge. Operators, technicians, repair and maintenance personnel, engineers, supervisors, and managers understand yields sufficient understanding of system behavior. Persons familiar with the system, or any subsystem, should be questioned to take advantage of their special knowledge. Operators, technicians, repair and maintenance pers new questions may arise and the model developers will return to this step of learning true system structure and behavior. model building consists of observing the real system and the interactions amomy
omponents and of collecting data on their behavior. But observation alone sele
understanding of system behavior. Persons familiar with the sys

The second step in model building is the construction of a conceptual model-a collection of assumptions about the components and the structure of the system, plus hypotheses about the values of model input parameters. As is illustrated by Figure, conceptual validation is the comparison of the real system to the conceptual model. ions may arise and the model developers will return to this step of learning true system
and behavior.
In a step in model building is the construction of a conceptual model-a collection of
ns about the components and the s

The third step is the implementation of an operational model, usually by using simulation software and incorporating the assumptions of the conceptual model into the worldview and concepts of the simulation software. In actuality, model building is not a linear process with three steps. Instead; the model builder will return to each of these steps many times while building, verifying, and validating the model. pts of the simulation software. In actuality, model building is not a linear process with
steps. Instead; the model builder will return to each of these steps many times while
ng, verifying, and validating the model.
pove

The above figure, depicts the ongoing model building process, in which the need for· verification and validation causes continual comparison of the real system to the conceptual model and to the operational model and induces repeated modification of the model to improve its accuracy. on causes continual comparison of the real system to the conceptual model and
model and induces repeated modification of the model to improve its accuracy.
 r and Finger formulated a three step approach which has been wi

10.b)Naylor and Finger formulated a **three step approach** which has been widely followed:-

1. Build a model that has high face validity.

2. Validate model assumptions.

3. Compare the model input-output transformations to corresponding input-output transformations for the real system.

1. Face Validity

• The first goal of the simulation modeler is to construct a model that appears reasonable on its face to model users and others who are knowledgeable about the real system being simulated.

• The users of a model should be involved in model construction from its conceptualization to its implementation to ensure that a high degree of realism is built into the model through reasonable assumptions regarding system structure, and reliable data.

• Another advantage of user involvement is the increase in the models perceived validity or credibility without which manager will not be willing to trust simulation results as the basis for decision making.

• Sensitivity analysis can also be used to check model's face validity.

• The model user is asked if the model behaves in the expected way when one or more input variables is changed.

• Based on experience and observations on the real system the model user and model builder would probably have some notion at least of the direction of change in model output when an input variable is increased or decreased.

• The model builder must attempt to choose the most critical input variables for testing if it is too expensive or time consuming to: vary all input variables.

2. Validation of Model Assumptions

• Model assumptions fall into two general classes: structural assumptions and data assumptions.

Structural assumptions involve questions of how the system operates and usually involve simplification and abstractions of reality.

• For example, consider the customer queuing and service facility in a bank. Customers may form one line, or there may be an individual line for each teller. If there are many lines, customers may be served strictly on a first-come, first-served basis, or some customers may change lines if one is moving faster. The number of tellers may be fixed or variable. These structural assumptions should be verified by actual observation during appropriate time periods together with discussions with managers and tellers regarding bank policies and actual implementation of these policies.

• Data assumptions should be based on the collection of reliable data and correct statistical analysis of the data.

3. Validating Input-Output Transformation

In this phase of validation process the model is viewed as input –output transformation : That is, the model accepts the values of input parameters and transforms these inputs into output measure of performance. It is this correspondence that is being validated.

• Using historical input data : Instead of validating the model input-output transformation by predicting the future, the modeler may use past historical data which has been served for validation purposes that is, if one set has been used to develop calibrate the model, its recommended that a separate data test be used as final validation test.

 Using Turing test: When no statistical test is readily applicable then persons knowledgeable about system behavior can be used to compare model output with system output. This type of test is called Turing test used in detecting model inadequacies and to increase the model credibility.