

# CBCS SCHEME

USN **ICRIGEEO7Z**

15EE62

## Sixth Semester B.E. Degree Examination, June/July 2019 Power System Analysis - I

Time: 3 hrs.

Max. Marks: 80

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

1. a. Show that per unit impedance of two winding transformer will remain same referred to primary as well as secondary. (06 Marks)
- b. A 300 MVA, 20 KV, 3-phase generator has subtransient reactance of 20%. The generator supplies two synchronous motors through a 64 KVA transmission line having transformers at both ends as shown in Fig.Q1(b).  $T_1$  is a 3-phase transformer and  $T_2$  is composed of 3-single phase transformers of rating 100 MVA each, 127/13.2 KV, 10% reactance, series reactance of transmission line is 0.5 ohm/km. Draw the reactance diagram with all reactances marked in per unit. Select generator rating on base values.

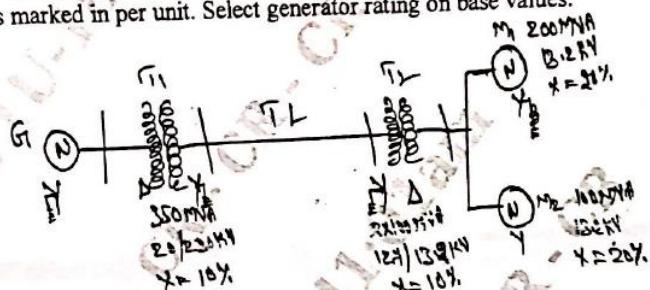


Fig.Q1(b)

(10 Marks)

### OR

2. a. Define per unit quantity. Mention the advantages of per unit system (04 Marks)
- b. The one line diagram of an unloaded generator is shown in Fig.Q2(b). Draw the PU reactance diagram. Choose a base of 50 MVA, 13.8 KV in the circuit of generator G1. The ratings are as follows:  
 $G_1 : 20 \text{ MVA, } 13.8 \text{ KV, } X'' = 20\%$        $T_1 : 25 \text{ MVA, } 13.8/220 \text{ KV, } X = 10\%$   
 $G_2 : 30 \text{ MVA, } 18 \text{ KV, } X'' = 20\%$        $T_2 : 30 \text{ MVA, } 220/18 \text{ KV, } X = 10\%$   
 $G_3 : 30 \text{ MVA, } 20 \text{ KV, } X'' = 20\%$        $T_3 : 35 \text{ MVA, } 220/22 \text{ KV, } X = 10\%$

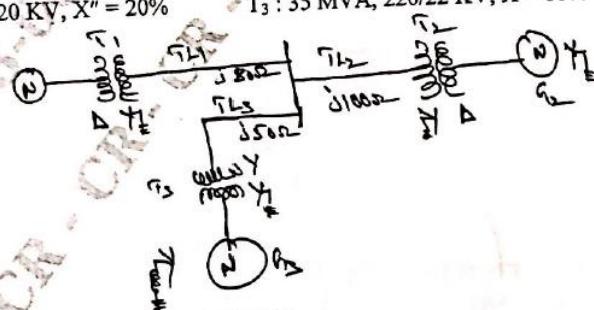


Fig.Q2(b)

(12 Marks)

3. a. With the help of waveform at the time of three phase ~~symmetrical~~ <sup>MAX</sup> generator define steady state, transient and subtransient reactances. (08 Marks)
- b. A generator is connected to a synchronous motor through transformer. Reduced to a common base, the per unit subtransient reactances of generator and motor are 0.15 and 0.35 PU respectively. The leakage reactance of the transformer is 0.1 PU. A 3-phase star circuit fault occurs at terminals of the motor when terminal voltage of generator is 0.9 P.U and output current of generator is 1 P.U at 0.8 pf leading. Find the subtransient current in the fault, generator and motor.

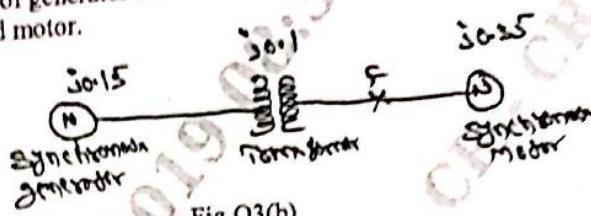


Fig.Q3(b)

(08 Marks)

OR

4. a. Explain clearly, how circuit breaker are rated? (06 Marks)
- b. A synchronous generator and motor are rated 30 MVA, 13.2 KV, both have subtransient reactance of 20%. The line connecting them has a reactance of 20%, on the base of machine rating. The motor is drawing 20 MW at 0.8 pf (lead). The terminal voltage of motor is 12.8 KV, when a symmetrical fault occurs at motor terminals, find subtransient current in generator, motor and at the point of fault? (10 Marks)

### Module-3

5. a. Obtain the relationship between line and phase sequence components of voltages in star connection. Give the relevant phasor diagrams. (08 Marks)
- b. Draw the positive, negative and zero sequence network for the power system shown in Fig.Q5(b). Choose a base of 50 MVA, 220 KV in the  $50\Omega$  transmission lines and marks all reactances in PU. The ratings of the generator and transformers are:  
 $G_1$ : 25 MVA, 11 KV,  $X'' = 20\%$ ;  $G_2$ : 25 MVA, 11 KV,  $X'' = 20\%$   
3φ transformers (each) : 20 MVA, 11/220 KV,  $X = 15\%$   
The negative sequence reactance of each synchronous machine is equal to the sub-transient reactance. The zero sequence reactance of each machine is 8%. Assume that the zero sequence reactances of lines are 250% of their positive sequence reactances.

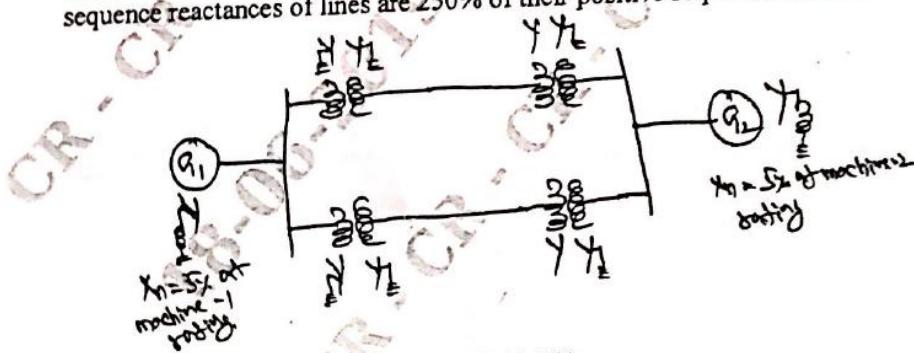


Fig.Q5(b)

(08 Marks)

OR

6. a. Draw the zero sequence impedance networks of a transformer for the following connections:
- i)  $\Delta - \Delta$       ii)  $\Delta - Y$       iii)  $Y - Y$

(06 Marks)

- c. The positive, negative and zero sequence components of line currents are  $20|10^\circ$ ,  $6|60^\circ$  and  $3|30^\circ$  A respectively. Determine the line currents. (04 Marks)
- c. In a  $3\phi$ , 4 wire system, the sequence voltages and currents are:  
 $V_{a1} = 0.9|10^\circ$  PU;  $V_{a2} = 0.25|110^\circ$  PU;  $V_{a0} = 0.12|300^\circ$  PU;  
 $I_{a1} = 0.75|25^\circ$  PU;  $I_{a2} = 0.15|170^\circ$  PU;  $I_{a0} = 0.1|330^\circ$  PU  
Find the complex power in PU. If the neutral gets disconnected, find the new power. (06 Marks)

**Module-4**

- 7 a. An unloaded fully excited three phase alternator is subjected to an L-G fault at its terminals. Find the fault current. Using symmetrical components by showing the interconnection of all sequence networks. (08 Marks)
- b. Draw the sequence networks for the system shown in Fig.Q7(b). Determine the fault current if a line to line occurs at F. The PU reactances all referred to the same base are as follows. Both the generators are generating 1.0 PU.

Component	$X_0$	$X_1$	$X_2$
$G_1$	0.05	0.30	0.20
$G_2$	0.03	0.25	0.15
Line-1	0.70	0.30	0.30
Line-2	0.70	0.30	0.30
$T_1$	0.12	0.12	0.12
$T_2$	0.10	0.10	0.10

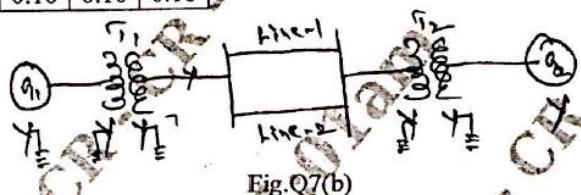


Fig.Q7(b)

(08 Marks)

**OR**

- 8 a. Derive expression for fault current if Line-Line-Ground (LLG) fault occurs through fault impedance  $Z_f$  in power system. Show the connection of sequence networks to represent the fault. (08 Marks)
- b. A three phase generator with an open circuit voltage of 400 V is subjected to an LG fault through a fault impedance of  $j2\Omega$ . Determine the fault current if  $Z_1 = j4\Omega$ ,  $Z_2 = j2\Omega$  and  $Z_0 = j1\Omega$ . Repeat the problem for LL fault. (08 Marks)

**Module-5**

- 9 a. Explain 'equal area criteria' concept when a power system is subjected to sudden loss of one of the 'parallel lines'. (08 Marks)
- b. Define stability pertaining to a power system and classify the different types of stability. (04 Marks)
- c. A 2 pole, 50 Hz, 11 KV turbo alternator has a rating of 100 MW, 0.85 p.f. lagging. The rotor has moment of inertia of  $10000 \text{ kg-m}^2$ . Calculate H and M. (04 Marks)

**OR**

- 10 a. Derive the power angle equation of a salient pole synchronous machine connected to an infinite bus. Draw the power angle curve. (08 Marks)
- b. Derive an expression for the swing equation. (08 Marks)

\*\*\*\*\*

Solen    Base Values

$(MVA)_B$  = rated MVA of the transformer

$(KV_1)_B$  = Base voltage in the primary side.

$(KV_2)_B$  = Base voltage in secondary side.

Also, let  $Z_{eq1}$  be the impedance of the transformer ref. to primary side and  $Z_{eq2}$  w.r.t secondary side.

$$(Z_{eq1})_{p.u} = Z_{eq1}(n) \times \frac{(MVA)_B}{(KV_1)_B^2} \quad \text{--- (1)}$$

$$(Z_{eq2})_{p.u} = Z_{eq2}(n) \times \frac{(MVA)_B}{(KV_2)_B^2} \quad \text{--- (2)}$$

$$Z_{eq2}(n) = Z_{eq1}(n) \times \frac{(KV_2)_B}{(KV_1)_B^2} \quad \text{--- (3)}$$

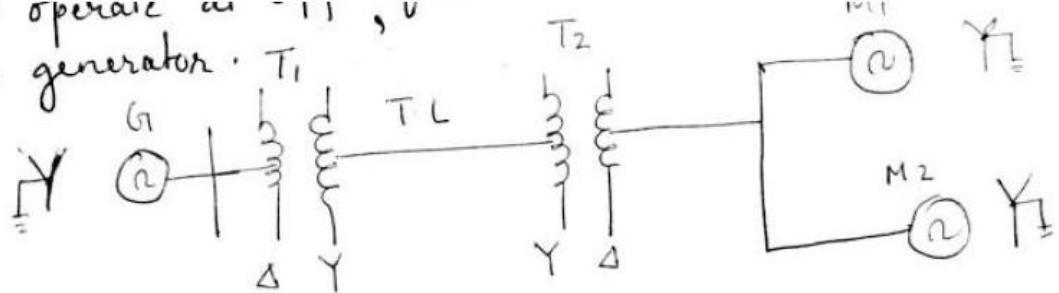
Substituting (3) in (2),

$$(Z_{eq2})_{p.u} = Z_{eq1}(n) \times \frac{(KV_2)_B^2}{(KV_1)_B^2} \times \frac{(MVA)_B}{(KV_2)_B^2}$$

$$(Z_{eq2})_{p.u} = (Z_{eq1})_{p.u}$$

1b

and operate as "11", v  
the generator.



Solution	Base values	$(MVA)_B, \text{old}$	$(MVA)_B, \text{new}$	$(kV)_{B, \text{old}}$	$(kV)_{B, \text{new}}$
Component					
Generator G	300	300	20	20	
Transformer T <sub>1</sub>	350	300	230 (HT)	$\frac{26 \times 230}{20} = 230$	
T <sub>L</sub>	-	300	-	$20 \times \frac{230}{20} = 230$	
T <sub>2</sub>	$100\sqrt{3} = 300$	300	127.3 (HT)	230	
M <sub>1</sub>	200	300	13.2	$\frac{230 \times 13.2}{127.3} = 13.8$	
M <sub>2</sub>	100	300	13.2	13.8	

Reactance of generator G

$$X_{G, \text{new}} = X_{G, \text{old}} \times \frac{(MVA)_B, \text{new}}{(MVA)_B, \text{old}} \times \frac{(kV)_{B, \text{old}}^2}{(kV)_{B, \text{new}}^2}$$

$$= j0.2 \times \frac{(300)}{(300)} \times \frac{(20)^2}{(20)^2} = j0.2 \text{ p.u.}$$

Reactance of transformer T<sub>1</sub>

$$X_{T1, \text{new}} = X_{T1, \text{old}} \times \frac{(MVA)_B, \text{new}}{(MVA)_B, \text{old}} \times \frac{(kV)_{B, \text{old}}^2}{(kV)_{B, \text{new}}^2}$$

$$= j0.1 \times \frac{(300)}{(350)} \times \frac{230^2}{230^2} = j0.085 \text{ p.u.}$$

Reactance of T<sub>L</sub>

$$X_{T_L, \text{new}} = X_{T_L, \text{old}} \times \frac{(MVA)_B}{(kV)_B^2}$$

$$= j0.5 \times 60 \times \frac{300}{230^2} = j0.181 \text{ p.u.}$$

Reactance of Transformer, T<sub>2</sub>

$$X_{T2, \text{new}} = X_{T2, \text{old}} \times \frac{(MVA)_B, \text{new}}{(MVA)_B, \text{old}} \times \frac{(kV)_{B, \text{old}}^2}{(kV)_{B, \text{new}}^2}$$

$$= j0.1 \times \frac{300}{300} \times \frac{(127.3)^2}{230^2} = j0.093 \text{ p.u.}$$

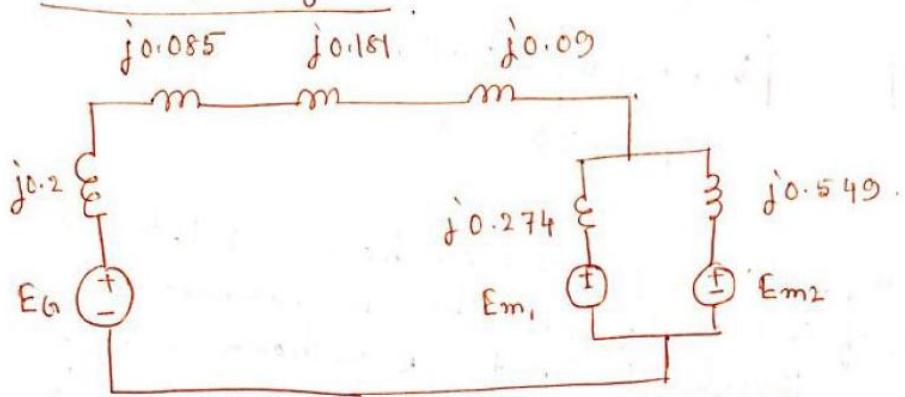
Reactance of motor M<sub>1</sub>

$$X_{M1, \text{new}} = X_{M1, \text{old}} \times \frac{(MVA)_B, \text{new}}{(MVA)_B, \text{old}} \times \frac{(KV)^2_{B, \text{old}}}{(KV)^2_{B, \text{new}}} \\ = j0.2 \times \frac{300}{200} \times \frac{13.2^2}{13.8^2} = j0.274 \text{ p.u}$$

Reactance of motor M<sub>2</sub>

$$X_{M2, \text{new}} = X_{M2, \text{old}} \times \frac{(MVA)_B, \text{new}}{(MVA)_B, \text{old}} \times \frac{(KV)^2_{B, \text{old}}}{(KV)^2_{B, \text{new}}} \\ = j0.2 \times \frac{300}{100} \times \frac{13.2^2}{13.8^2} = j0.549 \text{ p.u.}$$

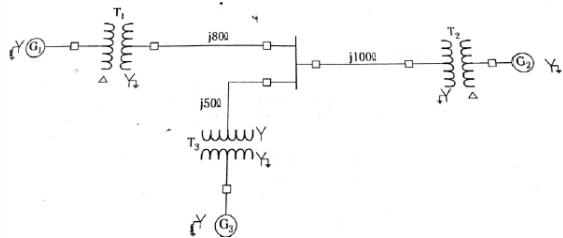
Reactance diagram



### Advantages of p.u computations:

- The greatest advantage of using p.u values is that it considerably simplified the calculations thus making the analysis of the system easier. Other advantages are:
  - (1) Per unit impedance of transformers is the same ref. to either side of it.
  - (2) The method of connection of transformers ( $\Delta-\Delta$ ,  $\Delta-Y$  etc) do not effect the p.u. impedance of the transformer.
  - (3) Manufacturers usually specify the impedance of an apparatus in p.u or percent value on the name plate based on the power rating and voltage rating of the apparatus. Rated impedance can be used directly in any analysis if the base chosen are the same as the name plate ratings of the apparatus.
  - (4) In case of machines absolute values (Ohmic) values of impedance may differ widely based on the constructing materials and the ratings of the machine. p.u impedance will lie within a narrow range. Therefore where actual values are not known, good approx value can be used.

2b



Reactance of  $T_{L1}$

$$j80 \text{ ohm line}, X_{T_{L1}} = X_{T_{L1}} (\Omega) \times \frac{(MVA)_B}{KV_B^2} \\ = j80 \times \frac{50}{220^2} = j0.083 \text{ p.u}$$

$$j100 \text{ ohm line } X_{T_{L2}} = X_{T_{L2}} (\Omega) \times \frac{(MVA)_B}{KV_B^2}$$

$$= j100 \times \frac{50}{220^2} = j0.1033 \text{ p.u.}$$

$$j50 \text{ ohm line } X_{T_{L3}} = X_{T_{L3}} (\Omega) \times \frac{50}{220^2} = j0.0516 \text{ p.u}$$

Reactance of transformer  $T_2$

This is a bank of 3 1φ transformer,  $(MVA)_B, \text{old} = 3 \times 10$

$$\therefore X_{T_2, \text{new}} = X_{T_2, \text{old}} \times \frac{50}{30} \times \frac{220^2}{220^2} = j0.1667 \text{ p.u.}$$

Reactance of Generator  $G_2$

This is connected to LT side of  $T_2$

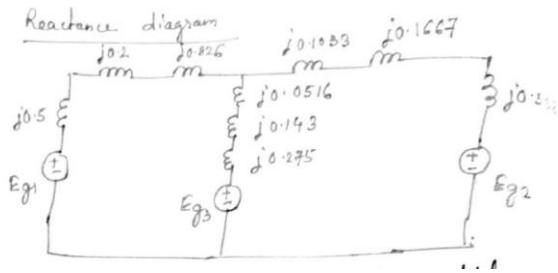
$$\therefore X_{G_2, \text{new}} = X_{G_2, \text{old}} \times \frac{50}{30} \times \frac{18^2}{18^2} = \\ = j0.2 \times \frac{50}{30} \times \frac{18^2}{18^2} = j0.333 \text{ p.u.}$$

Reactance of transformer  $T_3$

$$X_{T_3, \text{new}} = X_{T_3, \text{old}} \times \frac{(MVA)_B, \text{new}}{(MVA)_B, \text{old}} \times \frac{(KV)_B^2, \text{old}}{(KV)_B^2, \text{new}} \\ = j0.1 \times \frac{50}{35} \times \frac{22^2}{22^2} = j0.143 \text{ p.u}$$

Reactance of generator  $G_3$

$$X_{G_3, \text{new}} = X_{G_3, \text{old}} \times \frac{(MVA)_B, \text{new}}{(MVA)_B, \text{old}} \times \frac{(KV)_B^2, \text{old}}{(KV)_B^2, \text{new}} \\ = j0.2 \times \frac{50}{30} \times \frac{20^2}{22^2} = j0.275 \text{ p.u.}$$



3a

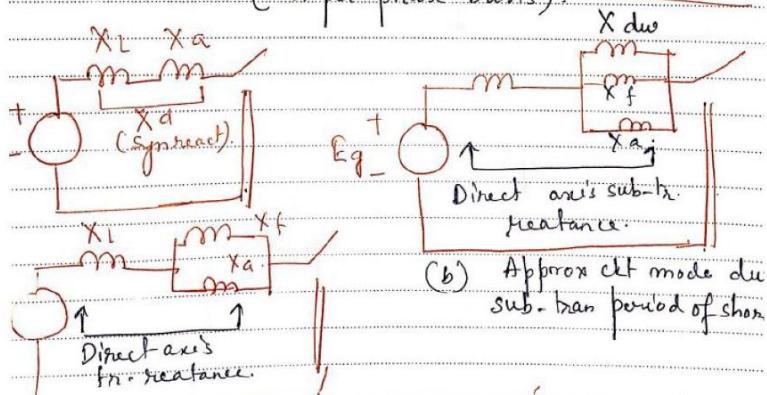
Under steady state short circuit conditions, the armature reaction of a synchronous generator produces a demagnetizing flux.

In terms of a circuit this effect is modelled as a reactance  $X_a$  in series with the induced emf.

This reactance when combined with leakage reactance  $X_L$  of the machine is called synchronous reactance  $X_d$  (direct axis's syn. reactance for salient pole machine).

Armature resistance being small can be neglected.

The steady state short circuit model of Syn M/C  
(on per phase basis)



Approx. clt model during tr. period of short ckt.

Let us consider, sudden short circuit (three phase) of a synchronous generator initially operating under open circuit conditions.

M/c undergoes a transient in all phases finally ending up in steady state.

CB must interrupt the current before steady conditions are reached.

Immediately upon short circuit, the off-set currents appear in all the three phases with a different magnitude since the form of the voltage wave at which short circuit occurs is different for each phase. These D.C offset currents are accounted for separately on an empirical basis and therefore concentrate on symmetrical (sinusoidal) short circuit currents.

Immediately in the event of a short circuit the symmetrical short circuit current is limited by the leakage reactance of the machine.

Since the air gap flux can not change instantaneously (theorem of constant flux linkage), to counter the demagnetization of the armature short circuit current, currents appear in the field windings as well as in the damper winding in a direction

to help the main flux. These currents decay in accordance with the winding time constants. The time constant of the damper winding which has low leakage inductance is much less than that of the field winding which has high leakage inductance. Thus during the initial part of the short circuit, the damper and field windings have transformer currents indeed in them so that in the circuit model their reactances —  $X_f$  for field winding  $X_{dw}$  — damper winding  $\rightarrow$  appear in parallel with  $X_a$ .

As the  $X_{dw}$  currents are first to die out,  $X_{dw}$  effectively becomes open circuited, at a later  $X_f \rightarrow$  becomes open circuited

The machine reactance thus changes from the parallel combination of  $X_a$ ,  $X_f$  and  $X_{dw}$  during the initial period of the short circuit to  $X_a$  and  $X_f$  in ||, in the middle period of short circuit and finally  $X_a$  in steady state.

The reactance presented by the machine in the initial period of the short circuit

$$X_d + \frac{1}{\left(\frac{1}{X_a} + \frac{1}{X_f} + \frac{1}{X_{dw}}\right)} = X_d''$$

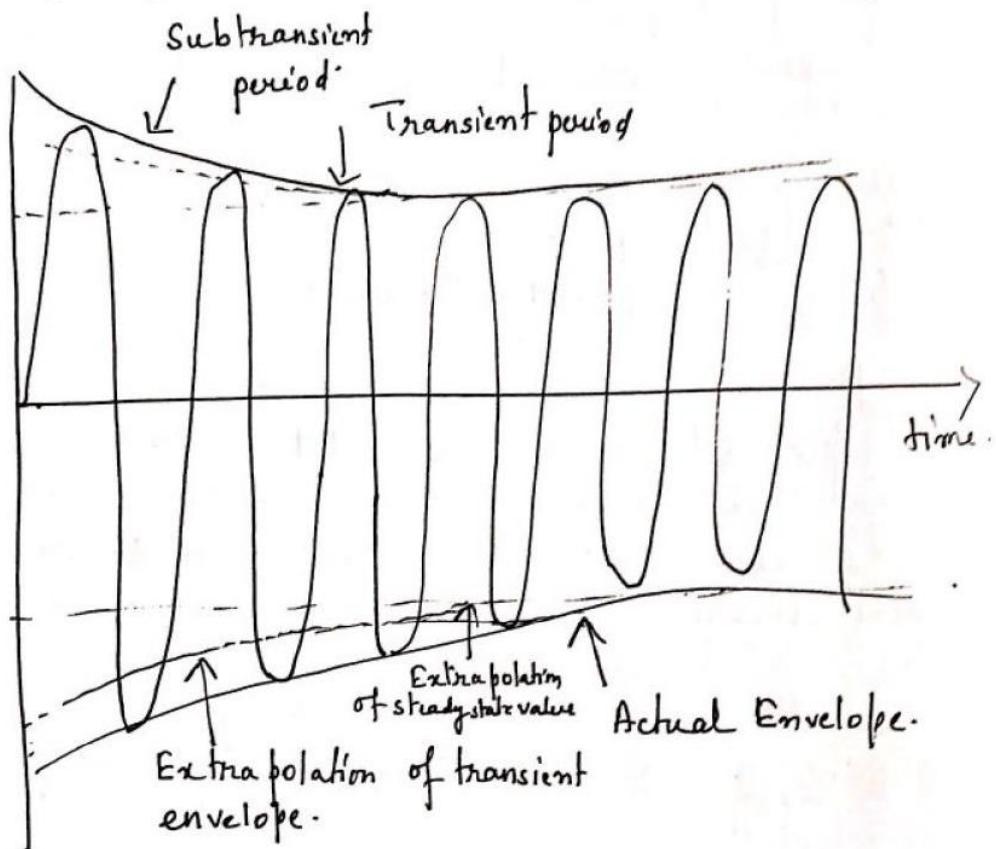
$X_d''$  = sub transient reactance of the machine.

After the damper winding currents have died,  
 $X'_d = X_L + (X_{al} \parallel X_f) \Rightarrow$  transient reactance

The reactance under steady conditions is the  
synchronous reactance.

$$X_d'' < X_d' < X_d$$

Machine offers a time varying reactance which  
 changes from  $X_d''$  to  $X_d'$  and finally to  $X_d$ .



- a) Symmetrical short circuit armature current in synchronous machine.

If we examine the oscillogram of the short current of a synchronous machine after the DC off-set currents have been removed from it, current wave form is as given in fig(1), envelope of current wave shape is fig(2).

The short circuit current can be divided into three periods — initial subtransient period, when the current is large as the machine offers subtransient reactance.

⇒ the middle transient period where the machine offers transient reactance

⇒ steady state period when the machine offers synchronous reactance.

\* If transient envelope is extrapolated backwards in time, the difference between the transient and subtransient envelopes is the current  $\Delta i''$  corresponding to damper winding ⇒ which decays first acc. to damper winding time constant.

\* If the difference  $\Delta i'$  between the steady state and transient envelopes decays in accordance with the field time constant.

$$|I| = \frac{|E_g|}{X_d} - i \quad I'' = \frac{|E_g|}{X_d''} - iii. \quad i)$$

$$|I'| = \frac{|E_g|}{X_{d'}} \rightarrow ii)$$

where  $|I| \Rightarrow$  steady state current (r.m.s)

$|I'| \Rightarrow$  transient current (r.m.s) excluding DC component.

$|I''| \Rightarrow$  subtransient current (r.m.s) excluding DC component.

$X_d \Rightarrow$  direct axis synchronous reactance.

$X_d' \Rightarrow$  " transient reactance.

$X_d'' \Rightarrow$  " subtransient reactance.

$|E_g| \Rightarrow$  per phase no load voltage (r.m.s)

$\Rightarrow$  Though machine reactance depends upon magnet saturation corresponding to excitation, the values normally lie within certain predictable limits for different types of machines.

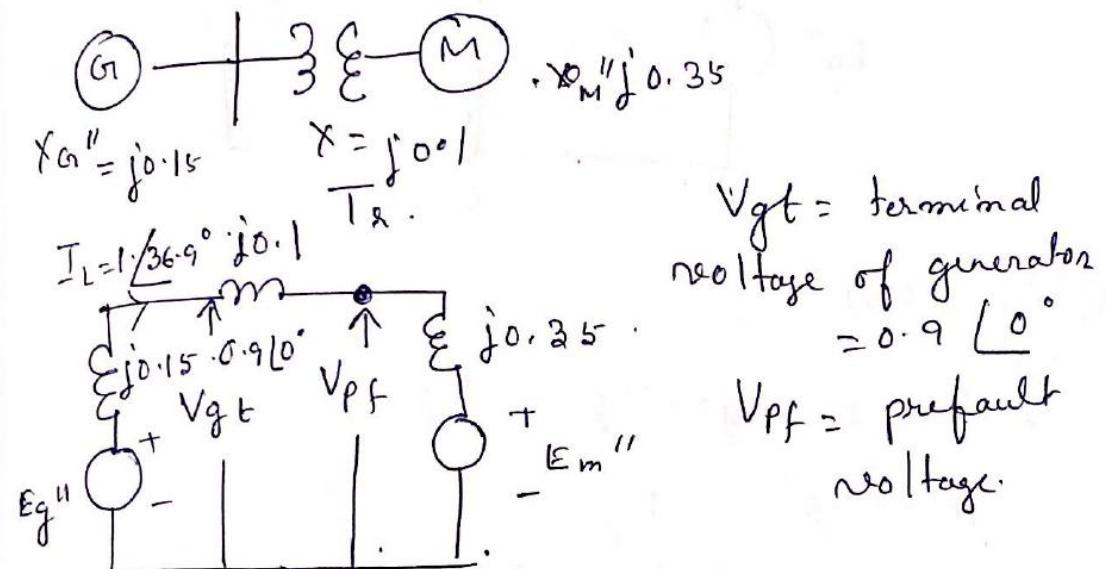
$\Rightarrow$  For both generator and motor  $X_d''$  are used to determine momentary current flowing on occurrence of a short circuit.

$\Rightarrow$  To decide interrupting capacity of circuit breakers except those which open instantaneously,  $X_d''$  for gen and  $X_d'$  for motors.

$X_d \Rightarrow$  for stability studies.

3b

Solution: SLD and circuit model to compute  
subtransient fault current



Prefault Condition

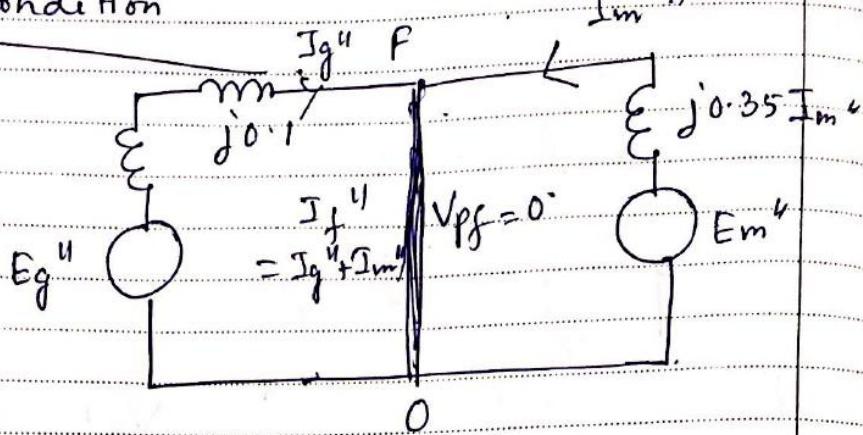
Applying KVL

$$\begin{aligned}
 E_g'' &= j0.15 I_L + V_{gt} \\
 &= 0.15 \angle 90^\circ \times 1 \angle 36.9^\circ + 0.9 \angle 0^\circ \\
 &= 0.8187 \angle 8.4^\circ \text{ p.u.}
 \end{aligned}$$

$$E_m'' + j0.35 I_L + j0.1 I_L = V_{gt}$$

$$\begin{aligned}
 E_m'' &= V_{gt} - 0.45 I_L \\
 &= 1.2243 \angle -17.1^\circ \text{ p.u.}
 \end{aligned}$$

### Fault Condition



Applying KVL.

$$j^{0.15} I_g'' + j^{0.1} I_g'' = E_g''$$

$$j^{0.25} I_g'' = E_g''$$

$$I_g'' = \frac{E_g''}{j^{0.25}} = \frac{0.8187 \angle 8.4^\circ}{0.25 \angle 90^\circ} = 3.2748 \angle -81.6^\circ$$

$$j^{0.35} I_m'' = E_m''$$

$$I_m'' = \frac{E_m''}{j^{0.35}} = 3.498 \angle -107.1^\circ \text{ p.u.}$$

$$I_f'' = I_g'' + I_m''$$

$$= 3.2748 \angle -81.6^\circ + 3.498 \angle -107.1^\circ$$

$$\approx 6.606 \angle -94.8^\circ \text{ p.u.}$$

Therefore CB has to withstand fault current from the instant of initiation of the fault to the time the arc is extinguished.

Two factors are of utmost importance of selection of CB.

- 1) The maximum instantaneous current that a breaker must withstand - (called momentary duty of CB)
- 2) The total current when the breaker contacts part (called interrupting current)



Two of the CB ratings which require the computation of SC current are : rated momentary current and rated symmetrical interrupting current.

- Symmetrical SC current is obtained by using subtransient reactance for synchronous machines.

Momentary (r.m.s) current is then calculated by multiplying the symmetrical momentary current by a factor of 1.6 to account for the presence of DC offset current.

Symmetrical current to be interrupted is computed by using  $x_d''$  for syn gen and  $x_d'$  for syn motors — induction motors are neglected.

DC offset value to be added to obtain the current to be interrupted is accounted for by multiplying the symmetrical SC current by a factor as follows:

CB speed-	Multiplying factor.
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8 cycles or slower

1.0

5 cycles

1.1

3 cycles

1.2

2 cycles

1.4.

speed of CB is the time between occurrences of the fault to the extinction of arc.

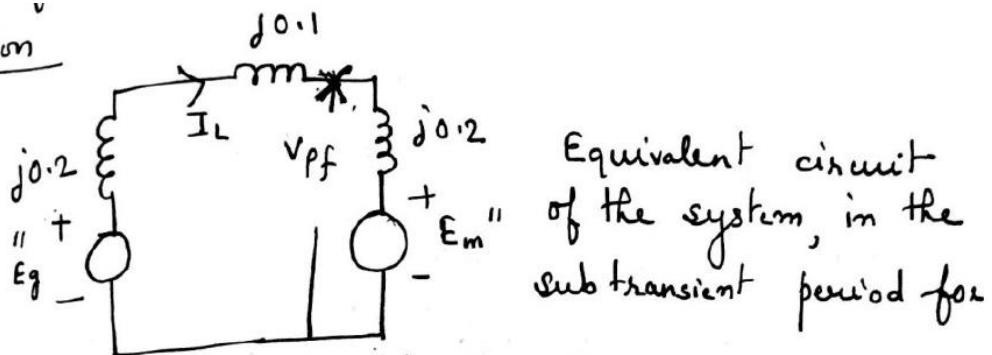
Normally specified in cycles of power-freq.

One cycle of for 50 Hz power freq is 0.02 ms.

The standard speed of CBs are 8, 5, 3, 2, 1½ cycles.

4b

solution



the value of the voltage sources are subtransient internal voltages.

Let the base values be  $(MVA)_B = 30 \text{ MVA}$  &  $(kV)_B = 13.2$

The prefault voltage at the fault point

$$V_{pf} = 12.8 \text{ kV.}$$

Let us use this as the ref. phasor  $\psi_0$  value of the prefault voltage.  $V_{pf} = \frac{12.8}{13.2} = 0.97 \angle 0^\circ$

$$\text{Base current } I_B = \frac{\text{Base Power}}{\sqrt{3} \times (\text{Base voltage})}$$

$$= \frac{30 \times 10^6}{\sqrt{3} \times 13.2 \times 10^3} = 1312 \text{ A.}$$

$$\text{The load current, } I_L = \frac{20,050}{\sqrt{3} \times 12.8 \times 0.8} / \cos^{-1} 0.8$$

$$= 1312.8 / 36.9^\circ \text{ A.}$$

$$\text{The load current } I_L \text{ in p.u.} = \frac{1,128}{1312} / 36.9^\circ$$

$$= 0.8594 / 36.9^\circ \text{ p.u.}$$

Method 1 : Using Kirchoff's Law.

The subtransient voltages  $E_g''$  &  $E_m''$  is calculated

$$E_g'' = j0.2 I_L + j0.1 I_L + V_{pf}$$

$$= j0.2 \times 0.8594 / 36.9^\circ + 90^\circ + 0.1 \times 0.8594 / 36.9^\circ$$

$$= 0.2578 / 126.9^\circ + 0.97 / 0^\circ + 0.97$$

$$= -0.1548 + j0.2062 + 0.97$$

$$= 0.8152 + j0.2062 = 0.84 / 14.2^\circ \text{ p.u}$$

$$E_m'' = V_{pf} - j0.2 I_L$$

$$= 0.97 - j0.2 \times 0.8594 / (36.9^\circ + 90^\circ)$$

$$= 0.97 - 0.17188 / 126.9^\circ$$

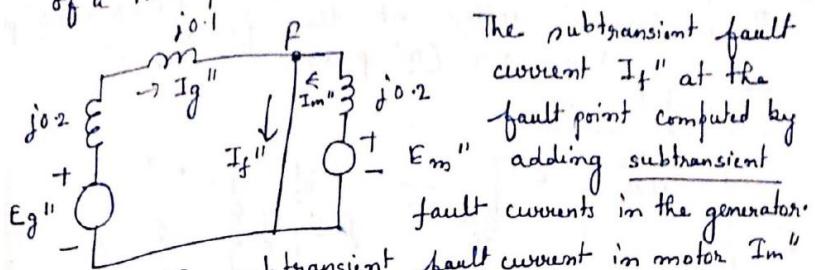
$$= 0.97 - (-0.1032 + j0.1374)$$

$$= 1.0732 - j0.1374$$

$$= 1.0819 / -7.3^\circ \text{ p.u.}$$

$$\begin{aligned}
 &= 0.97 - \underline{(-0.1032 + j0.1374)} \\
 &= 1.0782 - j0.1374 \\
 &= 1.0819 \underline{/-7.3^\circ} \text{ p.u.}
 \end{aligned}$$

The equivalent circuit of the system on the occurrence of a three phase fault.



The subtransient fault current  $I_f''$  at the fault point computed by adding subtransient fault currents in the generator.

$I_g''$  and the subtransient fault current in motor  $I_m''$

$$I_f'' = I_g'' + I_m''$$

Applying KVL in the circuit

$$\begin{aligned} j0.2 I_g'' + j0.1 I_g'' &= E_g'' \\ I_g'' &= E_g'' / j0.3 = \frac{0.84}{0.3} \underline{14.2^\circ} \end{aligned}$$

$$\begin{aligned} I_g'' &= 2.8 \underline{-75.8^\circ} \text{ p.u.} \\ \text{Also } j0.2 I_m'' &= E_m'' \quad \therefore I_m'' = \frac{E_m''}{j0.2} = 1.0819 \underline{-7.3^\circ} \\ &= 0.2 \underline{190^\circ} \end{aligned}$$

$$= \frac{1.0819}{0.2} \underline{-7.3^\circ}$$

$$= 5.4095 \underline{-97.3^\circ} \text{ p.u.}$$

$$\begin{aligned}
 \therefore \text{current at fault point } I_f'' &= I_g'' + I_m'' \\
 &= 2.8 \underline{-75.8^\circ} + 5.4095 \underline{-97^\circ} \\
 &= 0.687 - j2.7 - 0.687 - j5.365 \\
 &= -j8.065 = 8.065 \underline{-90^\circ} \text{ p.u.}
 \end{aligned}$$

Thus Absolute values:

$$\text{Subtransient fault current in generator } I_g'' = 2.8 \angle -75.8^\circ \times 1312 \\ = 3673.6 \angle -75.8^\circ \text{ A}$$

$$\text{Subtransient fault current in motor } I_m'' = 5.4095 \angle -97.3^\circ \times 1312 \\ = 7097.2 \angle -97.3^\circ \text{ A}$$

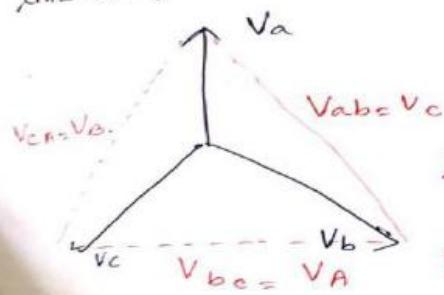
Subtransient fault current at the fault point

$$I_f'' = 8.065 \angle -90^\circ \times 1312 = 10581.3 \angle -90^\circ$$

5a

### Q) Line voltages?

- \* By sequence voltages if it is always meant phases of a star or an equivalent star connected system.
- \* star connected system phase voltage differs from line voltage:  $V_{LL} = \sqrt{3} V_p$ .



Net phase seq = ABCabc  
 $V_a, V_b, V_c \Rightarrow$  phases,

line voltages  $V_{bc}$ ,  $V_{ca}$ , and  $V_{ab}$ .

$$V_{bc} = V_c - V_b$$

$$V_{ca} = V_a - V_c$$

$$V_{ab} = V_b - V_a$$

Let  $V_{bc} = V_A$  (opposite to vertex A).

$V_{ca} = V_B$  (n n n B)

$V_{ab} = V_C$  (n n n C)

$$V_A = V_{BC} = V_C - V_B$$

$$V_B = V_{CA} = V_A - V_C$$

$$V_C = V_{AB} = V_B - V_A$$

+ve seq. component of line voltages are.

$$V_{A1} = \frac{1}{3} (V_A + \alpha V_B + \alpha^2 V_C)$$

$$= \frac{1}{3} (V_C - V_B + \alpha (V_A - V_C) + \alpha^2 (V_B - V_A))$$

$$= \frac{1}{3} [\alpha [V_A + \alpha V_B + \alpha^2 V_C] - \alpha^2 [V_A + \alpha V_B + \alpha^2 V_C]]$$

$$= \frac{1}{3} (\alpha - \alpha^2) (V_A + \alpha V_B + \alpha^2 V_C)$$

$$= \frac{1}{3} (\alpha - \alpha^2) \times 3 V_{a1}$$

$$\therefore V_{A1} = \frac{1}{3} (j\sqrt{3}) (3 V_{a1}) \quad \alpha - \alpha^2 = j\sqrt{3}$$

$$\therefore V_{A1} = j\sqrt{3} V_{a1}$$

Hence +ve seq. component of line voltage is  $\sqrt{3}$  times the +ve seq. component of phase voltage and leads the corresponding phase voltage by  $90^\circ$ .

The -ve seq. of line voltage is

$$\begin{aligned}
 V_{A2} &= \frac{1}{3} (V_A + \alpha^2 V_B + \alpha V_C) \\
 &= \frac{1}{3} [(V_C - V_B) + \alpha^2 (V_A - V_C) + \alpha (V_B - V_C)] \\
 &= \frac{1}{3} [\alpha^2 (V_A + \alpha^2 V_B + \alpha V_C) - \alpha (V_A + \alpha^2 V_B + \\
 &\quad \alpha^2 - \alpha) (V_A + \alpha^2 V_B + \alpha V_C) \\
 &= \frac{1}{3} (-j\sqrt{3}) (2V_{a2}),
 \end{aligned}$$

$$V_{A2} = -j\sqrt{3} V_{a2}.$$

-ve seq. component of line voltage is  $j\sqrt{3}$  to the negative seq. comp. of phase voltage. Lag the corresponding phase voltage by 90°.

$$\begin{aligned}
 V_{A2} &= \frac{1}{3} [V_A + \alpha^2 V_B + \alpha V_C] \\
 &= \frac{1}{3} [(V_C - V_B) + \alpha^2 (V_A - V_C) + \alpha (V_B - V_C)] \\
 &= \frac{1}{3} [\alpha^2 (V_A + \alpha^2 V_B + \alpha V_C) - \\
 &\quad \alpha (V_A + \alpha^2 V_B + \alpha V_C)] \\
 &= \frac{1}{3} (\alpha^2 - \alpha) [V_A + \alpha^2 V_B + \alpha V_C] \\
 &= \frac{1}{3} (-j\sqrt{3}) (2V_{a2})
 \end{aligned}$$

$$V_{A2} = -j\sqrt{3} V_{a2}.$$

Finally the zero seq. component of line voltage

$$\begin{aligned}
 V_{A0} &= \frac{1}{3} (V_A + V_B + V_C) \\
 &= \frac{1}{3} [(V_C - V_B) + (V_A - V_C) + (V_B - V_A)] \\
 &= 0
 \end{aligned}$$

$V_{A0} = 0$ .  $\Rightarrow$  zero seq. component of line voltage is zero.

Solution

The base power for the entire system is

$$(MVA)_{B, \text{new}} = 50 \text{ MVA}$$

Base voltage on Transmission lines = 220 KV.

$$\text{Gen 1} = 220 \times \frac{11}{220} = 11 \text{ KV}$$

$$\text{Gen 2} = 220 \times \frac{11}{220} = 11 \text{ KV}$$

Sequence reactance of gen:

Since the ratings of the machines are same, their reactance are also same.

+ve seq. reactance =  $X_{G1} = \text{Subtransient reactance on new base.}$

$$= X'' \times \frac{(MVA)_{B, \text{new}}}{(MVA)_{B, \text{old}}} \times \frac{KV^2_{B, \text{old}}}{KV^2_{B, \text{new}}}$$

$$= j^{0.2} \times \frac{50}{25} \times \frac{11^2}{11^2} = j^{0.4} \text{ p.u.}$$

-ve seq. reactance  $X_{G2} = \text{subtransient reactance on new base.} = j^{0.4} \text{ p.u.}$

Z ero sequence reactance = 8% on new base.

$$= j^{0.08} \times \frac{50}{25} \times \frac{11^2}{11^2} = j^{0.16} \text{ p.u.}$$

Since the ratings of all transformer are identical, their seq. reactance are identical.  
All the 3 seq. reactance of transformer are same.

+ve seq. reactance = -ve seq. reactance = zero seq. react.

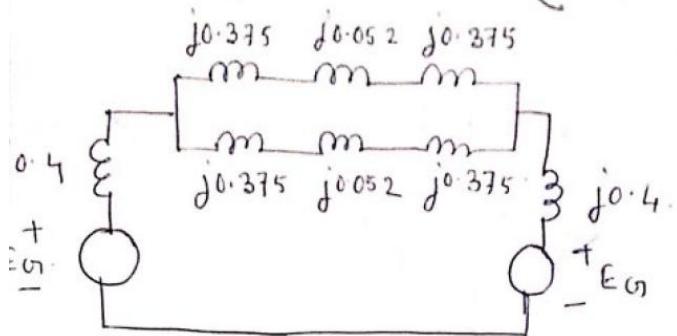
$$X_{pu} = j0.15 \times \frac{50}{20} \times \frac{220^2}{220^2} = j0.375 \text{ pu}$$

Sequence reactance of transmission lines.

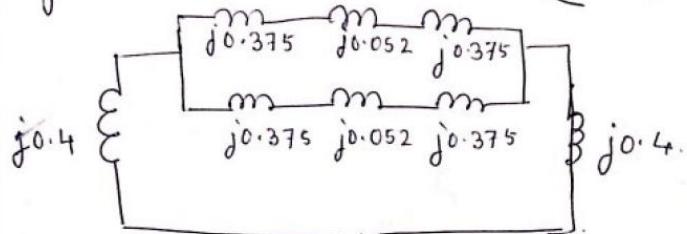
static device. Hence  $X_{TL1} = X_{TL2}$ .

$$X_{TL1} = X_{TL2} = X_{TL}(p.u) \times (\text{MVA})_{G, new}$$

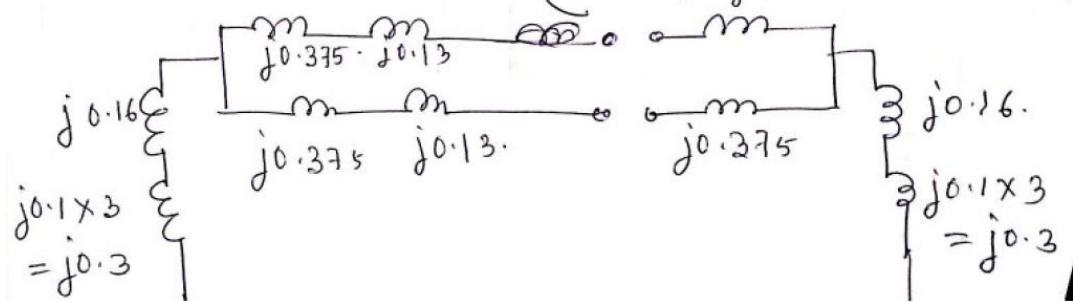
Positive Sequence Network (PSN).

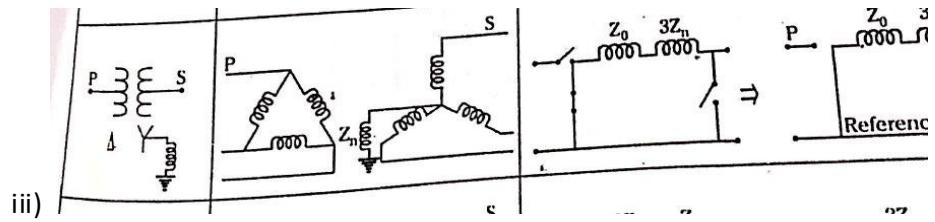
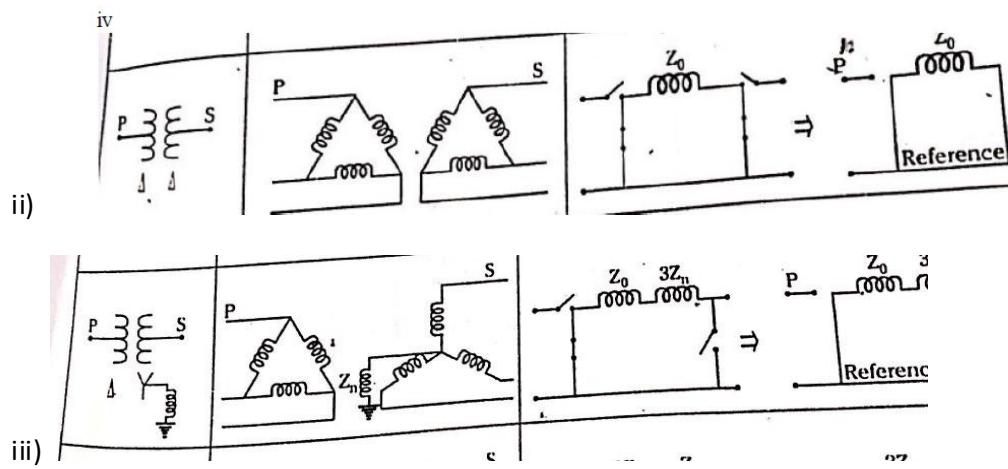
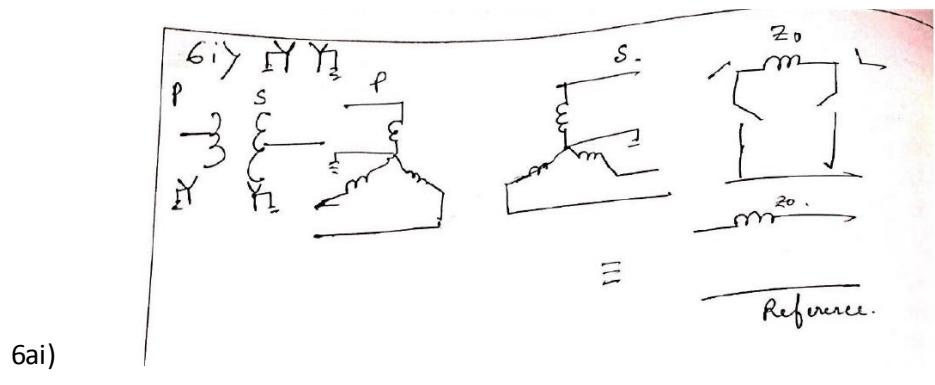


Negative Sequence Network (NSN).



Zero Sequence Network (ZSN).  $j0.375$





Solution

$$\text{Given: } I_{a1} = 20 \angle 10^\circ = (19.69 + j3.47) A$$

$$I_{a2} = 6 \angle 60^\circ = (3 + j5.19) A$$

$$I_{ao} = 3 \angle 30^\circ = (2.6 + j1.5) A.$$

Line current

$$I_a = I_{ao} + I_{a1} + I_{a2}$$

$$= (2.6 + j1.5) + (19.69 + j3.47) + (3 + j5.19)$$

$$= 27.25 \angle 21.88^\circ A$$

$$\begin{aligned} I_b &= I_{ao} + a^2 I_{a1} + a I_{a2} \\ &= 3 \angle 30^\circ + 20 \angle (10^\circ + 240^\circ) + 6 \angle (60^\circ + 120^\circ) \\ &= 2.59 + j1.5 + (-6.84 - j18.79) + (-6) \\ &= -10.25 - j17.29 A = 20.1 \angle -120.7^\circ A \end{aligned}$$

$$\begin{aligned} I_c &= I_{ao} + a I_{a1} + a^2 I_{a2} \\ &= 3 \angle 30^\circ + 20 \angle (10^\circ + 120^\circ) + 6 \angle (60^\circ + 240^\circ) \\ &= (2.59 + j1.5) + (-12.86 + j15.32) + (3 - j5.2) \\ &= (-7.27 + j11.62) A = 13.7 \angle 122^\circ A. \end{aligned}$$

3.21 The total 3φ power in p.u is given as

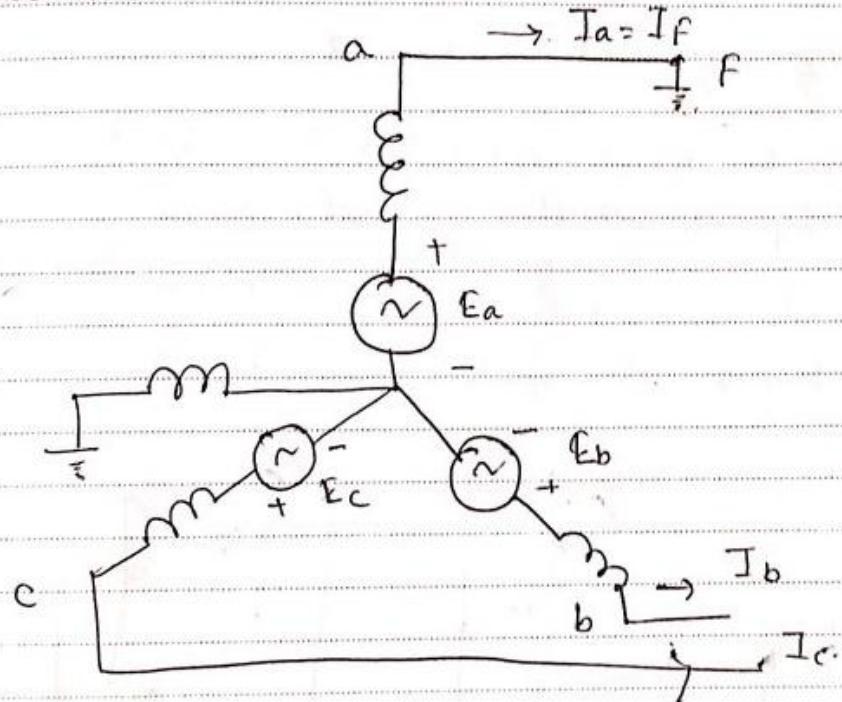
$$\begin{aligned}
 S_{pu} &= V_{ao} I_{ao}^* + V_{ai} I_{ai}^* + V_{a2} I_{a2}^* \\
 &= 0.12 \left[ 300^\circ \times 0.1 \right] \underbrace{-330^\circ}_{\text{}} + 0.9 \left[ 110^\circ \times 0.75 \right] \underbrace{-25^\circ}_{\text{}} \\
 &\quad + 0.25 \left[ 110^\circ \times 0.15 \right] \underbrace{-170^\circ}_{\text{}} \\
 &= 0.012 \left[ -30^\circ \right] + 0.675 \left[ -15^\circ \right] + 0.0375 \left[ -60^\circ \right] \\
 &= 0.68 - j0.212 \text{ p.u.}
 \end{aligned}$$

If neutral get disconnected new  $I_{ao} = 0$ ,  
new power is.

$$\begin{aligned}
 S_{pu}' &= V_{ai} I_{ai}^* + V_{a2} I_{a2}^* \\
 &= 0.9 \left[ 10^\circ \times 0.75 \right] \underbrace{-25^\circ}_{\text{}} + 0.25 \left[ 110^\circ \times \right. \\
 &\quad \left. 0.15 \right] \underbrace{-170^\circ}_{\text{}} \\
 &= 0.67 - j0.206 \text{ p.u.}
 \end{aligned}$$

7a

Single line to Ground (LG<sub>1</sub>) fault on an unloaded generator:



Terminal Conditions:  $V_a = 0$

$$I_b = 0$$

$$I_c = 0$$

## Symmetrical Component Relations.

$$I_{ao} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3}I_a$$

$$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c) = \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3}I_a$$

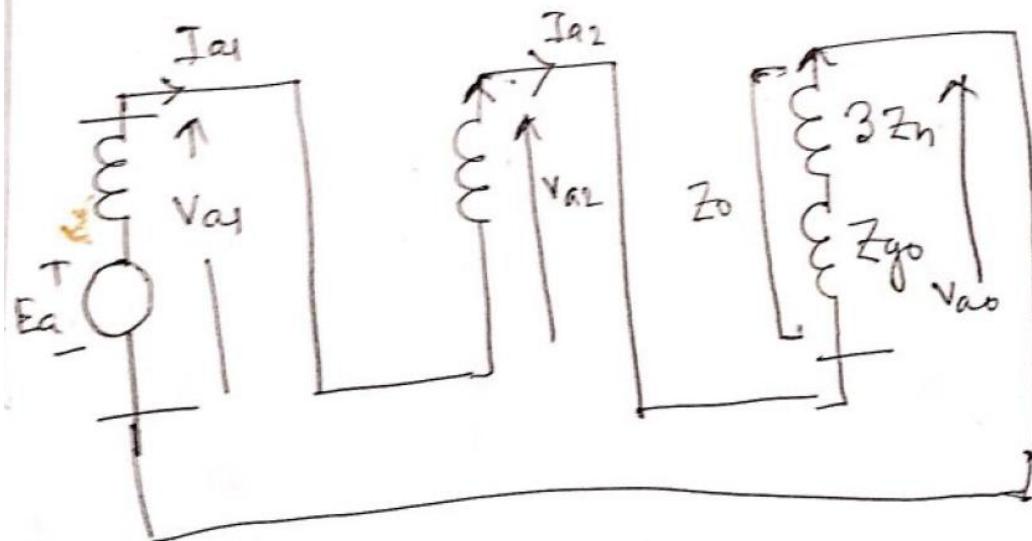
$$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c) = \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3}I_a$$

$$I_{a1} = I_{a2} = I_{ao} = \frac{1}{3}I_a \quad \text{--- (1)}$$

$$V_a = 0 \Rightarrow V_{ao} + V_{a1} + V_{a2} = 0 \quad \text{--- (2)}$$

As all sequence currents are equal and as the sum of seq. voltage equals zero.

Equation suggest a series connection of seq. networks through a short circuit



Interconnection of Sequence Networks

### Sequence quantities.

$$I_{a1} = I_{a2} = I_{ao} = \frac{E_a}{(z_1 + z_2 + z_0)}.$$

$$\begin{aligned} V_{a1} &= E_a - I_{a1} z_1 \\ &= E_a - \frac{E_a}{z_1 + z_2 + z_0} \times z_1 \\ &= E_a \times \left( \frac{z_2 + z_0}{z_1 + z_2 + z_0} \right). \end{aligned}$$

$$V_{a2} = I_{a2} z_2 = - \left( \frac{E_a z_2}{z_1 + z_2 + z_0} \right).$$

$$V_{ao} = - I_{ao} z_0 = - \left( \frac{E_a z_0}{z_1 + z_2 + z_0} \right).$$

### Fault Current

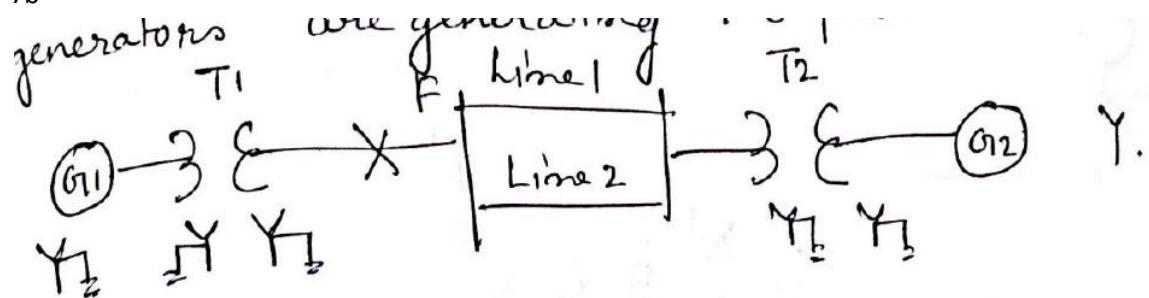
$$I_f = I_a = 3 I_{ao} = 3 \times \frac{E_a}{z_1 + z_2 + z_0}.$$

$$Z_o = Z_{go} + 3 Z_n = Z_{go} + \infty = \infty$$

$$I_f = 3 \times \frac{E_a}{z_1 + z_2 + \infty} = 0.$$

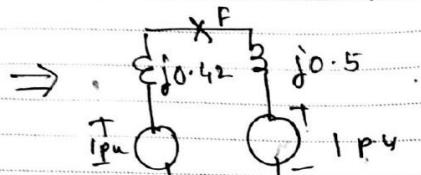
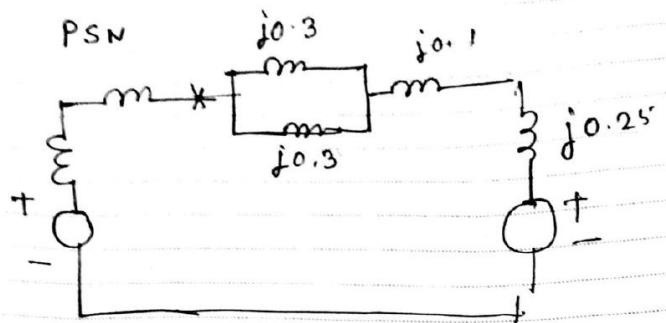
fault current in the sys is zero if the neutral is not grounded in case of an L<sub>G</sub> fault.

7b

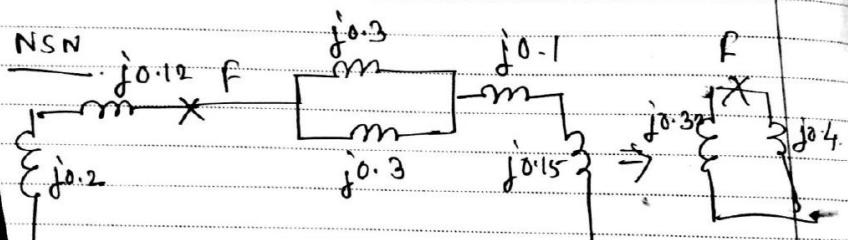


Component	$X_0$	$X_1$	$X_2$
$G_{11}$	0.05	0.30	0.20
$G_{12}$	0.03	0.25	0.15
Line - 1	0.70	0.30	0.30
Line - 2	0.70	0.30	0.30
$T_1$	0.12	0.12	0.12
$T_2$	0.10	0.10	0.10

Solution :



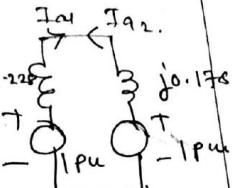
$$Z_{1\text{Th}} = \frac{(j0.42) \times (j0.5)}{j0.42 + j0.3} = j0.228 \text{ p.u.}$$



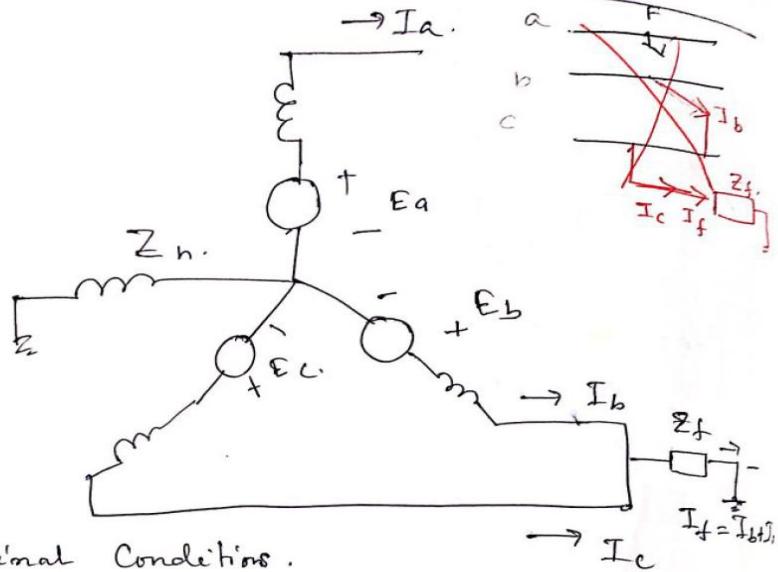
$$Z_{2\text{Th}} = \frac{(j0.32) \times (j0.4)}{j0.32 + j0.4} = j0.178 \text{ p.u.}$$

The interconnection of seq. m/w during LL fault

$$I_{A1} = -I_{A2} = \frac{1}{j0.228 + j0.178} \cdot j0.228 \times \frac{j0.228}{j0.228 + j0.178} = -j2.463 \text{ p.u.}$$



Double line to ground fault (LLG<sub>f</sub>) on an unloaded generator through a fault impedance.



Terminal Conditions.

$$I_a = 0$$

$$V_b = (I_b + I_c) Z_f$$

$$V_c = (I_b + I_c) Z_f$$

### Symmetrical component theorem

$$V_{a1} = \frac{1}{2} (V_a + dV_b + d^2 V_c).$$

$$\Rightarrow \frac{1}{2} [V_a + (d+d^2)V_b] = \frac{1}{2} [V_a - V_b]$$

$$V_{a2} = \frac{1}{2} [V_a + d^2 V_b + d V_c].$$

$$\Rightarrow \frac{1}{2} [V_a + (d^2+d)V_b] = \frac{1}{2} (V_a - V_c)$$

$$V_{ao} = \frac{1}{3} (V_{a1} + V_{a2} + V_c) = \frac{1}{3} (V_a + 2V_b)$$

$$V_{a1} = V_{a2}$$

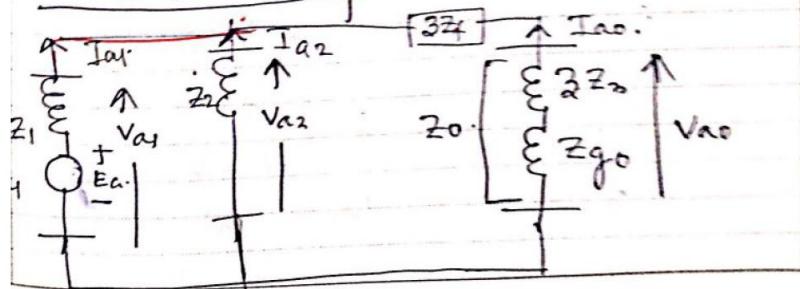
$$V_{ao} - V_{a2} = \frac{1}{3} (2V_b) = V_b.$$

$$= (I_b + I_c) Z_f = 3 I_{ao} Z_f \\ = I_{ao} + 3 Z_f$$

$$V_{ao} = V_{a2} + 3 I_{ao} Z_f.$$

$$I_a = 0, \quad I_{ao} + I_{a1} + I_{a2} = 0$$

### Interconnection of sequence networks.



### Sequence Quantities

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2 (3Z_f + Z_0)}{Z_2 + 3Z_f + Z_0}}$$

$$I_{a2} = -\frac{I_{a1} (Z_0 + 3Z_f)}{(Z_0 + Z_2 + 3Z_f)}, \quad I_{ao} = -\frac{I_{a1} Z_2}{(Z_0 + Z_2 + 3Z_f)}, \quad \left. \begin{array}{l} \text{Current} \\ \text{division} \\ \text{formula.} \end{array} \right\}$$

### Fault Current

$$\begin{aligned} I_f &= I_b + I_c \\ &= (I_{ao} + \alpha^2 I_{a1} + \alpha I_{a2}) + (I_{ao} + \alpha I_{a1} + \alpha^2 I_{a2}) \\ &= 2I_{ao} + (\alpha^2 + \alpha) I_{a1} + (\alpha + \alpha^2) I_{a2} \\ &= 2I_{ao} - I_{a1} - I_{a2} \\ &= 2I_{ao} - (I_{a1} + I_{a2}) \\ &= 2I_{ao} - (I_{ao}) = 3I_{ao}. \end{aligned}$$

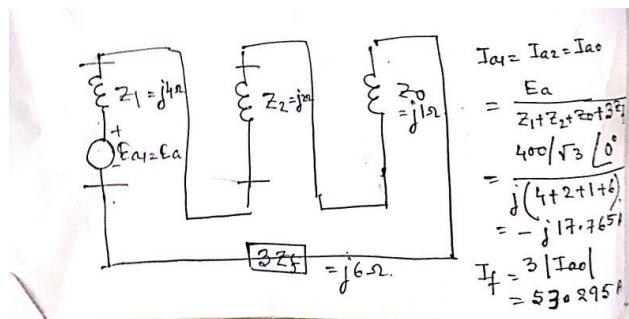
$$I_f = -3I_{a1} \left( \frac{Z_2}{Z_0 + Z_2 + 3Z_f} \right)$$

$Z_n = \infty, \quad Z_0 \Rightarrow \infty, \quad I_f = 0$

Solution

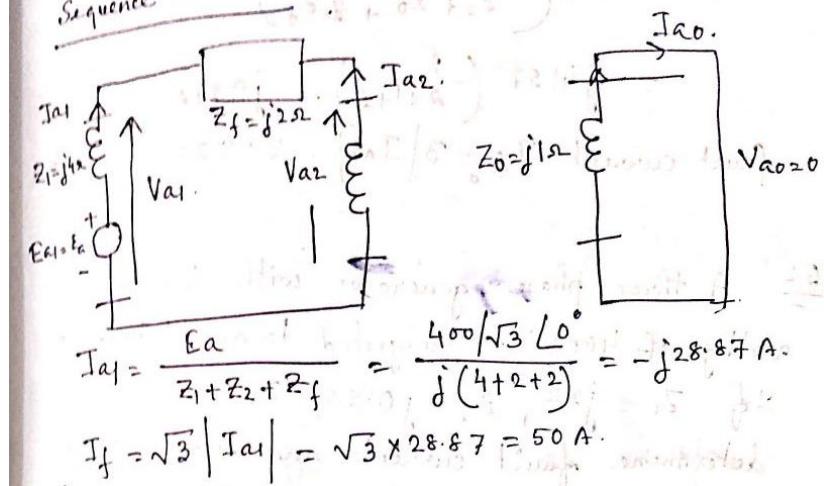
i) L G<sub>1</sub> fault

Sequence network



ii) LL fault

Sequence Network



### Equal Area Criterion (EAC).

Transient stability is the ability of the system to remain stable under large disturbances like short circuits, line outage, generation or load los.

⇒ evaluation of T.S required for planning designing power system.

⇒ Analysis deals with actual solution of non-linear differential equations describing the dynamics of m/fcs and their controls.

⇒ stability for a given fault can be established by solving the swing equation.

⇒ But laborious.

⇒ Simple systems like Single M/C connected to an Infinite Bus bar (SMIB) transient stability analysis can be carried out by

### Equal Area Criterion (EAC).

⇒ It provides qualitative analysis of stability of syn m/fc. without

swing equation

Consider swing equation of a single machine connected to an infinite bus.

$$M \frac{d^2\delta}{dt^2} = P_a.$$

Multiplying both sides of the equation by

$$\frac{2}{M} \frac{d\delta}{dt}$$
, we get-

$$2 \frac{d\delta}{dt} \times \frac{d^2\delta}{dt^2} = \frac{2}{M} P_a \frac{d\delta}{dt}$$

$$\frac{d}{dt}(x^2) = 2x \frac{dx}{dt}.$$

$$\text{or } \frac{d}{dt} \left( \frac{d\delta}{dt} \right)^2 = \frac{2}{M} P_a \frac{d\delta}{dt}$$

$$\left( \frac{d\delta}{dt} \right)^2 = \frac{2}{M} \int_{\delta_0}^{\delta} P_a \frac{d\delta}{dt} \cdot dt$$

$$= \frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta}$$

$$\frac{d\delta}{dt} = 0 \cdot i.e. \sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta} = 0$$

$$\int_{\delta_0}^{\delta} P_a d\delta = 0.$$

The physical meaning of integration is the estimation of the area under the curve.

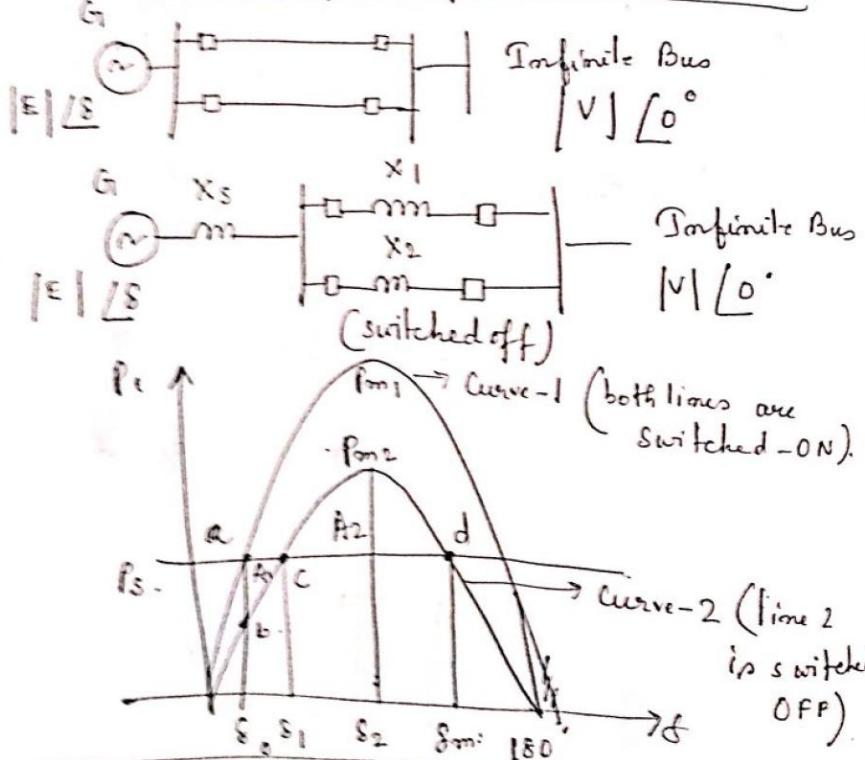
\* If the system is stable, the area under  $P_a - s$  curve reduces to zero for some value of  $s$ .

\* Possible when  $P_a$  has both +ve (acceleration) and (-ve) decelerating powers.

$$+ve \text{ area} = -ve \text{ area}$$

$\Rightarrow$  Equal area criterion for stability

b) Sudden Loss of one of the Parallel Lines :



Case (1) Before switching OFF, the power angle equation is

$$P_{e1} = \frac{|E||V|}{X_s + (X_1 || X_2)} \sin\delta = P_m \sin\delta \dots \text{curve 1.}$$

Case (2) on switching off, line 2.

$$P_{e2} = \frac{|E||V|}{X_s + X_1} \sin\delta = P_m \sin\delta \rightarrow \text{curve 2.}$$

$$P_{m2} < P_{m1}, \text{ as } (X_s + X_1) > (X_s + (X_1 || X_2)).$$

As soon as line 2 was switched off, original operating point a on curve -1 is shifted to point b on curve -2.

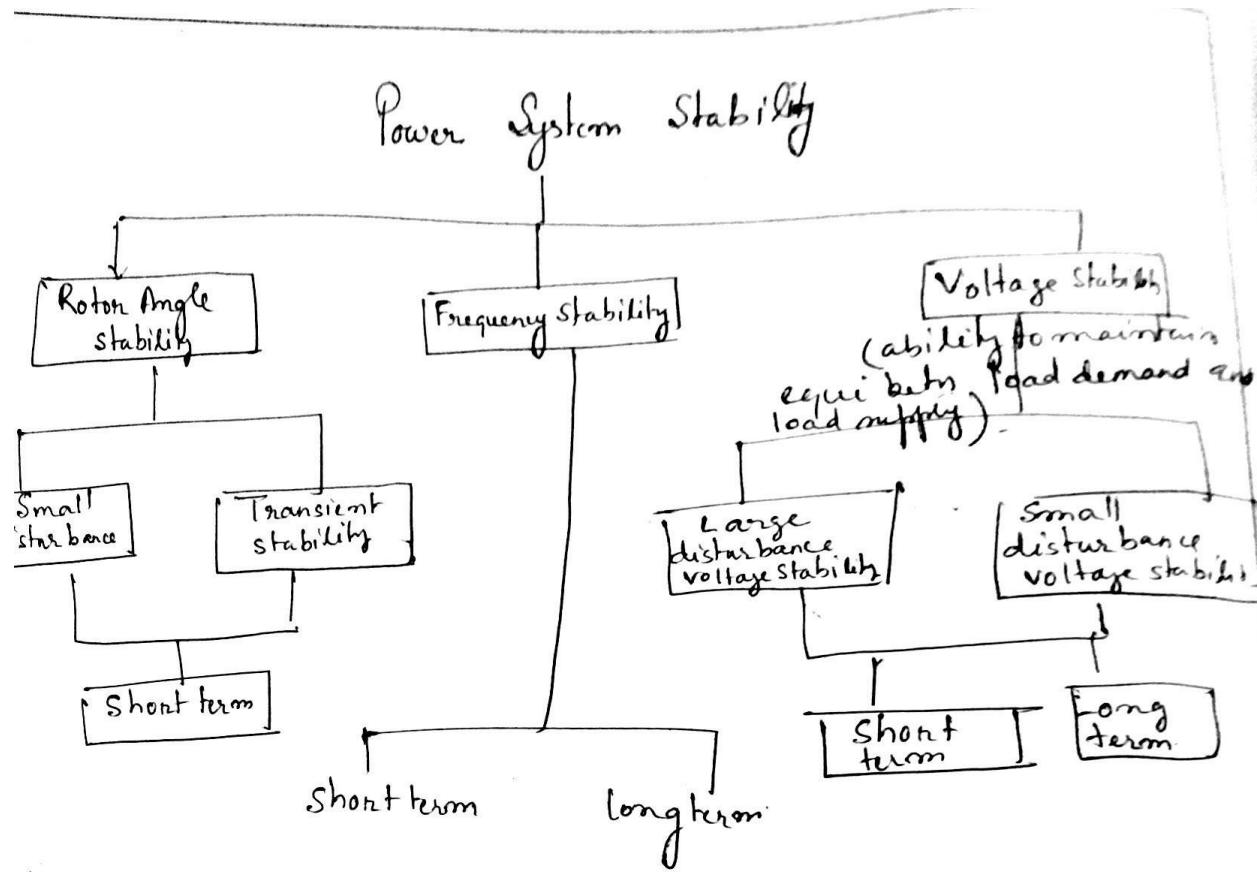
Accelerating energy corresponding to area A, is put into rotor followed by decelerating energy.

If area  $A_2 = A_1$ , then stable finally operates at C  $\Rightarrow \delta_2 = \delta_1$ .

$$\delta_1 > \delta_0$$

$$\delta_2 = \delta_{2n} = 180^\circ - \delta_1$$

**Power system stability** is the ability of an electric power system for a given initial operating condition to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded, so that practically the entire system remains intact.



**Rotor Angle Stability:** Under steady state conditions there is equilibrium between mechanical I/P and electrical o/p of generator and its speed remains constant.

- Major disturbance upsets this equilibrium.
- Generator accelerates acc. to the mechanics of rotating body

Rotor angle stability refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after subjected to a disturbance. Further classified as :

(a) Small Signal Rotor angle Stability / steady state stability.

It is the ability of the power system to maintain synchronism under small disturbances.

• Stability depends only on initial operating state of the system and not on the disturbance.

(b) Large Signal Rotor angle stability or Transient stability This refers to the ability of the power system to maintain synchronism under sudden large disturbances due to sudden load change, heavy switching operation, loss or generation or faults in system

9c

Solution The MVA rating of the alternator:

$$= G_1 = \frac{100}{0.85} = 117.65 \text{ MVA}$$

$$\text{Kinetic energy} = G_1 H = \frac{1}{2} I \omega^2$$

$$I = 10,000, \quad \omega = 2\pi \times 50 = 100\pi \text{ rad/sec}$$

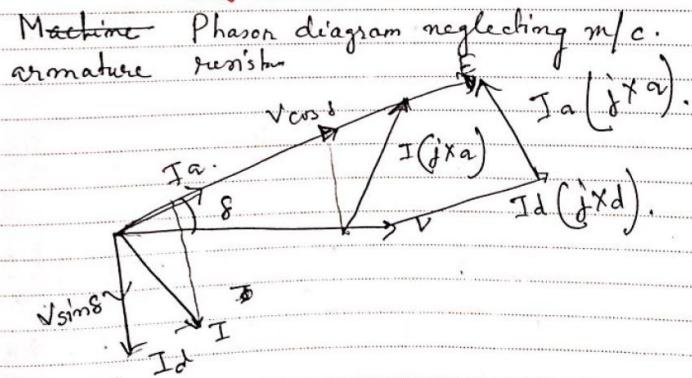
$$G_1 = 117.65 \text{ MVA}$$

$$(117.65) \times H = \frac{1}{2} \times \frac{10,000 \times (100\pi)^2}{10^6} \text{ MJ}$$

$$H = 41.9 \text{ MJ/MVA.}$$

$$\begin{aligned}
 M &= G_1 H / 180f = \frac{117.65 \times 41.9}{180 \times 50} \\
 &= 0.0548 \text{ MJ s / Elect. deg.}
 \end{aligned}$$

Power Angle Equation of a Salient Pole Synchronous Machine



$E \angle \alpha \Rightarrow$  Generated emf in the syn m/c.  
 $V \angle \theta$  = Bus bar voltage (taken as ref).

$X_d$  = direct axis syn reactance.

$X_q$  = quadrature axis syn reactance.

I = current delivered at lagging Pt. Q.

$$P = |V| \cos \theta |I_d| + |V| \sin \theta |I_q|.$$

$$|I_d X_{d\text{ref}}| = |V \sin \theta|.$$

$$|I_d| = \frac{|V| \sin \theta}{X_d}.$$

$$|I_d X_d| = |E - V \cos \theta|.$$

$$|I_d| = \frac{|E| - |V| \cos \theta}{X_d}.$$

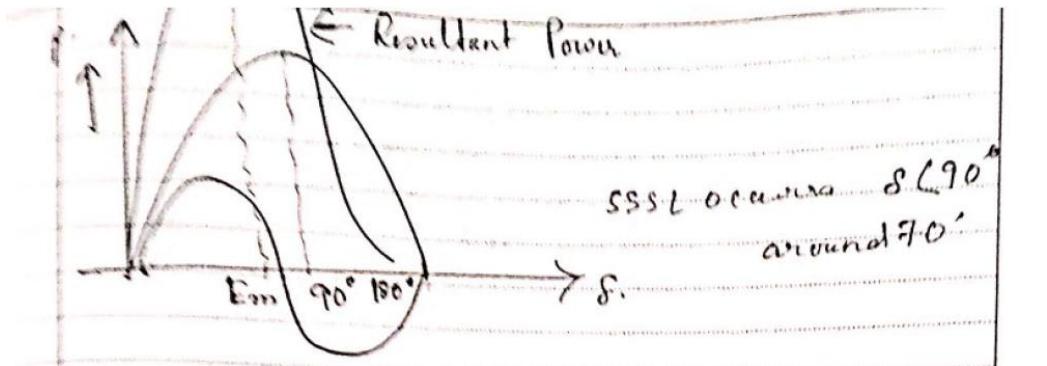
$$P = |V| \cos \theta \left( \frac{|V| \sin \theta}{X_d} \right) + |V| \sin \theta + \frac{(|E| - |V| \cos \theta)}{X_d}.$$

$$= \frac{|V|^2 \sin 2\theta}{2 X_d} + \frac{|V| |E| \sin \theta}{X_d} \quad \leftarrow \frac{\sqrt{2} \sin 2\theta}{2 X_d}$$

$$= |V|^2 \frac{\sin 2\theta}{2} \left( \frac{1}{X_d} - \frac{1}{X_q} \right) + |V| |E| \frac{\sin \theta}{X_d}$$

$$P = \frac{|V| |E| \sin \theta}{X_d} + \frac{|V|^2 \sin 2\theta}{2} \left( \frac{X_d - X_q}{X_d \cdot X_q} \right)$$

Reluctance Power.



10b

### Rotor Dynamics and swing Equation

Mechanical Power I/P =  $P_s$  at torque  $T_s$   
 motor speed  $\omega$  via shaft from prime mover.

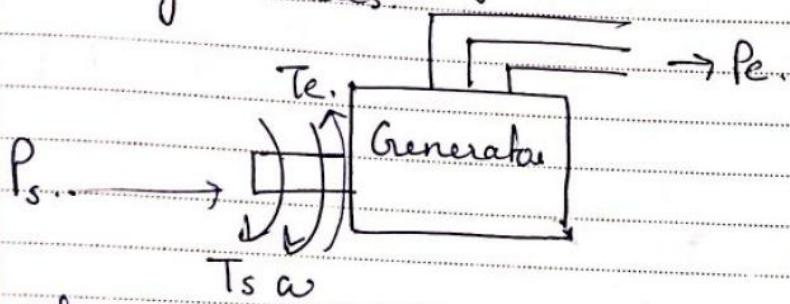
Electrical power  $P_e$  delivered to power system  
 via bus bars

Generator develops electro mechanical torque  
 $T_e$  in opposition to  $T_s$ .

when  $T_e = T_s \Rightarrow$  rotor would be in stable running position at syn speed.  $\Rightarrow$  phase sync with the grid bus.

$\Rightarrow$  When syn done gen can be connected to the system bus.

generator is in synchronism with grid system bus



When there is a torque difference, resultant torque will accelerate the rotor of the generator.

winding and friction losses  $\rightarrow$  negligible.

$$T_a = T_s - T_e$$

$$\text{on } \omega T_a = \omega T_s - \omega T_e. \quad P_a = \omega T_a \Rightarrow \text{acc. power.}$$

$$P_s = \omega T_s \Rightarrow \text{mech power I/p}$$

$$P_e = \omega T_e \Rightarrow \text{electrical n opp.}$$

assuming Power loss negligible.

$$P_a = P_s - P_e.$$

Under steady state  $P_s = P_e, P_a = 0.$

When balance between  $P_s$  and  $T_e$  is disturbed, m/c dynamics is governed by

$$P_a = T_a \omega = I \alpha \omega = M \frac{d^2 \theta}{dt^2}$$

$$\alpha = \frac{d^2\theta}{dt^2} \Rightarrow \text{angular acc. of the rotor.}$$

Since the angular position of the rotor is continually varying with time. It is more convenient to measure angular position and  $\omega$  w.r.t synchronously rotating axis's.

$$\delta = \theta - \omega_0 t.$$

Where  $\omega_0$  = angular vel. of ref.  
rotating axis  
taking time derivatives.

$$\frac{ds}{dt} = \frac{d\theta}{dt} - \omega_0.$$

$$\text{and } \frac{d^2s}{dt^2} = \frac{d^2\theta}{dt^2}$$

$$M \frac{d^2s}{dt^2} = P_a = P_s - P_e$$

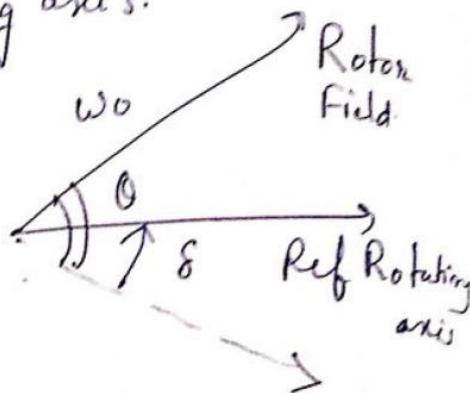
$$\text{On } \frac{G_i H}{180 f} \frac{d^2s}{dt^2} = P_a = P_s - P_e. \quad \textcircled{A}$$

as  $G_i$  is rating of m/c in MVA

$$\frac{H}{180 f} \frac{d^2s}{dt^2} = P_a = P_e - P_c \text{ p.u.} \quad \textcircled{B}$$

Solving Equation

Relative motion of rotor (load / torque / power etc)  
w.r.t stator field as a function of time.



## Controlling rotor dynamics of the syn m/c

When  $P_s = P_e \Rightarrow$  no acc/deacc power  $\rightarrow m/e$   
at syn speed.

If load an gen  $\uparrow \Rightarrow P_e \uparrow \Rightarrow$  rotor would tend  
to slow down.  $\Rightarrow$  slowing action would be  
sensed by speed governor, so  $P_s$  will increase  
to bring the rotor back to syn speed.

The reverse would happen if  $P_e \downarrow$ .

Swing Curve : As swing eqn. 2nd  
order diff eqn. Solution will give in.  
a curve  $s$  vs  $t$ .  $\Rightarrow$  solved by

Euler's method / Runge Kutta's method.  
 $\Rightarrow$  swing curve provides information  
regarding stability.

