

Modified

# CBCS SCHEME

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15ME81

## Eighth Semester B.E. Degree Examination, June/July 2019 Operations Research

Time: 3 hrs.

Max. Marks: 80

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. Use of statistical tables is permitted.

### Module-1

- 1 a. List and explain briefly the phases of operations research. (06 Marks)  
b. A paper manufacturing company produces two grades of papers grade 'R' and grade 'S'. Because of raw material restrictions, not more than 450 tonnes of grade R and 240 tonnes of grade S papers can be produced per week. It requires 0.2 hours to produce 1 tonne of grade R paper and 0.4 hours to produce 1 tonne of grade S paper. There are 108 production hours per week. The profit per tonne of grade R paper is Rs 400 and per tonne of grade S paper it is Rs. 500. Formulate a mathematical model to determine how many tones of grade R and grade S papers the company has to produce per week to maximize its profit. Solve graphically. (10 Marks)

OR

- 2 a. Discuss the limitations of operations research. (06 Marks)  
b. Solve the following LPP by graphical method and indicate the solution :  
Maximize  $Z = 2x_1 + 3x_2$   
Subject to constraints :  $x_1 - 2x_2 \leq 0$   
 $2x_1 - x_2 \geq 0$   
 $x_1 - x_2 \leq 0$   
with  $x_1, x_2 \geq 0$ . (10 Marks)

### Module-2

- 3 a. What is the significance of introducing slack, surplus and artificial variables in LPP? (04 Marks)  
b. Solve the following LPP by Simplex Method :  
Maximize  $Z = 6x_1 + 4x_2$   
Subject to constraints:  $-2x_1 + x_2 \leq 2$   
 $x_1 - x_2 \leq 2$   
 $3x_1 + 2x_2 \leq 9$   
with  $x_1, x_2 \geq 0$ . (12 Marks)

OR

- 4 a. Solve the following LPP by either Big-M method or two phase method :  
Minimize  $Z = x_1 - 2x_2 - 3x_3$   
Subject to constraints :  $-2x_1 + x_2 + 3x_3 = 2$   
 $2x_1 + 3x_2 + 4x_3 = 1$   
with  $x_1, x_2, x_3 \geq 0$ . (08 Marks)  
b. Solve the following by Dual Simplex Method :  
Maximize  $Z = -2x_1 - 2x_2 - 4x_3$   
Subject to constrains:  $2x_1 + 3x_2 + 5x_3 \geq 2$   
 $3x_1 + x_2 + 7x_3 \leq 3$   
 $x_1 + 4x_2 + 6x_3 \leq 5$   
with  $x_1, x_2, x_3 \geq 0$ . (08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and/or equations written eg.  $42+8=50$ , will be treated as malpractice.

**Module-3**

- 5 a. What is degeneracy in transportation problem? Discuss its consequence and how it is overcome. (04 Marks)
- b. Obtain the optimum solution to the following transportation problem to minimize the total transportation cost. Initial solution by Vogel's approximation method. (VAM).

		Destination				Supply
		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
Origin	O <sub>1</sub>	42	48	38	37	16
	O <sub>2</sub>	40	49	52	51	15
	O <sub>3</sub>	39	38	40	43	19
Demand		8	9	11	16	

(12 Marks)

**OR**

- 6 a. Explain the differences between assignment problem and transportation problem. (05 Marks)
- b. A small machine shop has five jobs to be assigned to five machines. The following matrix indicates the cost of assigning each of the five jobs to each of the five machines. Obtain the optimum assignment of jobs to machines, in order to minimize the total assignment cost.

		Machines				
		1	2	3	4	5
Jobs	A	11	17	8	16	20
	B	9	7	12	6	15
	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	15

Q6(b) Cost Matrix

(11 Marks)

**Module-4**

- 7 a. Explain the Kendall and Lee's notations for representing queuing models. (04 Marks)
- b. A small project consists of activities from 'A' to 'I'. The following table indicates the precedence relationship among activities and the three time estimates – optimistic, most – likely and pessimistic time for each activity in days.

Activity	Predecessor Relationship	Optimistic time 't <sub>0</sub> '	Most likely time 't <sub>m</sub> '	Pessimistic time 't <sub>p</sub> '
A	–	2	5	8
B	A	6	9	12
C	A	6	7	8
D	B, C	1	4	7
E	A	8	8	8
F	D, E	5	14	17
G	C	3	12	21
H	F, G	3	6	9
I	H	5	8	11

- i) Draw the project network. Determine the expected time and variance for each activity
- ii) Obtain the total expected duration of the project and critical path
- iii) What is the probability of completing the project in 50 days? (12 Marks)

OR

- 8 a. For the following set of activities of a project, draw the network and obtain Early Start [ES], Early Finish [EF], Late Start [LS] and Late Finish [LF] for each activity. Also, identify the critical path and project duration.

Activity	Predecessor	Duration in days
A	–	5
B	A	8
C	A	6
D	C	5
E	B, D	9

(08 Marks)

- b. The mean arrival rate to a service centre is 3 per hour. The mean service time is found to be 10 minutes per service. Assuming Poisson arrival and exponential service time, find :
- Utilization factor for the service facility
  - Probability of two units in the system
  - Queue length
  - Expected waiting time in the system

(08 Marks)

**Module-5**

- 9 a. Apply the rules of dominance to reduce the game to  $(2 \times 2)$  and solve the game to obtain game value and optimum strategies for both the players.

		Player B		
		1	2	3
Player A	1	2	-2	4
	2	-1	4	2
	3	2	1	6

(08 Marks)

- b. Solve the following  $(2 \times 4)$  game graphically.

		Player B		
		1	2	3
Player A	1	1	3	12
	2	8	6	02

(08 Marks)

OR

- 10 a. There are seven jobs to be processed on a single machine. The following table indicates the jobs and corresponding processing time in hours. Obtain the optimum sequence of jobs by Shortest Processing Time [SPT] rule that minimizes the mean flow time. Also obtain average in process inventory.

(06 Marks)

Jobs (j)	A	B	C	D	E	F	G
Processing time ( $t_j$ ) in hr	8	3	5	4	3	9	6

- b. There are six jobs to be processed on three machines A, B and C in the order CAB. The following table indicates the processing time in hours for the six jobs on the three machines. Obtain optimum sequence of jobs that minimizes the total elapsed time for completing all the jobs on the three machines. Also indicate the idle time of each machine.

Jobs	1	2	3	4	5	6
Processing time in hours on M/C A	4	6	7	4	5	3
Processing time in hours on M/C B	8	10	7	8	11	8
Processing time in hours on M/C C	5	6	2	3	4	9

(10 Marks)

Verified by



Approved by



Subject Title : OPERATIONS RESEARCH

Subject Code : ISME81

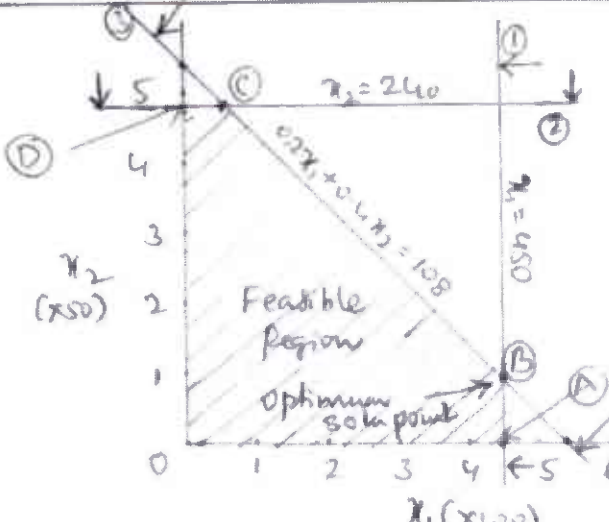
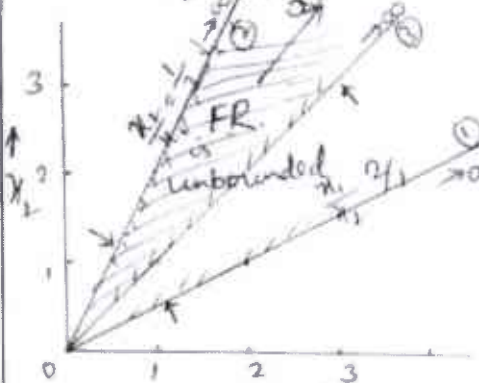
(1)

Question Number	Solution	Marks Allocated
1 a	<p align="center"><u>Module - 1</u></p> <p>Phases of OR</p> <p>i) Formulation of the problem (ii) Construction of Mathematical model (iii) Deriving solution to model (iv) Validating the solution (v) Controlling the solution (vi) Implementation.</p> <p align="right">Listing — 01</p> <p align="right">Explanation in brief, the above phases. — 05</p> <p align="right"><b>(06)</b></p>	
1 b.	<p>Maximize <math>Z = 400x_1 + 500x_2</math></p> <p>where, <math>x_1 =</math> No of tonnes of Grade R paper produced/week</p> <p><math>x_2 =</math> — " — Grade S — " — "</p> <p>Subject to constraints</p> <p><math>x_1 \leq 450</math> tonnes/week</p> <p><math>x_2 \leq 240</math> tonnes/week</p> <p><math>0.2x_1 + 0.4x_2 \leq 108</math> hours/week</p> <p>with <math>x_1, x_2 \geq 0</math></p> <p>Plotting of graph (shown in next page) — 02</p> <p><math>x_1 = 450 \rightarrow \textcircled{1}</math></p> <p><math>x_2 = 240 \rightarrow \textcircled{2}</math></p> <p><math>0.2x_1 + 0.4x_2 = 108 \rightarrow \textcircled{3}</math></p> <p><math>x_1 = 540 ; x_2 = 270</math></p>	02 02

\* APPROVED \*

*Bunni*

Registrar (Evaluation)  
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Question Number	Solution	Marks Allocated
	 <p>At (A) <math>x_1 = 450</math>,  <math>Max Z_A = 1,80,000/-</math></p> <p>At (B) <math>x_1 = 450</math>  <math>x_2 = 45</math> ✓  <math>Max Z_B = 2,02,500/-</math></p> <p>At (C) <math>x_1 = 60</math>  <math>x_2 = 240</math>  <math>Max Z_C = 1,44,000/-</math></p> <p>At (D) <math>x_1 = 240</math>  <math>Max Z_D = 1,20,000/-</math></p> <p>Optimum solution at (B)  <math>x_1 = 450</math>  <math>x_2 = 45</math> <math>Max Z = Rs 2,02,500/-</math></p>	<p>04</p>
<p>2 a.</p>	<p>Limitations of OR</p> <p>(i) Dependency on computer (ii) Non-Quantifiable factors. (iii) Distance between Manager and OR incharge (iv) Money and time costs (v) Implementation</p> <p>Brief Explanation of the above</p>	<p>10</p> <p>01</p> <p>05</p>
<p>b.</p>	<p><math>x_1 \leq 2x_2</math>      <math>2x_1 \geq x_2</math>      <math>x_1 \leq x_2</math></p> <p><math>\frac{x_1}{x_2} = \frac{2}{1} \rightarrow (1)</math>      <math>\frac{x_2}{x_2} = \frac{1}{2} \rightarrow (2)</math>      <math>\frac{x_1}{x_2} = \frac{1}{1} \rightarrow (3)</math></p>  <p>Plotting of Graph — 04</p> <p>Solution:          Feasible Region is unbounded  <math>\therefore</math> Solution is unbounded</p>	<p>06</p> <p>04</p> <p>04</p> <p>02</p> <p>10</p>

Question Number	Solution	Marks Allocated
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Module - 2

3a.

Slack and surplus variables are added in the constraint inequalities in the LHS to convert them to equality form.

01

As slack variable has +ve coefficient of 1 it can be used as one of the basic variables as it provides feasible solution. Surplus variable has -ve coefficient of 1 & therefore makes initial solution infeasible, if it is used as Basic variable.

02

Therefore Artificial variable is introduced in such constraint equations that do not contain a slack variable to generate IBFS

01

(04)

3b

Introducing slack variables in each of the three constraints to generate IBFS

IBFS

$$BV \begin{cases} PS_1 = 2 \\ S_2 = 2 \\ S_3 = 9 \end{cases}$$

02

Table 1

$C_b$	PBV	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$	RHS	Min/ve Ratio
0	$S_1$	-2	1	1	0	0	2	X
0	$S_2$	1	-1	0	1	0	2	2 ← K
0	$S_3$	3	2	0	0	1	9	3
(-4)		-6	-4	0	0	0	0	

03

Question Number	Solution	Marks Allocated
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Table 2

BV	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	RHS	Min Ratio
$s_1$	0	-1	1	2	0	6	X
$x_1$	1	-1	0	1	0	2	X
$s_3$	0	5	0	-3	1	3	3/5 ✓ ← ER
(Zj - Cj)	0	-10	0	6	0	12	

↓ EV  
↑ MC

Table 3

BV	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	RHS
$s_1$	0	-1	1	2	0	6
$x_1$	1	0	0	2/5	1/5	13/5
$x_2$	0	1	0	-3/5	1/5	3/5
(Zj - Cj)	0	0	0	0	2	18

In Table 3 all (Zj - Cj)  $\geq 0$  for optimality  
(Zj - Cj) row element must be zero or positive

∴ Solution is optimum

$x_1 = 13/5 ; x_2 = 3/5 \text{ Max } Z = 18$

4a.

Solution by Big-M method

Max Z =  $-x_1 + 2x_2 + 3x_3 - MA_1 - MA_2$

s.t.c.  $-2x_1 + x_2 + 3x_3 + A_1 = 2$

$2x_1 + 3x_2 + 4x_3 + A_2 = 1$

$A_1, A_2 \rightarrow$  Artificial variables

LBFS  $\Rightarrow A_1 = 2$

$A_2 = 1$

12

01

Question Number	Solution	Marks Allocated
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Table 1

$C_j$	BV	$x_1$	$x_2$	$x_3$	$A_1$	$A_2$	RHS	Min. Ratio
	$A_1$	-2	1	3	1	0	2	$2/3 \rightarrow 6/3$
	$A_2$	2	3	4	0	1	1	$1/4 \rightarrow 1/4$
	$(Z_j - C_j)$	1	$(4M - 2)$	$(-3M - 3)$	0	0	$-3M$	

↑ EV  
↑ MC

Table 2

$C_j$	$x_1$	$x_2$	$x_3$	$A_1$	$A_2$	RHS	Min. Ratio
	$A_1$	$-7/2$	$-3/4$	0	1	X	$3/2$
	$x_3$	$1/2$	$3/4$	1	0	X	$1/4$
	$(Z_j - C_j)$	$(7M + 5)/2$	$(5M + 1)/4$	0	0	X	$(-5M + 3)/4$

In Table 2, All  $(Z_j - C_j) \geq 0$ . Solution is optimum. However  $A_1$  an artificial var, is present as BV in final solution with the value  $A_1 = 3/2$ .  $\therefore \text{Max } Z' = \frac{(-5M + 3)}{4}$  becomes -ve & Infeasible.

$\therefore$  The problem has Infeasible Solution - 01

OR Solve by Two phase Method - 08

In Phase I, Table 2 All  $(Z_j - C_j) \geq 0$  But  $\text{Max } Z' < 0$  & Artificial variable appears as BV with the value  $\therefore$  Cannot proceed to Phase II. Solution is 'INFEASIBLE'



Question Number	Solution	Marks Allocated																																																																																					
4.6	<p>Solution by Dual Simplex Method</p> $\text{Max } Z = -2x_1 - 2x_2 - 4x_3$ $-2x_1 - 3x_2 - 5x_3 + s_1 = -2 \quad (*)$ $3x_1 + x_2 + 7x_3 + s_2 = 3$ $x_1 + 4x_2 + 6x_3 + s_3 = 5$ <p>Initial <u>infeasible</u> Basic Solution</p> $s_1 = -2, s_2 = 3, s_3 = 5$ <p>Table 1 <span style="margin-left: 100px;">non Basic variables</span></p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th><math>C_b</math></th> <th>BV</th> <th><math>x_1</math></th> <th><math>x_2</math></th> <th><math>x_3</math></th> <th><math>s_1</math></th> <th><math>s_2</math></th> <th><math>s_3</math></th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>0</td> <td><math>s_1</math></td> <td>-2</td> <td>-3</td> <td>-5</td> <td>1</td> <td>0</td> <td>0</td> <td>-2 ← KR</td> </tr> <tr> <td>0</td> <td><math>s_2</math></td> <td>3</td> <td>1</td> <td>7</td> <td>0</td> <td>1</td> <td>0</td> <td>3</td> </tr> <tr> <td>0</td> <td><math>s_3</math></td> <td>1</td> <td>4</td> <td>6</td> <td>0</td> <td>0</td> <td>1</td> <td>5</td> </tr> <tr> <td></td> <td><math>(Z_j - C_j)</math></td> <td>2</td> <td>2</td> <td>4</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> </tbody> </table> <p>Min <math>\left  \frac{(Z_j - C_j)}{KR} \right  \left  \frac{2}{-2} \right  \left  \frac{2}{-3} \right  \left  \frac{4}{-5} \right  \times \times \times -</math></p> <p>KR &lt; 0      1      0.67      0.8</p> <p style="margin-left: 100px;">↑ WC</p> <p>Table 2</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th>BV</th> <th><math>x_1</math></th> <th><math>x_2</math></th> <th><math>x_3</math></th> <th><math>s_1</math></th> <th><math>s_2</math></th> <th><math>s_3</math></th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td><math>x_2</math></td> <td><math>2/3</math></td> <td>1</td> <td><math>5/3</math></td> <td><math>-4/3</math></td> <td>0</td> <td>0</td> <td><math>2/3</math></td> </tr> <tr> <td><math>s_2</math></td> <td><math>7/3</math></td> <td>0</td> <td><math>16/3</math></td> <td><math>4/3</math></td> <td>1</td> <td>0</td> <td><math>7/3</math></td> </tr> <tr> <td><math>s_3</math></td> <td><math>-9/3</math></td> <td>0</td> <td><math>-2/3</math></td> <td><math>4/3</math></td> <td>0</td> <td>1</td> <td><math>7/3</math></td> </tr> <tr> <td><math>(Z_j - C_j)</math></td> <td><math>2/3</math></td> <td>0</td> <td><math>2/3</math></td> <td><math>2/3</math></td> <td>0</td> <td>0</td> <td><math>-4/3</math></td> </tr> </tbody> </table> <p>In Table 2 All RHS elements are <math>\geq 0</math> (i.e.)  <math>\therefore</math> Solution is Feasible and Optimum</p>	$C_b$	BV	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	RHS	0	$s_1$	-2	-3	-5	1	0	0	-2 ← KR	0	$s_2$	3	1	7	0	1	0	3	0	$s_3$	1	4	6	0	0	1	5		$(Z_j - C_j)$	2	2	4	0	0	0	0	BV	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	RHS	$x_2$	$2/3$	1	$5/3$	$-4/3$	0	0	$2/3$	$s_2$	$7/3$	0	$16/3$	$4/3$	1	0	$7/3$	$s_3$	$-9/3$	0	$-2/3$	$4/3$	0	1	$7/3$	$(Z_j - C_j)$	$2/3$	0	$2/3$	$2/3$	0	0	$-4/3$	<p>02</p> <p>02</p> <p>02</p>
$C_b$	BV	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	RHS																																																																															
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$(Z_j - C_j)$	$2/3$	0	$2/3$	$2/3$	0	0	$-4/3$																																																																																

Question Number	Solution	Marks Allocated
	<p>∴ final Feasible and optimum solution is  <math>x_1 = 0, x_2 = 2/3 \text{ Max } Z = -4/3</math></p>	<p>02  <u>08</u></p>
5a	<p style="text-align: center;"><u>Module-3</u></p> <p>For optimality test by MODI method, the two conditions to be satisfied are</p> <p>i) Number of Allocations = <math>(m+n-1)</math>  <math>m = \text{Number of origins}</math>  <math>n = \text{Number of destinations}</math></p> <p>ii) All these allocations must be in independent position.</p> <p>Therefore, if the number of allocations is <u>less than</u> <math>(m+n-1)</math>, then it is called degeneracy in Transportation problem.</p> <p>Consequence is that we cannot obtain net evaluation values for some of the non-basic cells and cannot proceed further towards optimality.</p> <p>To overcome degeneracy, Add a small +ve allocation 'ε' (<math>\epsilon \rightarrow 0</math>) in a non basic independent cell. ∴ Allocation = <math>(m+n-1)</math></p>	<p>→ 02</p> <p>— 01</p> <p>— 01</p> <p><u>04</u></p>

Question Number	Solution	Marks Allocated
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56

Problem is Unbalanced

Supply > Demand. Create Dummy destination

ERFS by VAM

Table 1

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub> <sup>Dummy</sup>	Supply
O <sub>1</sub>	42	48	38	37	0	16
O <sub>2</sub>	40	49	52	51	0	15
O <sub>3</sub>	39	38	40	43	0	19
Demand	8	9	11	16	6	50

Allocation



Unit Cost

Legend

ERFS ⇒ Rs 1706/-

Optimality Test by MODI method or U/V method

No. of allocations = (3+5-1) = 7 ✓

all are independent

Solve for Basic cells  $u_i + v_j = C_{ij}$

Non Basic cells Net Evaluation =  $C_{ij} - (u_i + v_j)$

If all NE  $\geq 0$  Allocation optimum.

Table 2

Table 3 (Final Table)

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply	u <sub>i</sub>	Allocation
O <sub>1</sub>	42	48	38	37	0	16	-13	O <sub>1</sub> -D <sub>4</sub> → 16
O <sub>2</sub>	40	49	52	51	0	15	0	O <sub>2</sub> -D <sub>1</sub> → 8 O <sub>2</sub> -D <sub>2</sub> → 1 O <sub>2</sub> → D <sub>5</sub> → 6 Dummy
O <sub>3</sub>	39	38	40	43	0	19	-11	O <sub>3</sub> → D <sub>2</sub> → 8 O <sub>3</sub> → D <sub>3</sub> → 11
v <sub>j</sub>	40	49	51	50	0			Optimum cost = Rs 1706/-

All net evaluations  $NE = C_{ij} - (u_i + v_j) \geq 0$

Optimum allocation.

(12)

Question Number	Solution	Marks Allocated																																																																								
6a	Indicate atleast 3 differences	→ 05																																																																								
6b	<p>After Row and Column Manipulations</p> <p>Table 2</p> <table border="1" data-bbox="446 448 798 806"> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>A</td><td>2</td><td>9</td><td>10</td><td>8</td><td>8</td></tr> <tr><td>B</td><td>2</td><td>1</td><td>6</td><td>10</td><td>5</td></tr> <tr><td>C</td><td>10</td><td>4</td><td>3</td><td>1</td><td>0</td></tr> <tr><td>D</td><td>3</td><td>7</td><td>0</td><td>11</td><td>5</td></tr> <tr><td>E</td><td>3</td><td>10</td><td>2</td><td>1</td><td>1</td></tr> </table> <p>Table 3 Modified Matrix</p> <p>Table 4 (Final Table)</p> <table border="1" data-bbox="319 1075 718 1433"> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>A</td><td>10</td><td>8</td><td>0</td><td>8</td><td>6</td></tr> <tr><td>B</td><td>0</td><td>0</td><td>6</td><td>10</td><td>3</td></tr> <tr><td>C</td><td>0</td><td>5</td><td>5</td><td>2</td><td>10</td></tr> <tr><td>D</td><td>1</td><td>6</td><td>10</td><td>11</td><td>3</td></tr> <tr><td>E</td><td>2</td><td>10</td><td>3</td><td>2</td><td>0</td></tr> </table> <p>Optimum Assignment</p> <p>A - 1 → 11            B - 4 → 6            C → 5 → 16            D → 3 → 17            E → 2 → 10</p> <p><del>13 (SD) 60</del> → <del>60</del></p> <p>No. of Assignment = Matrix order = 5            It is optimum</p>		1	2	3	4	5	A	2	9	10	8	8	B	2	1	6	10	5	C	10	4	3	1	0	D	3	7	0	11	5	E	3	10	2	1	1		1	2	3	4	5	A	10	8	0	8	6	B	0	0	6	10	3	C	0	5	5	2	10	D	1	6	10	11	3	E	2	10	3	2	0	<p>→ 04</p> <p>→ 02</p> <p>→ 05</p> <p>→ 11</p>
	1	2	3	4	5																																																																					
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7a.	<p>Kendall and Lee's Notations for Queue models. (a/b/c) : (d/e)</p> <p>Where a = enter arrival distribution            b = departure or service time distribution            c = no. of channels            d = capacity of the system            e = queue discipline</p>	<p>→ 06.</p>																																																																								

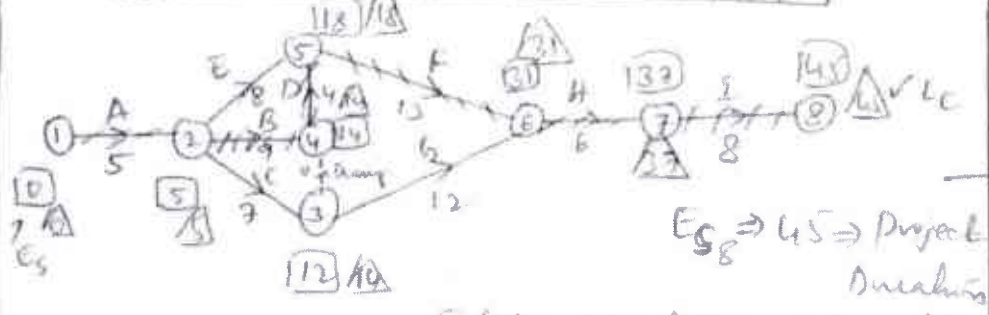
Module - 4

Question Number	Solution	Marks Allocated
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7b.

Find First  $t_e =$  expected time of completion for each activity & Variance  
 $t_e = \frac{t_o + 4t_m + t_p}{6}$  &  $\sigma^2 = \frac{(t_p - t_o)^2}{6}$

Activity	A	B	C	D	E	F	G	H	I
$t_e$	5	9	7	4	8	13	12	6	8
$\sigma^2$	1	1	0.44	1	-	4	9	1	1



Total expected time of completion

$T_e = 45 \text{ days}$

Critical path  $A \rightarrow B \rightarrow D \rightarrow F \rightarrow H \rightarrow I$

Variance  $\sigma^2$  for C.P.  $\rightarrow 1 + 1 + 1 + 4 + 1 + 1$

$\sigma^2 = 9$

$\therefore \sigma = \sqrt{9} = 3$

Prob. of completing project in 50 days

$T_s = 50 \text{ days}$

Normal variate  $Z = \frac{T_s - T_e}{\sigma} = \frac{50 - 45}{3}$

$= 1.67$

From Normal distribution Table for

$Z = 1.67$  probability = 95.25%

03

03

01

02

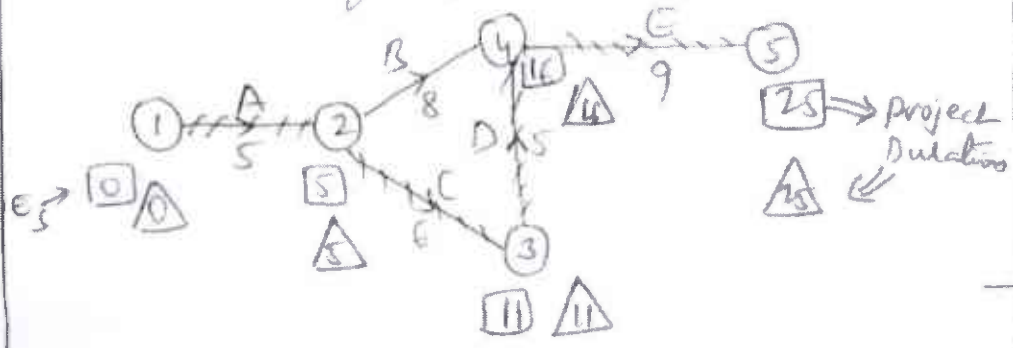
03

12

Question Number	Solution	Marks Allocated
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8 a

network of the project



Project Duration = 25 days

Activity	Pre-activity	Duration days	ES	EF	LS	LF
A	-	5	0	5	0	5
B	A	8	5	13	8	16
C	A	6	5	11	5	11
D	C	5	11	16	11	16
E	B, D	9	16	25	16	25

03

01

04

08

8 b

Mean arrival rate =  $\lambda = 3/hr$

Mean service rate =  $\mu = \frac{1}{10} \times 60 = 6/hr$

i) utilization factor  $\rho = \frac{\lambda}{\mu} = \frac{3}{6} = 0.5$

ii) probability of two units in the system

$$P_2 = P_0 (\frac{\lambda}{\mu})^2 = 0.5 \times (0.5)^2$$

$$P_0 = 1 - \rho = 0.5$$

$$P_2 = 0.125 = \frac{1}{8}$$

iii) Queue length =  $L_q = \frac{(\lambda/\mu)^2}{(1-\lambda/\mu)} = \frac{0.5^2}{(1-0.5)} = 0.5$

(iv) Expected waiting time in the system

$$= W_s = W_q + \frac{1}{\mu}$$

$$= \frac{L_q}{\lambda} + \frac{1}{\mu} = \frac{0.5}{3} + \frac{1}{6} = \frac{1}{3}$$

02

02

02

02

08

OR  

$$\frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$= \frac{3^2}{6(6-3)}$$

$$= 0.5$$

OR  $W_s = \frac{1}{\mu - \lambda} = \frac{1}{(6-3)} = \frac{1}{3} = 20 \text{ minutes}$

Question Number	Solution	Marks Allocated																																																			
9.a	<p>Applying Rules of Dominance Rowwise and Column wise</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td>1</td> <td>2</td> <td>3</td> <td>Row Min</td> <td></td> </tr> <tr> <td rowspan="3">A</td> <td>1</td> <td>2</td> <td>-2</td> <td>4</td> <td>-2</td> <td rowspan="3">No Saddle point Maximin ≠ Minimax</td> </tr> <tr> <td>2</td> <td>-1</td> <td>4</td> <td>2</td> <td>-1</td> </tr> <tr> <td>3</td> <td>2</td> <td>1</td> <td>6</td> <td>1</td> </tr> </table> <p>Columns: ② Minimax 4 6</p> <p><u>Row Dominance</u> Compare <math>R_1</math> with <math>R_3</math>          Element in <math>R_3 \gg</math> Element in <math>R_1</math>  <math>R_3</math> dominates <math>R_1</math> Eliminate <math>R_1</math></p> <p>Compare <math>R_2</math> &amp; <math>R_3</math>          - NO Dominance</p> <p><u>Column Dominance</u>          Compare Column <math>C_1</math> with <math>C_2</math> - NO Dominance          Compare Column <math>C_1</math> with <math>C_3</math>          Element in Column <math>C_1 \leq</math> Element in <math>C_3</math>  <math>\therefore C_1</math> dominates <math>C_3</math> Delete <math>C_3</math></p> <p>Reduced Matrix</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td>1</td> <td>2</td> <td>Prob.</td> </tr> <tr> <td rowspan="2">A</td> <td>2</td> <td>-1</td> <td>4</td> <td><math>x_2</math></td> </tr> <tr> <td>3</td> <td>2</td> <td>1</td> <td><math>x_3</math></td> </tr> <tr> <td>Prob</td> <td></td> <td><math>y_1</math></td> <td><math>y_2</math></td> <td></td> </tr> </table> <p>Oddment Matrix</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td><math>a_{21}</math></td> <td><math>a_{22}</math></td> </tr> <tr> <td></td> <td></td> <td><math>a_{31}</math></td> <td><math>a_{32}</math></td> </tr> </table> <p> <math display="block">\text{Prob. } x_2 = \frac{a_{32} - a_{31}}{(a_{21} + a_{32}) - (a_{31} + a_{22})} = \frac{1 - 2}{(1+1) - (2+4)} = \frac{-1}{-6} = \frac{1}{6}</math> <math display="block">x_3 = 1 - \frac{1}{6} = \frac{5}{6} \quad (x_1, x_2, x_3) = (0, \frac{1}{6}, \frac{5}{6})</math> <math display="block">\text{Prob } y_1 = \frac{a_{32} - a_{22}}{(a_{21} + a_{32}) - (a_{31} + a_{22})} = \frac{1 - 4}{(1+1) - (2+4)} = \frac{-3}{-6} = \frac{1}{2}</math> <math display="block">y_2 = 1 - \frac{1}{2} = \frac{1}{2}</math> </p>			1	2	3	Row Min		A	1	2	-2	4	-2	No Saddle point Maximin ≠ Minimax	2	-1	4	2	-1	3	2	1	6	1			1	2	Prob.	A	2	-1	4	$x_2$	3	2	1	$x_3$	Prob		$y_1$	$y_2$				$a_{21}$	$a_{22}$			$a_{31}$	$a_{32}$	<p>01</p> <p>02</p> <p>02</p>
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Question Number	Solution	Marks Allocated																	
	<p>Prob. <math>(x_1, x_2, x_3) = (0, 1/6, 5/6)</math></p> <p>Prob <math>(y_1, y_2, y_3) = (1/2, 1/2, 0)</math></p> $\text{Value} = \frac{(a_{21} \cdot a_{32}) - (a_{22} \cdot a_{31})}{(a_{21} + a_{32}) - (a_{31} + a_{22})}$ $= \frac{(-1 \times 1) - (4 \times 2)}{(-1+1) - (2+4)} = \frac{-1-8}{-6-6} = \frac{-9}{-6} = 3/2$ <p style="text-align: center;"><span style="border: 1px solid black; padding: 2px;"><math>V = 3/2</math></span> ✓</p>	-03																	
96.	<p>Expected gain Matrix for A</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Player B Strategy</th> <th>Expected gain for A</th> </tr> </thead> <tbody> <tr> <td>1</td> <td><math>1x_1 + 8x_2</math> ①</td> </tr> <tr> <td>2</td> <td><math>3x_1 + 6x_2</math> ②</td> </tr> <tr> <td>3</td> <td><math>12x_1 + 2x_2</math> ③</td> </tr> </tbody> </table> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>Player A Graph</p> </div> <div style="text-align: center;"> <p>Graph &amp; Reduced Matrix</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td><math>x_1</math></td> <td><math>x_2</math></td> </tr> <tr> <td>1</td> <td>3</td> <td>12</td> </tr> <tr> <td>2</td> <td>6</td> <td>2</td> </tr> </table> <p> <math>\text{Prob } (x_1, x_2) = (\frac{4}{13}, \frac{9}{13})</math>  <math>\text{Prob } (y_1, y_2) = (\frac{10}{13}, \frac{3}{13})</math>  <math>\text{Value} = \frac{66}{13} = 5.1</math> </p> </div> </div>	Player B Strategy	Expected gain for A	1	$1x_1 + 8x_2$ ①	2	$3x_1 + 6x_2$ ②	3	$12x_1 + 2x_2$ ③		$x_1$	$x_2$	1	3	12	2	6	2	-02
Player B Strategy	Expected gain for A																		
1	$1x_1 + 8x_2$ ①																		
2	$3x_1 + 6x_2$ ②																		
3	$12x_1 + 2x_2$ ③																		
	$x_1$	$x_2$																	
1	3	12																	
2	6	2																	

08

08



Question Number	Solution	Marks Allocated
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10a.

optimum job sequence based on SPT rule

Sequence I

B	E	D	C			
---	---	---	---	--	--	--

Sequence II

E	B	D	C			
---	---	---	---	--	--	--

→ 01

Job Sequence	P.T. $t_j$	$t_i$	$t_o$	Completion time $C_j$ - flow time $F_j$
B	3	0	3	3
E	3	3	6	6
D	4	6	10	10
C	5	10	15	15
G	6	15	21	21
A	8	21	29	29
F	9	29	38	<u>38</u>

$\sum t_j = 38$

$\sum F_j = 122$

$\sum P.T. = \sum t_j = 38$

→ 02

Mean Flow Time  $\bar{F} = \frac{1}{n} \sum_{j=1}^n F_j$      $n = 7$

$= \frac{1}{7} \times [122]$

$= \underline{17.43 \text{ hrs}}$

→ 02

Average in process inventory =  $\frac{\sum F_j}{\sum t_j}$

$= \frac{122}{38} = 3.21 \text{ or } \underline{4 \text{ jobs}}$

→ 01

06

Question Number	Solution	Marks Allocated																																										
106	<p>Problem on <math>n</math> jobs 3 machines</p> <p><math>n = 6</math> &amp; 3 Machines A B C in the order <u>CAB</u></p> <p>To convert this problem into <math>n</math> jobs 2 m/c any one of the following conditions to be satisfied.</p> <p>I) Min processing time on 1<sup>st</sup> Machine 'C' <math>\nabla</math> Maximum processing time on 2<sup>nd</sup> machine 'A' <math>\nabla</math> 7 <math>\nabla</math> 7 X NOT Satisfied</p> <p>II) Min processing time on 3<sup>rd</sup> M/c i.e. B <math>\nabla</math> Max processing time on 2<sup>nd</sup> M/c 'A' <math>\nabla</math> 7 <math>\nabla</math> 7 Both are equal</p> <p>✓ <math>\nabla</math> <math>\nabla</math> <math>\nabla</math> Both are equal</p> <p>Create two imaginary Machines G and H.</p> <table border="1" data-bbox="343 1422 837 1892"> <thead> <tr> <th>M/c G = (C+A)</th> <th>M/c H = (A+B)</th> </tr> </thead> <tbody> <tr><td>9</td><td>12</td></tr> <tr><td>12</td><td>16</td></tr> <tr><td>9</td><td>14</td></tr> <tr><td>7</td><td>12</td></tr> <tr><td>9</td><td>16</td></tr> <tr><td>12</td><td>11</td></tr> </tbody> </table> <p>Optimum Job Sequence</p> <table border="1" data-bbox="893 1601 1292 1870"> <tbody> <tr><td>1.</td><td>4</td><td>3</td><td>1</td><td>5</td><td>2</td><td>6</td></tr> <tr><td>2.</td><td>4</td><td>1</td><td>3</td><td>5</td><td>2</td><td>6</td></tr> <tr><td>3.</td><td>4</td><td>5</td><td>1</td><td>3</td><td>2</td><td>6</td></tr> <tr><td>4.</td><td>4</td><td>1</td><td>3</td><td>5</td><td>6</td><td>2</td></tr> </tbody> </table> <p>→ 02</p> <p>→ 03</p>	M/c G = (C+A)	M/c H = (A+B)	9	12	12	16	9	14	7	12	9	16	12	11	1.	4	3	1	5	2	6	2.	4	1	3	5	2	6	3.	4	5	1	3	2	6	4.	4	1	3	5	6	2	
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4.	4	1	3	5	6	2																																						

*[Signature]*

Question Number	Solution						Marks Allocated	
	To find Total Elapsed Time							
	Job Sequence	Machine C $t_i$   $t_o$		Machine A $t_i$   $t_o$		Machine B $t_i$   $t_o$		
	4	0	3	3	7	7	15	
	3	3	5	7	14	15	22	
	1	5	10	14	18	22	30	
	5	10	14	18	23	30	41	
	2	14	20	23	29	41	51	
	6	20	29	29	32	51	59	
	$t_o$ of job 6 (last job) on M/C B (last machine) = 59 hrs = Total Elapsed Time (TET)						→ 03	
	Idle Time of M/C C ⇒ TET - P <sub>T</sub> on M/C C = 59 - 29 = 30 hrs							
	— h — M/C A ⇒ TET - P <sub>T</sub> on M/C A = 59 - 29 = 30 hrs							
	— h — M/C B ⇒ 59 - 52 = 7 hrs						→ 02	
<p><b>*APPROVED*</b>  <i>Bunni</i>            Registrar (Evaluation)            Visvesvaraya Technological University            BELAGAVI - 530016</p>	<p>Approved by <i>[Signature]</i> Verified by <i>[Signature]</i></p>						<p>10</p>	