CBCS SCHEME

USN 1CR17ME056 **17ME42**

Fourth Semester B.E. Degree Examination, June/July 2019
Kinematics of Machinery

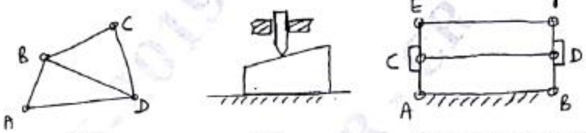
Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define 'degree of freedom' and find degree of freedom for the chains shown in Fig.Q1(a).



(i) (ii) (iii) AB = CD = EF & AE = BF

Fig.Q1(a) **(10 Marks)**

b. Define 'inversion of a kinematic chain'. A four bar mechanism has links of lengths 150mm, 250mm, 300mm and frame L_0 mm. Find the range of L_0 if the mechanism has to work as (i) Double crank mechanism (ii) Crank-rocker mechanism. **(10 Marks)**

OR

2 a. Sketch a neat, proportionate 'Peaucellier's mechanism'. State geometric relationships among links. Identify the point tracing the straight line and prove that the point traces straight line. **(10 Marks)**

b. Draw 'Crank and Slotted lever' type of quick return motion mechanism showing the positions of crank clearly for extreme positions of lever. If the crank and frame are 200 mm, 800mm, find the ratio of time of return to time of cutting if the crank rotates uniformly. Also find angle of oscillation of lever. **(10 Marks)**

Module-2

3 In a four bar mechanism ABCD, AD is fixed link of 120 mm long. The crank AB is 30mm and rotates at 100 rpm clockwise, while CD = 60 mm oscillates about D. BC and AD are of same length. Find the angular velocity of link CD when angle BAD = 60° by
(i) relative velocity method (ii) instantaneous centre method. **(20 Marks)**

OR

4 a. State and prove Kennedy's theorem. **(08 Marks)**

b. Explain the procedure to construct 'Klein's construction' to determine the velocity and acceleration of a slider crank mechanism in which crank is rotating uniformly. **(12 Marks)**

Module-3

5 a. For the slider crank mechanism shown in Fig.Q5(a), write (i) loop closure equation (ii) differentiate loop closure equation with respect to time to get velocity equation (iii) differentiate velocity equation with respect to time to get acceleration equation.

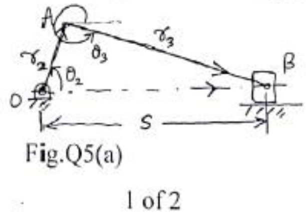


Fig.Q5(a) **(08 Marks)**

b. In Fig.Q5(a), if r_2 = 100mm r_3 = 350mm, θ_2 = 60°, find angular velocity and angular acceleration of connecting rod if crank rotates uniformly at 600 rpm in CCW direction. **(12 Marks)**

OR

6 a. For the 4-bar mechanism shown in Fig.Q6, obtain Frendenstein's equation. **(08 Marks)**

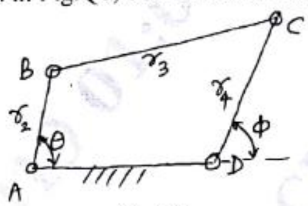


Fig.Q6

b. Find r_2, r_3 and r_4 to generate a function $y = x^3$, $1 \le x \le 3$ accurate at x = 1.1339, x = 2 and x = 2.866 if r_1 = 100mm, θ_S = 30°, θ_f = 90°, ϕ_s = 45° and ϕ_f = 135° with respect to Fig.Q6. **(12 Marks)**

Module-4

7 a. Define 'pitch circle', 'circular pitch', 'diametral pitch' and 'module'. **(08 Marks)**

b. Obtain an expression for the minimum number of teeth on pinion to avoid interference. **(12 Marks)**

OR

8 An epicyclic gear train consists of a sun-wheel S, a stationary internal gear E and three identical planet wheels P carried on a star shaped planet carrier C. The size of different tooth wheels are such that the planet carrier C rotates at $1/5^{th}$ of the speed of the sunwheel S. The no. of teeth on sun-wheel is 16. The driving torque on the sun-wheel is 100 N-m. Determine (i) no. of teeth on P and E. (ii) Torque required to keep the internal gear stationary. **(20 Marks)**

Module-5

9 From the following data draw the profile of a cam in which the follower moves with SHM during ascent while it moves with uniform acceleration and deceleration during descent.

Cam rotates in anticlockwise ; Lift of follower : 4 cm

Least radius of cam : 5 cm ; Angle of ascent : 48°

Angle of dwell between ascent and descent : 42° ;

Angle of descent = 60°

The diameter of roller = 3 cm

If cam rotates at 360 rpm, find maximum velocity and acceleration of the follower during descent. **(20 Marks)**

OR

10 a. Explain with sketch in brief 'radial cam' and 'cylindrical cam'. **(06 Marks)**

b. Obtain expressions for displacement, velocity and acceleration for a flat faced follower in contact with circular flank of a cam. **(14 Marks)**

* * * * *

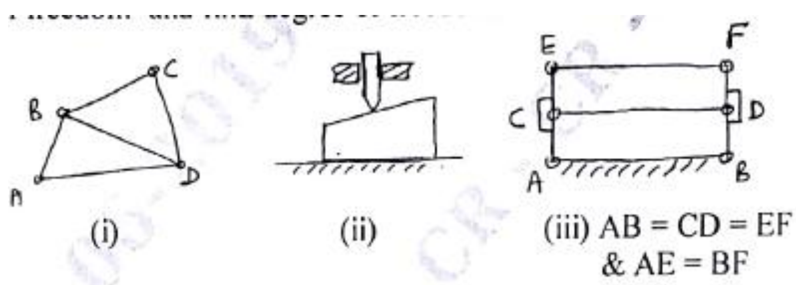
<table>
<tr><td>1
[a]</td><td>
Degrees of freedom: An unconstrained rigid body moving in space can have three translations and three rotational motions (that is six motions) about three mutually perpendicular axes. The number of degrees of freedom of a kinematic pair is defined as the number of independent relative motions, both translational and rotational that a kinematic pair can have.
Degrees of freedom = 6 – number of restraints


Fig. 1a
(i) $n=5$; $j_1=2$; $j_2=2$ $\therefore j = 1\times2 + 2\times2 = 6$; $h=0$; $n_r=0$; $j_r=0$; $F_r=0$
$F = 3(n-n_r-1) - 2(j-j_r) - h - F_r = 0$
(ii) $n=3$; $j_1=2$ $\therefore j=2$; $n_r=0$; $j_r=0$; $h=1$; $F_r=0$
$F = 3(3-0-1) - 2(2-0) - 1 - 0 = 1$
(iii) $n=5$; $n_r=1$; $j_r=2$; $j_1=6$ $\therefore j=6$; $h=0$; $F_r=0$.
$F = 3(5-1-1) - 2(6-2) - 0 - 0 = 1$.
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[b]</td><td>
Inversion of Kinematic chain: A kinematic chain becomes a mechanism when one of its links is fixed. Therefore, the number of mechanisms that can be obtained is as many as the number of links in the kinematic chain. This method of obtaining different mechanisms by fixing different links of a kinematic chain is called inversion of the mechanism.
The relative motion between various links is not altered as a result of inversion, but their absolute motion with respect to the fixed link may alter drastically.
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For double Crank

$L_o = s$; $l + s \le p + q$; $L_o + 300 \le 150 + 250$ $\therefore$ $L_o \le 100$ mm

For Crank rocker

$L_o \ne s$ $\therefore$ L_o may be l or may not be l.

$\therefore$ $150 + L_o \le 250 + 300$ $\therefore$ $L_o \le 400$.

or

$150 + 300 \le L_o + 250$ $\therefore$ $L_o \ge 200$.

$200 \le L_o \le 400$.

2 [a] ***Peaucellier mechanism***

It consists of a fixed link OO1 and the other straight links O1A, OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections. The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O_1A.

AC = CB = BD = DA : OC =OD: and $OO_1 = O_1A$

It may be proved that the product OA × OB remains constant, when the link O_1A rotates. Join CD to bisect AB at R.

Now from right angled triangles ORC and BRC, We have

$$OC^2 = OR^2 + RC^2 \quad ..(i)$$

and

$$BC^2 = RB^2 + RC^2 \quad ..(ii)$$

subtratcing equation (ii) from (i), we have

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)(OR - RB)$$

$$= OB \times OA$$

Since OC and BC are of constant length, therefore the product OB × OA remains constant. Hence the point B traces a straight path perpendicular to the diameter OP.

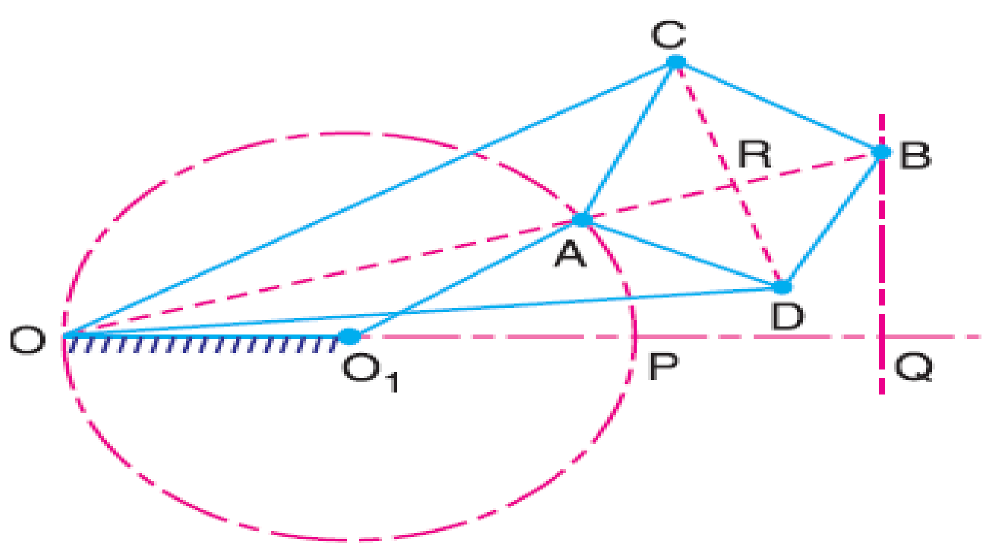


Fig 2a.: Peaucellier mechanism

2 [b]

Crank and slotted lever quick return motion mechanism

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines. In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine.

The driving crank CB revolves with uniform angular speed about the fixed centre C.

A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced.

In the extreme positions, AP1 and AP2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB1 to CB2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB2 to CB1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed,

Therefore,

$$\frac{\textit{Time of cutting stroke}}{\textit{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \ or \ \frac{360^\circ - \alpha}{\alpha}$$

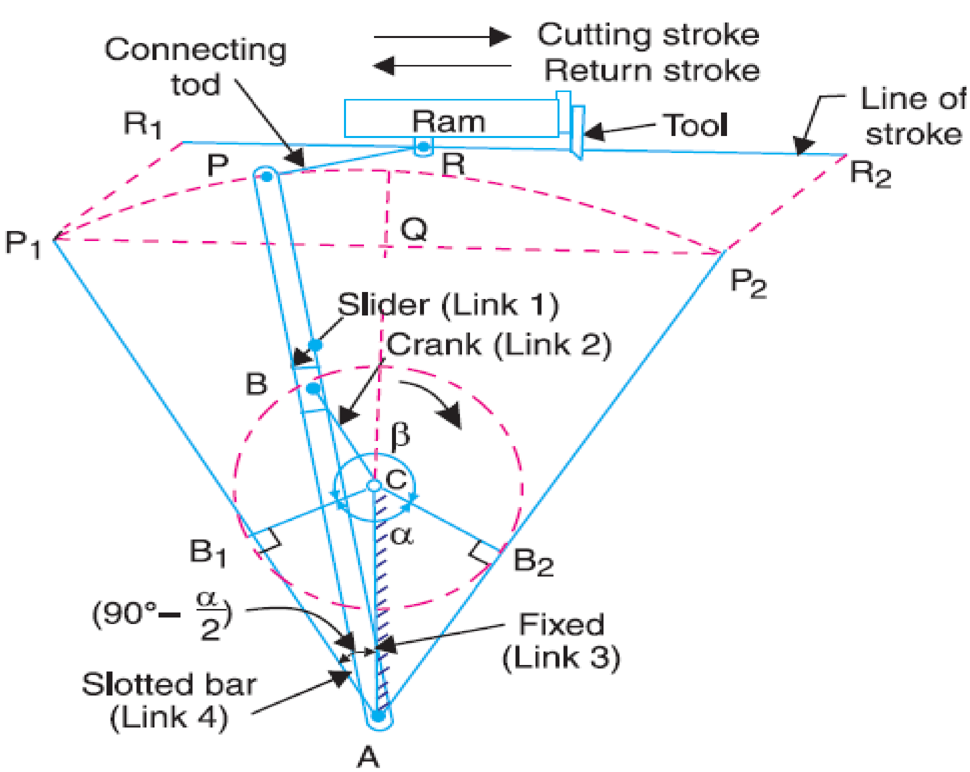


Fig.2b: Crank and Slotted lever quick return motion mechanism

$$\cos\alpha = \frac{200}{800} \quad \therefore \alpha = \cos^{-1} 0.25$$

$$\frac{T_R}{T_{cutt}} = \frac{2\alpha}{360 - 2\alpha} = 0.72$$

$$\theta = 90^\circ - \alpha.$$

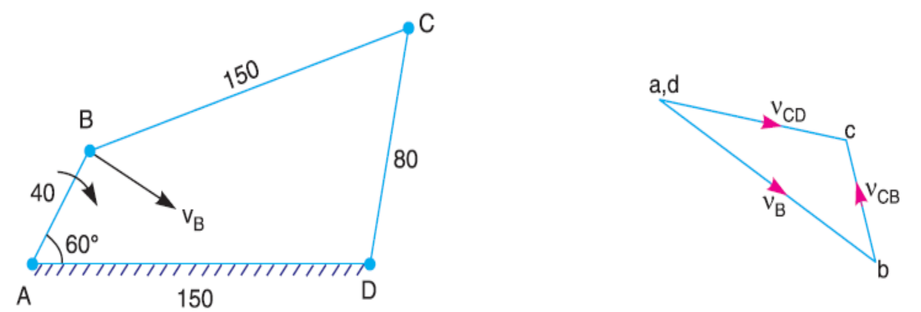
$$\text{Swing angle} = 2\theta = 28.96^\circ$$

3 [a]

Solution. Given : $N_{BA} = 120$ r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 12.568$ rad/s

Since the length of crank $AB = 40$ mm $= 0.04$ m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$



(*a*) Space diagram (All dimensions in mm). (*b*) Velocity diagram.

Fig. 7.7

First of all, draw the space diagram to some suitable scale, as shown in Fig. 7.7 (*a*). Now the velocity diagram, as shown in Fig. 7.7 (*b*), is drawn as discussed below :

1. Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to BA, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (*i.e.* v_{BA} or v_B) such that

$$\text{vector } ab = v_{BA} = v_B = 0.503 \text{ m/s}$$

2. Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (*i.e.* v_{CB}) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (*i.e.* v_{CD} or v_C). The vectors bc and dc intersect at c.

By measurement, we find that

$$v_{CD} = v_C = \text{vector } dc = 0.385 \text{ m/s}$$

We know that $CD = 80$ mm $= 0.08$ m

$\therefore$ Angular velocity of link CD,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about } D\text{)} \textbf{ Ans.}$$

4 [a]

Aronhold Kennedy (or Three Centres in Line) Theorem

The Aronhold Kennedy's theorem states that ***if three bodies move relatively to each other**, **they have three instantaneous centres and lie on a straight line.*** Consider three kinematic links *A*, *B* and *C* having relative plane motion. The number of instantaneous centres (*N*) is given by

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

Where n = Number of links = 3

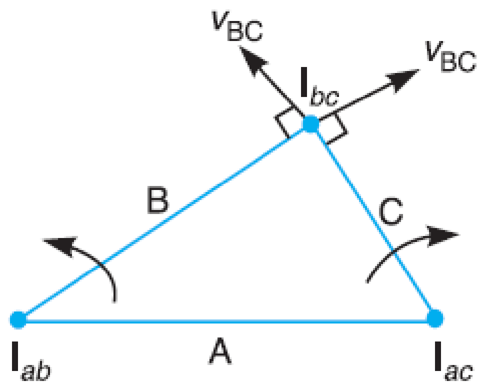


Fig.4a: **Aronhold Kennedy's theorem**

The two instantaneous centres at the pin joints of *B* with *A*, and *C* with *A* (*i.e.* I_{ab} and I_{ac}) are the permanent instantaneous centres. According to Aronhold Kennedy's theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this, let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in Fig.1.

The point I_{bc} belongs to both the links *B* and *C*. Let us consider the point I_{bc} on the link *B*. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc}. Now consider the point I_{bc} on the link *C*. Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc}.

The velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines I_{ab} I_{bc} and I_{ac} I_{bc} unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac}. Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line. The exact location of I_{bc} on line I_{ab} I_{ac} depends upon the directions and magnitudes of the angular velocities of *B* and *C* relative to *A*.

4 [b]

Klien's Construction

Let *OC* be the crank and *PC* the connecting rod of a reciprocating steam engine, as shown in Fig. 4. Let the crank makes an angle θ with the line of stroke *PO* and rotates with uniform angular velocity ω rad/s in a clockwise direction. The Klien's velocity and acceleration diagrams are drawn as discussed below:

Klien's velocity diagram

First of all, draw *OM* perpendicular to *OP*; such that it intersects the line *PC* produced at *M*. The triangle *OCM* is known as ***Klien's velocity diagram***. In this triangle *OCM*,

OM may be regarded as a line perpendicular to *PO*,

CM may be regarded as a line parallel to *PC*, and ...(It is the same line.)

CO may be regarded as a line parallel to CO.

We have already discussed that the velocity diagram for given configuration is a triangle *ocp* as shown in Fig. 4. If this triangle is revolved through 90°, it will be a triangle $oc_1\ p_1$, in which oc_1 represents v_{CO} (*i.e.* velocity of *C* with respect to *O* or velocity of crank pin *C*) and is paralel to *OC*, op_1 represents v_{PO} (*i.e.* velocity of *P* with respect to *O* or velocity of cross-head or piston *P*) and is perpendicular to *OP*, and c_1p_1 represents v_{PC} (*i.e.* velocity of *P* with respect to *C*) and is parallel to *CP*. A little consideration will show that the triangles oc_1p_1 and *OCM* are similar. Therefore,

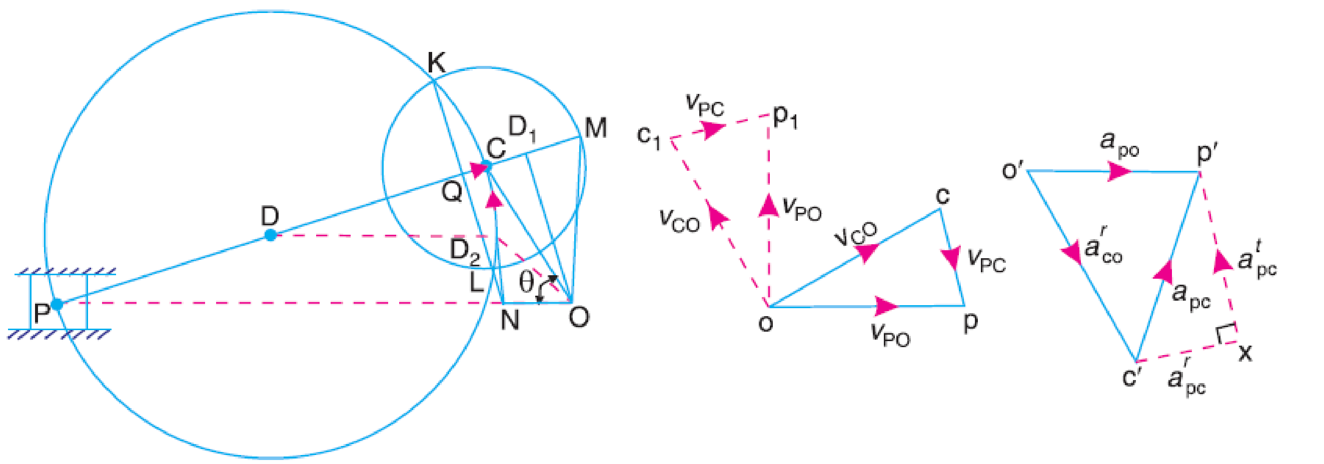
$$\frac{oc_1}{OC} = \frac{op_1}{OM} = \frac{c_1p_1}{CM} = \omega\ (a\ constant)$$

or

$$\frac{v_{CO}}{OC} = \frac{v_{PO}}{OM} = \frac{v_{PC}}{CM} = \omega$$

Therefore, $v_{co} = \omega \times OC$; $v_{PO} = \omega \times OM$ and $v_{PC} = \omega \times CM$

Thus, we see that by drawing the Klien's velocity diagram, the velocities of various points may be obtained without drawing a separate velocity diagram.



(*a*) Klien's acceleration diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig. 4b: Klein's construction

Klien's acceleration diagram

The Klien's acceleration diagram is drawn as discussed below:

1. First of all, draw a circle with *C* as centre and *CM* as radius.
2. Draw another circle with *PC* as diameter. Let this circle intersect the previous circle at *K* and *L*.
3. Join *KL* and produce it to intersect *PO* at *N*. Let *KL* intersect *PC* at *Q*. This forms the quadrilateral *CQNO*, which is known as ***Klien's acceleration diagram.***

We have already discussed that the acceleration diagram for the given configuration is as shown in Fig. 6 (*c*). We know that

i) *o'c'* represents a^r_{CO} (*i.e.* radial component of the acceleration of crank pin *C* with respect to *O*) and is parallel to *CO*;

ii) $c'x$ represents a^r_{PC} (*i.e.* radial component of the acceleration of crosshead or piston *P* with respect to crank pin *C*) and is parallel to *CP* or *CQ*;

iii) xp' represents a^t_{PC} (*i.e.* tangential component of the acceleration of *P* with respect to *C*) and is parallel to *QN* (because *QN* is perpendicular to *CQ*); and

iv) $o'p'$ represents a_{PO} (*i.e.* acceleration of *P* with respect to *O* or the acceleration of piston *P*) and is parallel to *PO* or *NO*.

A little consideration will show that the quadrilateral $o'c'x\,p'$ [Fig. 6 (*c*)] is similar to quadrilateral *CQNO* [Fig. 4]. Therefore,

$$\frac{o'c'}{OC} = \frac{c'x}{CQ} = \frac{xp'}{QN} = \frac{o'p'}{NO} = \omega^2 \ (a\ constant)$$

$$\frac{a^r_{CO}}{OC} = \frac{a^r_{PC}}{CQ} = \frac{a^t_{PC}}{QN} = \frac{a_{PO}}{NO} = \omega^2$$

Therefore, $a^r_{CO} = \omega^2 \times OC$; $a^r_{PC} = \omega^2 \times CQ$

$a^t_{PC} = \omega^2 \times QN$; and $a_{PO} = \omega^2 \times NO$

Thus we see that by drawing the Klien’s acceleration diagram, the acceleration of various points may be obtained without drawing the separate acceleration diagram.

6

(1) Freudenstein's Equation for four bar mechanism

A design problem where the link lengths of a four bar mechanism must be determined so that the rotations of the two levers within the mechanism, ϕ and ψ, are functionally related.

The desired relation is represented by $f(\phi, \psi) = 0$.

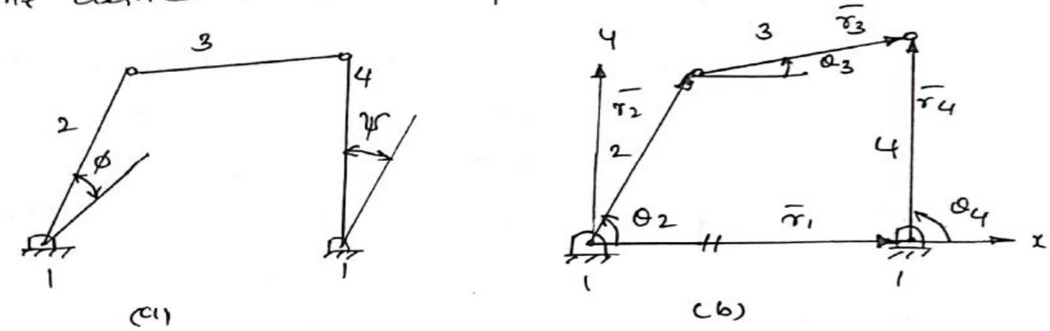


Fig.(b) shows the four bar mechanism and the vector loop necessary for the mechanism's analysis. The vector loop equation is,

$$\bar{r}_2 + \bar{r}_3 - \bar{r}_4 - \bar{r}_1 = 0 \quad -(1)$$

Considering the links to be vectors, displacement along the x- axis is,

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 = 0$$

$$\therefore\ r_3 \cos\theta_3 = -r_2 \cos\theta_2 + r_4 \cos\theta_4 + r_1 \quad -(2)$$

Squaring equation (2)

$$r_3^2 \cos\theta_3^2 = r_2^2 \cos\theta_2^2 + r_4^2 \cos\theta_4^2 + r_1^2 - 2r_2 r_4 \cos\theta_2 \cos\theta_4 - 2r_2 r_1 \cos\theta_2 + 2r_4 r_1 \cos\theta_4 \quad -(3)$$

Displacement along Y- axis is,

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$

$$\therefore\ r_3 \sin\theta_3 = -r_2 \sin\theta_2 + r_4 \sin\theta_4 \quad -(4)$$

Squaring equation(4)

$$r_3^2 \sin\theta_3^2 = r_2^2 \sin\theta_2^2 + r_4^2 \sin\theta_4^2 - 2r_2 r_4 \sin\theta_2 \sin\theta_4 \quad -(5)$$

	Equation (3) and (5) can be reduced to a single equation relating θ_2, θ_4 and the four link lengths by eliminating θ_3. To eliminate θ_3, add both sides of the eqn (3) and (5) $\therefore r_3^2\cos^2\theta_3 + r_3^2\sin^2\theta_3 = r_2^2\cos^2\theta_2 + r_4^2\cos^2_4 + r_1^2 - 2r_2r_4\cos\theta_2\cdot\cos\theta_4 - 2r_2r_1\cos\theta_2 + 2r_4r_1\cos\theta_4 + r_2^2\sin\theta_2^2 + r_4^2\sin\theta_4^2 - 2r_2r_4\sin\theta_2\sin\theta_4$ i.e., $r_3^2(\cos^2\theta_3+\sin^2\theta_3) = r_2^2(\cos^2\theta_2+\sin^2\theta_2) + r_4^2(\cos^2\theta_4+\sin^2\theta_4) + r_1^2 - 2r_2r_4\cos\theta_2\cos\theta_4 - 2r_2r_1\cos\theta_2 + 2r_4r_1\cos\theta_4 - 2r_2r_4\sin\theta_2\sin\theta_4$ i.e, $r_2^2 - r_3^2 + r_4^2 + r_1^2 + 2r_4r_1\cos\theta_4 - 2r_2r_1\cos\theta_2 = 2r_2r_4\cos\theta_2\cos\theta_4 + 2r_2r_4\sin\theta_2\sin\theta_4$ Dividing both sides by $2r_2r_4$ we get, $\frac{r_2^2-r_3^2+r_4^2+r_1^2}{2r_2r_4} + \frac{r_1}{r_2}\cos\theta_4 - \frac{r_1}{r_4}\cos\theta_2 = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4$ — (6) Let $\frac{r_1}{r_4} = R_1$; $\frac{r_1}{r_2} = R_2$ and $\frac{r_2^2-r_3^2+r_4^2+r_1^2}{2r_2r_4} = R_3$ Substituting these values in equation (6) we get, $R_3 + R_2\cos\theta_4 - R_1\cos\theta_2 = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4$ $\therefore \boxed{R_3 + R_2\cos\theta_4 - R_1\cos\theta_2 = \cos(\theta_2-\theta_4)}$ — (7) Eqn (7) is called Freudenstein's equation. * It is the relationship between input rotation θ_2 and output rotation θ_4 as determined by the link lengths r_1 through r_4. In function generation via Freudenstein's equation, the idea is to use equation (7) to determine a set of link lengths that will result in a $(\theta_2 - \theta_4)$ relationship that matches a desired function. 2
7 [a]	1. **Pitch circle**. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

	2. Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by Pc. Mathematically, Circular pitch, $P_c = \pi D/T$ Where, D = Diameter of the pitch circle, and T = Number of teeth on the wheel. A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch. **3. Diametral pitch.** It is the ratio of number of teeth to the pitch circle diameter in mm. It is denoted by Pd . Mathematically, Diametral pitch, $P_d = \frac{T}{D} = \frac{\pi}{P_c}$ But, $P_c = \frac{\pi D}{T}$ Where, T = *Number of teeth, and* D = *Pitch circle diameter*. **4. Module.** It is the ratio of the pitch circle diameter in mm to the number of teeth. It is usually denoted by m. Mathematically, m = D/T
7 [b]	**Minimum Number of Teeth on the Pinion in Order to Avoid Interference** In order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and gear wheel pass through points N and M (see Fig. 2) respectively. Let t = Number of teeth on the pinion, T = Number of teeth on the wheel, m = Module of the teeth, r = Pitch circle radius of pinion = $m.t / 2$ G = Gear ratio = $T / t = R / r$ ϕ= Pressure angle or angle of obliquity. From triangle O_1NP, $(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2\times O_1P\times PN.\cos O_1PN$ $= r^2 + R^2\sin^2\phi - 2r.R\sin\phi.\cos(90° + \phi)$ ....(since $PN = O_2P\sin\phi = R\sin\phi$)

$$= r^2\left[1+\frac{R^2 sin^2\emptyset}{r^2}+\frac{2Rsin^2\emptyset}{r}\right] = r^2\left[1+\frac{R}{r}\left(\frac{R}{r}+2\right)sin^2\emptyset\right]$$

Therefore, limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)sin^2\emptyset} = \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)sin^2\emptyset}$$

Let $A_m.m$ = Addendum of the pinion, where A_m is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion = $O_1N - O_1P$

Therefore,

$$A_m m = \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)sin^2\emptyset} - \frac{mt}{2} \qquad \left(\text{since } O_1P = r = \frac{mt}{2}\right)$$

$$= \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)sin^2\emptyset} - 1\right]$$

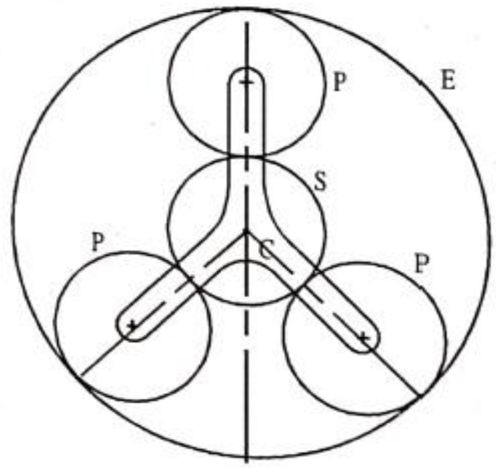
$$A_m = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)sin^2\emptyset} - 1\right]$$

Therefore,

$$t = \frac{2A_m}{\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)sin^2\emptyset} - 1}$$

$$t = \frac{2A_m}{\sqrt{1+G(G+2)sin^2\emptyset} - 1}$$

This equation gives the minimum number of teeth required on the pinion in order to avoid interference.

8	*Solution :*  *Fig : 7.31* **(i) Number of teeth on different wheels** The given arrangement is shown in Fig. 7.31 As the minimum number of teeth on any wheel is 16, take the number of teeth on sun wheel $Z_s = 16$. Since the pitch circle radius is proportional to number of teeth and the gears have same pitch $r_E = r_s + 2r_p$ i.e, $Z_E = Z_S + 2Z_P$ $\therefore Z_p = \dfrac{Z_E - Z_S}{2}$ ---- (i	

Tabular Column

Condition of motion	Planet carrier C	Sunwheel S	Planet wheel P	Internal gear E
Fix the planet carrier 'C' and give + 1 rev to sunwheel S	0	+ 1	$-\frac{Z_S}{Z_P}$	$-\frac{Z_S}{Z_P}.\frac{Z_P}{Z_E} = -\frac{Z_S}{Z_E}$
Multiply by x	0	x	$-\frac{Z_S}{Z_P}.x$	$-\frac{Z_S}{Z_E}.x$
Add y	y	$y + x$	$y - \frac{Z_S}{Z_P}.x$	$y - \frac{Z_S}{Z_E}.x$

Planet carrier C rotates at 1/5 of the speed of the Sunwheel S. i.e., For every 5 revolutions of the Sunwheel S, planet carrier C will make 1 revolution.

$$\therefore \quad y = 1 \text{ and } y + x = 5$$

$$\text{i.e.,} \quad 1 + x = 5, \ \therefore x = 4$$

Internal gear E is stationary

$$\text{i.e.,} \quad y - \frac{Z_S}{Z_E}.x = 0$$

$$\text{i.e.,} \quad 1 - \frac{Z_S}{Z_E}.4 = 0$$

$$\therefore Z_E = 4Z_S$$

$$= 4 \times 16 = 64$$

i.e., Number of teeth on internal gear E, $Z_E = 64$

From equation (i)

$$Z_P = \frac{Z_E - Z_S}{2} = \frac{64 - 16}{2} = 24$$

i.e., Number of teeth on planet wheel P, $Z_P = 24$

(ii) Torque necessary to keep the internal gear stationary.

From energy equation

$$T_S n_S + T_C n_C + T_E n_E = 0$$

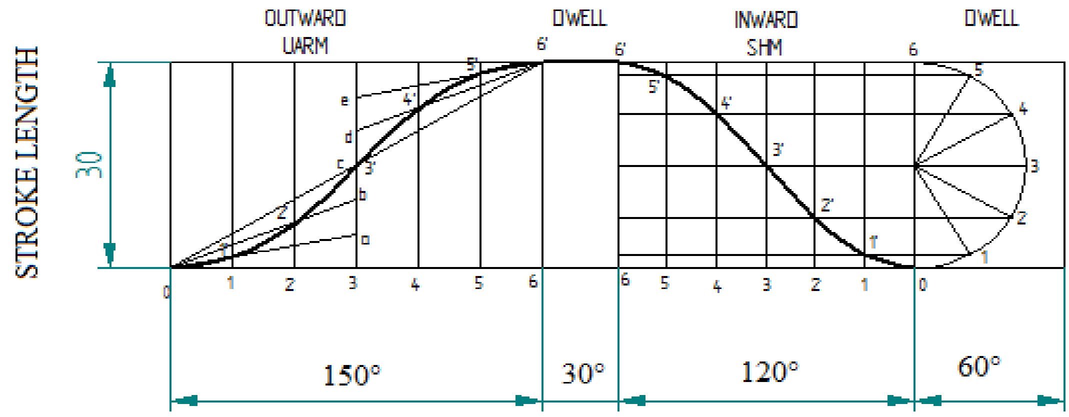
$$\text{i.e.,} \ T_S n_S + T_C n_C = 0 \quad (\because n_E = 0)$$

$$100 \times 5 + T_C \times 1 = 0$$

Torque = 500 Nm

9

DISPLACEMENT DIAGRAM



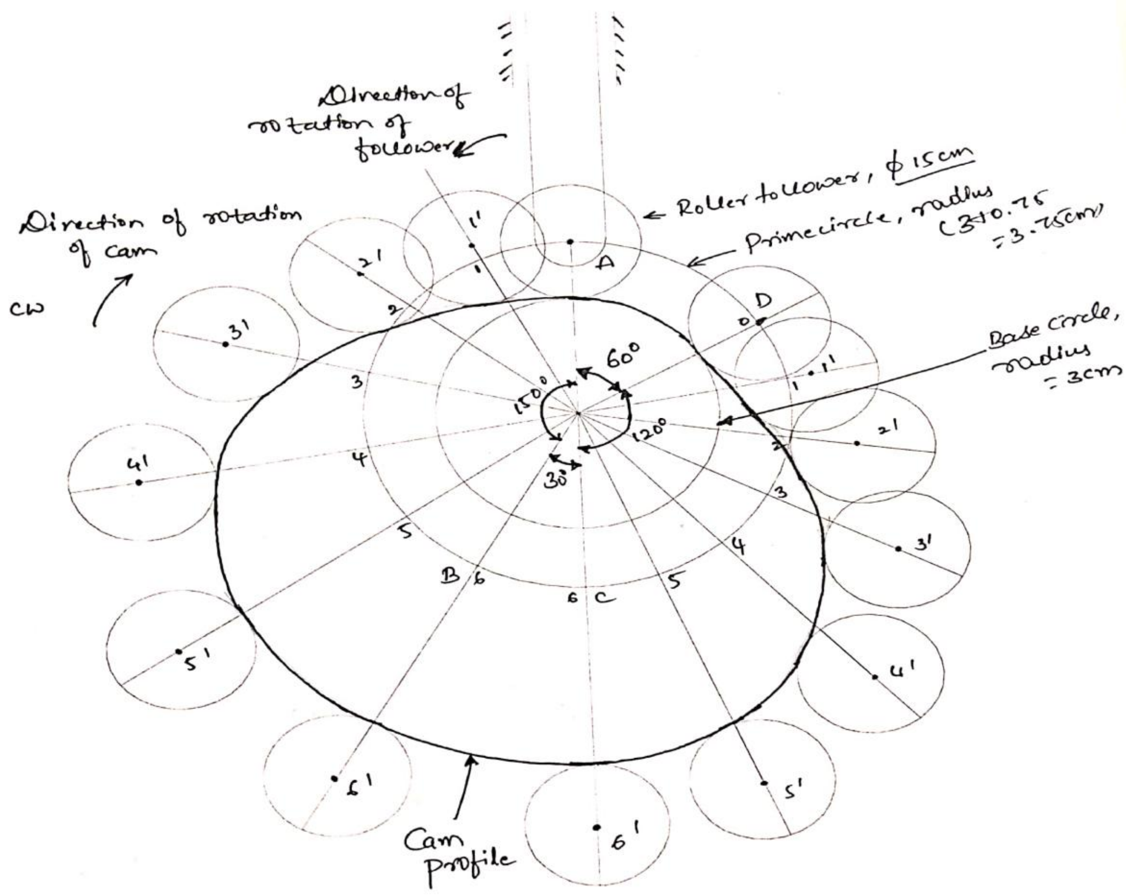


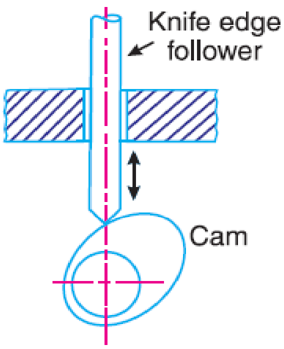
Fig.4 : Cam profile

$$V_{max} = \frac{2\omega x}{\theta_d} = \frac{2 \times 37.69 \times 4}{60 \times \frac{\pi}{180}} = 2.88 \text{ m/s}$$

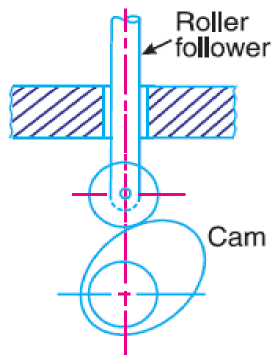
$$A_{max} = \frac{4\omega^2 x}{(\theta_d)^2} = 207.25 \text{ m/s}^2.$$

10 [a]

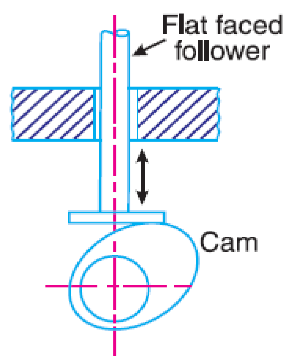
1. **Radial or disc cam:** In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in Fig. (a) to (f) are all radial cams.



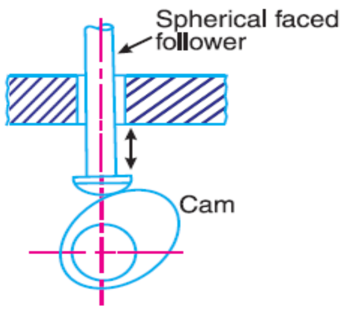
(*a*) Cam with knife edge follower.



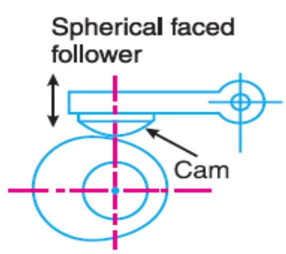
(*b*) Cam with roller follower.



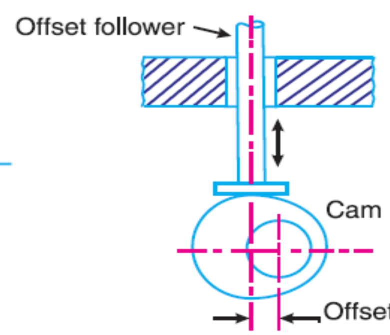
(*c*) Cam with flat faced follower.



(*d*) Cam with spherical faced follower.

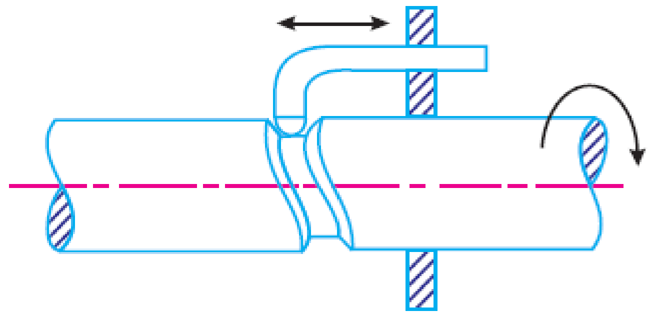


(*e*) Cam with spherical faced follower.

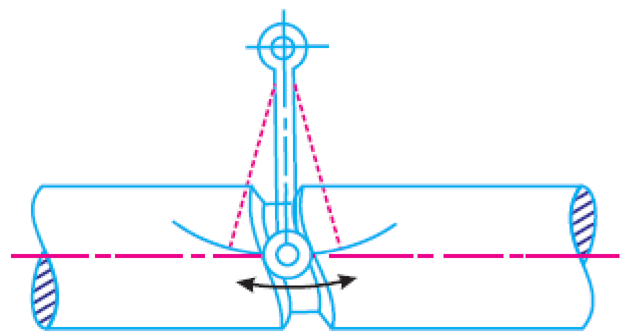


(*f*) Cam with offset follower.

2. **Cylindrical cam:** In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface.



(*a*) Cylindrical cam with reciprocating follower.



(*b*) Cylindrical cam with oscillating follower.

10 [b] When the flanks of the cam connecting the base circle and nose are of convex circular arcs, then the cam is known as ***circular arc cam.*** A symmetrical circular arc cam operating a flat-faced follower, in which *O* and *Q* are the centres of cam and nose respectively. *EF* and *GH* are two circular flanks whose centres lie at *P* and *P′* respectively. The centres *P* and *P′* lie on lines *EO* and *GO* produced.

Let r_1 = Minimum radius of the cam or radius of the base circle = *OE*,

r_2 = Radius of nose,

R = Radius of circular flank = *PE*,

2α = Total angle of action of cam = angle *EOG*,

α = Semi-angle of action of cam or angle of ascent = angle *EOK*, and

ϕ = Angle of action of cam on the circular flank.

Circular arc cam with flat face of the follower having contact with the circular flank.

We shall consider the following two cases :

1. When the flat face of the follower has contact on the circular flank, and

2. When the flat face of the follower has contact on the nose.

In deriving the expressions for displacement, velocity and acceleration of the follower for the above two cases, it is assumed that the cam is fixed and the follower rotates in the opposite sense to that of the cam. In Fig. 10b, the cam is rotating in the clockwise direction and the follower rotates in the counter-clockwise direction.

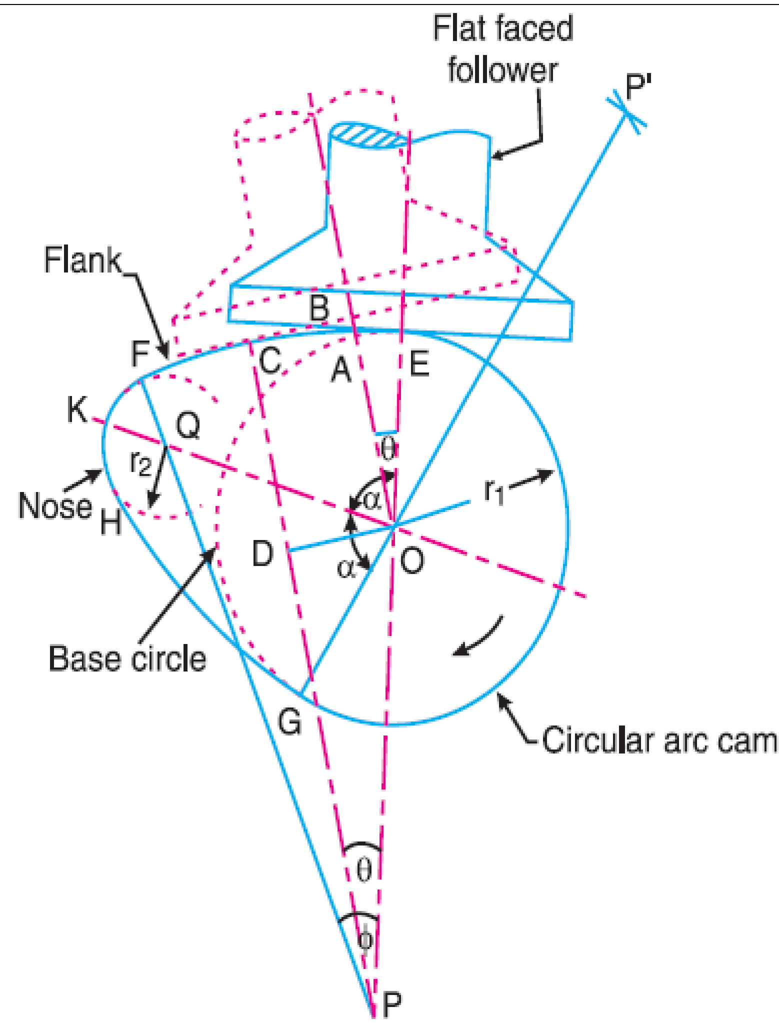


Fig. 10b

***When the flat face of the follower has contact on the circular flank*.** First of all, let us consider that the flat face of the follower has contact at *E* (*i.e.* at the junction of the circular flank and base circle). When the cam turns through an angle θ (less than ϕ) relative to the follower, the contact of the flat face of the follower will shift from *E* to *C* on the circular flank, such that flat face of the follower is perpendicular to *PC*. Since *OB* is perpendicular to *BC*, therefore *OB* is parallel to *PC*. From *O*, draw *OD* perpendicular to *PC*. From the geometry of the figure, the displacement or lift of the follower (*x*) at any instant for contact on the circular flank, is given by

$$x = BA = BO - AO = CD - EO \qquad \ldots (i)$$

We know that

$$CD = PC - PD = PE - OP\cos\theta$$

$$= OP + OE - OP\cos\theta = OE + OP(1-\cos\theta)$$

Substituting the value of CD in equation ***(i)***,

$$x = OE + OP(1-\cos\theta) - EO = OP(1-\cos\theta)$$

$$= (PE - OE)(1-\cos\theta) = (R - r_1)(1-\cos\theta) \qquad \ldots (ii)$$

Differentiating equation ***(ii)*** with respect to t, we have velocity of the follower,

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = \frac{dx}{d\theta} \times \omega \qquad \ldots \left(\text{substituting } \frac{d\theta}{dt} = \omega\right)$$

$$= (R - r_1)\sin\theta \times \omega = \omega(R - r_1)\sin\theta \qquad \ldots (iii)$$

From the above expression, we see that at the beginning of the ascent (*i.e.* when $\theta = 0$), the velocity is zero (because $\sin 0 = 0$) and it increases as θ increases. The velocity will be maximum when $\theta = \phi$, *i.e.* when the contact of the follower just shifts from circular flank to circular nose. Therefore maximum velocity of the follower,

$$v_{max} = \omega(R - r_1)\sin\phi$$

Now differentiating equation ***(iii)*** with respect to t, we have acceleration of the follower,

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} = \frac{dv}{d\theta} \times \omega$$

$$= \omega(R - r_1)\cos\theta \times \omega = \omega^2 (R - r_1)\cos\theta \qquad \ldots (iv)$$

From the above expression, we see that at the beginning of the ascent (*i.e.* when $\theta = 0$), the acceleration is maximum (because $\cos 0 = 1$) and it decreases as θ increases. The acceleration will be minimum when $\theta = \phi$.

$\therefore$ Maximum acceleration of the follower,

$$a_{max} = \omega^2 (R - r_1)$$

and minimum acceleration of the follower,

$$a_{min} = \omega^2 (R - r_1)\cos\phi$$