CBCS SCHEME

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17CS36

Third Semester B.E. Degree Examination, Aug./Sept.2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Let p and q be primitive statements for which the conditional $p \to q$ is false. Determine the truth value of the following compound propositions $p \wedge q$, $\neg p \vee q$, $q \to p$, $\neg q \to \neg p$ **(07 Marks)**

b. Show that $S \vee R$ is a tautology implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$ using rules of inference. **(07 Marks)**

c. Define Converse, Inverse and Contra positive with an illustration. **(06 Marks)**

OR

2 a. Define tautology. Show that for any proposition p, q, r the compound propositions

$[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(06 Marks)**

b. Prove the following logical equivalence

$\{(p \to q) \wedge [\neg q \wedge (r \wedge \neg q)]\} \Leftrightarrow \neg(q \vee p)$ **(07 Marks)**

c. Find whether the following argument is valid or not.

If a triangle has 2 equal sides, then it is isosceles
If a triangle is isosceles, then it has 2 equal angles
<u>A certain ΔABC does not have 2 equal angles</u>
$\therefore$ The ΔABC does not have 2 equal sides. **(07 Marks)**

Module-2

3 a. Prove by mathematical induction that, for all integer $n \geq 1$.

$$1 + 2 + 3 + \ldots\ldots\ldots\ldots + n = \frac{1}{2}n(n+1)$$ **(08 Marks)**

b. The Fibonacci numbers are designed recursively by $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Evaluate F_2 to F_{10}. **(04 Marks)**

c. Find the number of permutations of the letters of the word MASSASAUGA. In how many of these, all 4 A's are together? How many of them begin with S? **(08 Marks)**

OR

4 a. Prove by mathematical induction that $1^2 + 3^2 + 5^2 + \ldots\ldots\ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ for all integers $n \geq 1$. **(08 Marks)**

b. The Lucas number's are defined recursively by $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. Evaluate L_2 to L_{10} **(06 Marks)**

c. There are four bus routes between the places A and B, three bus routes between the places B and C. Find the number of ways a person can make a round trip from A to C via B, if he does not use a route more than once. **(06 Marks)**

Module-3

5 a. Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$

Determine $f(0)$, $f(-1)$, $f(5/3)$, $f^{-1}(-1)$, $f^{-1}(-3)$, $f^{-1}(6)$, $f^{-1}([-5, 5])$. **(07 Marks)**

b. ABC is an equilateral triangular, whose sides are of length 1 cm each. If we select 5 points inside the triangle, prove that atleast two of these points are such that the distance between then is less than ½ cm. **(06 Marks)**

c. Let A = {1, 2, 3, 4} and R be a relations on A defined by xRy if and only if "x divides y", written x/y. Write down R as a set of order pairs, draw the diagraph of R and determine indegree and outdegree of the vertices of the graph. **(07 Marks)**

OR

6 a. State pigeon hole principle. A bag contains 12 pairs of socks (each pair in different color). If a person drawn the socks one by one at random, determine atmost how many draws are required to get atleast one pair of matched socks. **(05 Marks)**

b. Let f, g, h be functions from z to z defined by f(x)= x – 1,g(x)= 3x, $h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$

Determine (fo(goh))(x) and ((fog)oh)(x) and verify that fo(goh) = (fog)oh. **(07 Marks)**

c. Let, R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 4), (2, 2), (3, 3), (4, 4)} be relation, verify that R is a partial ordering relation or not. If yes, draw the Hasse diagram for R. **(08 Marks)**

Module-4

7 a. Determine the number of positive integers n such that $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. **(07 Marks)**

b. Find the number of derangements of 1, 2, 3, 4 and list them. **(05 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases by 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day? **(08 Marks)**

OR

8 a. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, FUN or BYTE occurs? **(08 Marks)**

b. An Apple, a Banana, a Mango and an Orange are to be distributed to four boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have Apple, the boy B_3 does not want Banana or Mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(07 Marks)**

c. Solve the recurrence relation

$a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \ge 2$, given that $a_0 = 5$, $a_1 = 12$. **(05 Marks)**

Module-5

9 a. Define Isolated vertex, complete graph, Trail path with example. **(06 Marks)**

b. Explain Konigsberg bridge problem. **(07 Marks)**

c. Using the mergesort method, sort the list

7, 3, 8, 4, 5, 10, 6, 2, 9 **(07 Marks)**

OR

10 a. If G(V, E) is a simple graph, prove that

$2|E| \le |V|^2 - |V|$ **(06 Marks)**

b. Prove that a tree with n vertices has n – 1 edges. **(06 Marks)**

c. Obtain the prefix code represented by the following labeled complete binary tree shown in Fig.Q10(c) and also find the code for the words abc, cdb, bde. **(08 Marks)**

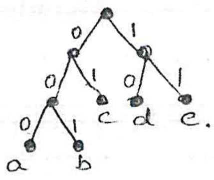

Fig.Q10(c)

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