USN

10CS42

Fourth Semester B.E. Degree Examination, Aug./Sept. 2020
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Show that there is no graph with 12 vertices and 28 edges where
   i) the degree of each vertex is either 3 or 4
   ii) the degree of each vertex is either 3 or 6. **(06 Marks)**

b. Define Isomorphism. Show that in a graph G, the number of odd degree vertices is even. **(07 Marks)**

c. A connected graph G has an Euler circuit, if and only if all vertices of G are of even degree. **(07 Marks)**

2 a. Define : i) Bipartite graph ii) Planar graph iii) Hamilton cycle . Give one example each. **(06 Marks)**

b. Find the dual graph for the following planar graph shown in Fig 2(b). Write down any four observations.

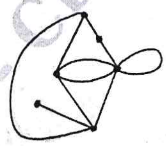

Fig 2(b) **(07 Marks)**

c. Find the Chromatic polynomial for the graph, shown in Fig 2(c). If 5 colors are available, in how many ways can the vertices of this graph be properly colored?

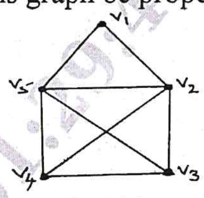

Fig 2(c) **(07 Marks)**

3 a. Define spanning tree. Draw all the spanning trees of the graph shown in Fig Q3(a).

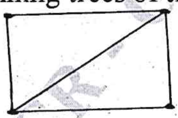

Fig Q3(a) **(07 Marks)**

b. Apply merge- sort to the list
$-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3.$ **(06 Marks)**

c. Obtain an optimal prefix code for the message "ROAD IS GOOD". Indicate the code. **(07 Marks)**

4 a. Explain Dijkstra's algorithm. **(06 Marks)**

b. Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown in Fig Q4(b).

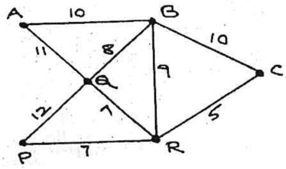

Fig Q4(b) **(07 Marks)**

c. Explain the max – flow – min – cut theorem, apply this to networks shown in Fig Q4(c) to find the maximum flow possible between the vertices A and E.

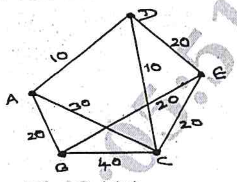

Fig Q4(c) **(07 Marks)**

PART – B

5 a. In how many ways one can distribute ten identical white marbles among six distinct containers? **(06 Marks)**

b. i) Evaluate $\binom{12}{5,\ 3,\ 2,\ 2}$

ii) Find the co-efficient of

(1) x^9y^3 in the expansion of $(2x - 3y)^{12}$ and

(2) xyz^2 in the expansion of $(2x - y - z)^4$ **(07 Marks)**

c. Define Catalan number In how many ways can one arrange three 1's and three –1's so that all six partial sums (starting with the first summand) are non negative? List all the arrangements. **(07 Marks)**

6 a. There are 30 students in a hostel In that 15 study history, 8 study economics and 6 study geography. It is known that 3 students study all these subjects, show that 7 or more students study none of these subjects. **(06 Marks)**

b. Determine the number of positive integers 'n' where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(07 Marks)**

c. What is the expansion formula for rook, polynomial? Find the rook polynomial for the board C shown below in Fig Q6(c)

		1
	2	3
4	5	6
7	8	

Fig Q6(c) **(07 Marks)**

7 a. Find the generating function for the following sequences

i) $0^2, 1^2, 2^2, 3^2, \ldots\ldots$ and

ii) numeric function $a_r = \begin{cases} 2^r, & \text{if r is even} \\ -2^r, & \text{if r is odd} \end{cases}$ **(06 Marks)**

b. In how many ways can we distribute 24 pencils to 4 children so that each child gets at least 3 pencils but not more than eight. **(07 Marks)**

c. Using exponential generating function, find the number of ways in which 4 of the letters in ENGINE be arranged. **(07 Marks)**

8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

b. Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200$, $n \geq 0$ and $a_0 = 3000$, $a_1 = 3300$. **(07 Marks)**

c. Solve the recurrence relation $a_{n+2} + 3a_{n+1} + 2a_n = 3^n$ for $n \geq 0$, given $a_0 = 0$, $a_1 = 1$. **(07 Marks)**

* * * * *