

CBCS SCHEME

USN

| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|

18MCA23

Second Semester MCA Degree Examination, Aug./Sept.2020 Discrete Mathematical Structures and Statistics

Time: 3 hrs.

Max. Marks: 100

Note: Answer FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Examine whether the following conditional is a tautology:
 $\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$ (06 Marks)
- b. Using the laws of logic prove the following logical equivalence:
 $\neg[\{(p \vee q) \wedge r\} \rightarrow \neg q] \Leftrightarrow \neg[\neg\{(p \vee q) \wedge r\} \vee \neg q] \Leftrightarrow q \wedge r$ (07 Marks)
- c. Write down the following proposition in symbolic form and find its negation:
"All integers are rational numbers and some rational numbers are not integers". (07 Marks)

OR

- 2 a. Test whether the following argument is valid:
$$\begin{array}{l} p \rightarrow r \\ \neg p \rightarrow q \\ \hline q \rightarrow s \\ \hline \therefore \neg r \rightarrow s \end{array}$$
 (06 Marks)
- b. Indicate how many rows are needed for the truth table of the compound proposition:
 $(p \vee \neg q) \leftrightarrow [(\neg r \wedge s) \rightarrow t]$
Find the truth value of this proposition if p and r are true and q, s, t are false. (07 Marks)
- c. Give (i) an indirect proof (ii) proof by contradiction of the following statement
For every integer n, if n^2 is odd then n is odd. (07 Marks)

Module-2

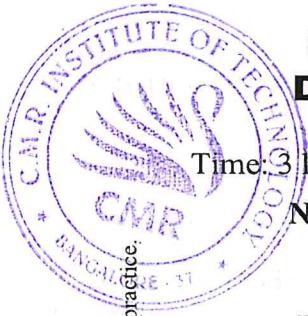
- 3 a. Using Venn diagram prove that for any three sets A, B, C
 $\overline{(A \cup B) \cap C} = (\overline{A} \cap \overline{B}) \cup \overline{C}$ (07 Marks)
- b. In a class of 52 students, 30 are studying C++, 28 are studying Java and 13 are studying both languages:
i) How many in this class are studying at least one of these languages?
ii) How many are studying neither of these languages? (06 Marks)
- c. State and prove Baye's theorem. (07 Marks)

OR

- 4 a. Using laws of set theory simplify the following:
 $\overline{\overline{(A \cup B) \cap C} \cup B}$ (07 Marks)
- b. If A and B are two events, then prove:
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (06 Marks)
- c. In a class 70% are boys and 30% are girls 5% of boys, 3% of the girls are irregular to the classes:
i) What is the probability of a student selected at random is irregular to the class?
ii) What is the probability that the irregular student is a girl? (07 Marks)

CMRIT LIBRARY
BANGALORE - 560 037

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.



Module-3

- 5 a. Prove by mathematical induction, $4n < n^2 - 7$ for all positive integers $n \geq 6$. (07 Marks)
 b. The Lucas numbers are defined recursively by $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. Evaluate L_2 to L_{10} . (06 Marks)
 c. How many positive integers 'n' we can form using the digits 3, 4, 4, 5, 5, 6 and 7 if we want 'n' to exceed 5,000,000? (07 Marks)

OR

- 6 a. Prove the following by using Mathematical induction for every positive integer $n \geq 1$:

$$1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{1}{3} n(n+1)(n+2)$$
 (07 Marks)
 b. Find the number of arrangements of the letter TALLAHASSEE which have no adjacent A's. (07 Marks)
 c. Write a recursive formula for the sequence 3, 8, 15, 24,..... (06 Marks)

Module-4

- 7 a. Derive the mean and standard deviation of the binomial distribution. (07 Marks)
 b. The probability distribution of a finite random variable X is given by

| | | | | |
|------|-----|-----|-----|-----|
| X | 10 | 20 | 30 | 40 |
| P(x) | 1/8 | 3/8 | 3/8 | 1/8 |

Find the mean and variance. (07 Marks)

- c. If x is normally distributed with mean 12 and standard deviation 4, find:
 i) $P(x \geq 20)$ ii) $P(x \leq 20)$ given $A(2) = 0.4772$ (06 Marks)

OR

- 8 a. When a coin is tossed 4 times, find the probability of getting
 i) exactly one head ii) at most 3 heads iii) at least 2 heads (07 Marks)
 b. The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of taxi drivers with
 i) no accident in a year ii) more than 3 accidents in a year. (07 Marks)
 c. If x is an exponential variate with mean 3. Find: i) $P(x > 1)$ ii) $P(x < 3)$ (06 Marks)

Module-5

- 9 a. Ten participants in a contest are ranked by two judges as follows:

| | | | | | | | | | | |
|-----|---|---|---|----|---|---|---|----|---|---|
| x : | 1 | 6 | 5 | 10 | 3 | 2 | 4 | 9 | 7 | 8 |
| y : | 6 | 4 | 9 | 8 | 1 | 2 | 3 | 10 | 5 | 7 |

Calculate the rank correlation coefficient ρ . (06 Marks)

- b. The two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - .50x$.

Find: i) Mean of x's ii) Mean of y's iii) Correlation coefficient between x and y (07 Marks)

- c. Fit a second degree parabola of the form $y = a + bx + cx^2$ to the following data:

| | | | | | |
|-----|---|-----|-----|-----|-----|
| x : | 0 | 1 | 2 | 3 | 4 |
| y : | 1 | 1.8 | 1.3 | 2.5 | 2.3 |

(07 Marks)

OR

- 10 a. Fit a curve of the form $y = ae^{bx}$ for the data

| | | | |
|-----|------|----|-------|
| x : | 0 | 2 | 4 |
| y : | 8.12 | 10 | 31.82 |

(10 Marks)

- b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

| | | | | | | | | | | |
|---|---|---|----|---|----|----|----|----|----|----|
| x | 1 | 3 | 4 | 2 | 5 | 8 | 9 | 10 | 13 | 15 |
| y | 8 | 6 | 10 | 8 | 12 | 16 | 16 | 10 | 32 | 32 |

(10 Marks)
