15MAT31

Third Semester B.E. Degree Examination, Aug./Sept. 2020 Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Obtain the Fourier series of $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$ and hence deduce that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

(08 Marks)

b. Express y as a Fourier series upto the second harmonics given:

х	0	1 🛷	2	3	4	5
y	4	8	15	7	6	2

(08 Marks)

OF

Obtain the Fourier series for $f(x) = e^{-x}$ in the interval 0 < x < 2.

(06 Marks)

b. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$.

(05 Marks)

c. Expand f(x) = 2x - 1 as a cosine half range Fourier series in $0 \le x < 1$.

(05 Marks)

Module-2

3 a. Find the Fourier transform of

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$$f(x) = \begin{cases} 1 - |x| & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

And hence deduce that $\int_{0}^{10} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$

(06 Marks)

b. Find the Fourier cosine transform of

$$f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & \text{else where} \end{cases}$$

(05 Marks)

c. Find the z - transform of : i) $\cos n\theta$ ii) $\sin n\theta$.

(05 Marks)

OR

4 a. Obtain the Fourier transform of $f(x) = xe^{-|x|}$.

(06 Marks)

b. If $u(z) = \frac{2z^2 + 3z}{(z+2)(z-4)}$, find the inverse z-transform.

(05 Marks)

c. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using z – transforms. (05 Marks)

Module-3

5 a. Compute the co-efficient of correlation and equation of lines of regression for the data:

х	1	2	3	4	5	6	7
у	9	8	10	12	11	13	14

(06 Marks)

b. Fit a best fitting parabola $y = ax^2 + bx + c$ for the following data:

X	1	2	3	4	5
у	10	12	13	16	19

(05 Marks)

c. Use the Regula – Falsi method to find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places. (05 Marks)

OR

6 a. Find the co-efficient of correlation for the following data:

Х	10	14	18	22	26	30
у	18	12	24	6	30	36

(06 Marks)

b. Fit a least square geometric curve $y = ae^{bx}$ for the following data:

X	₩ 0	2	4
У	8.12	10	31.82

(05 Marks)

c. Use Newton – Raphson method to find a real root of the equation : $x \log_{10}^{x} = 1.2$ correct to four decimal places that is near to 2.5. (05 Marks)

Module-4

- 7 a. From the following table find the number of students who have obtained:
 - i) Less than 45 marks
 - ii) Between 40 and 45 marks.

	C TORRY		a Table		
Marks	30 - 40	40 – 50	50 - 60	60 - 70	70 - 80
Number of students	3 1	42	51	35	31

(06 Marks)

- b. Find the Legrange's interpolation polynomial for the following values y(1) = 3, y(3) = 9, y(4) = 30 and y(6) = 132. (05 Marks)
- c. Evaluate $\int_{0}^{1} \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}$ th rule. (05 Marks)

OR

- 8 a. Give $u_{20} = 24.37$, $u_{22} = 49.28$, $u_{29} = 162.86$ and $u_{32} = 240.5$ find u_{28} by Newton's divided difference formula. (06 Marks)
 - b. Extrapolate for 25.4 given the data using Newton's backward formula:

х	19	20	21	22	23
у	91	100.25	110	120.25	131

(05 Marks)

c. Evaluate: $\int_{0}^{1} \frac{x}{1+x^2} dx$ by Weddle's rule taking seven ordinates.

(05 Marks)

Module-5

- 9 a. Verify Green's theorem for $\oint_C (xy + y^2) dx + x^2 dy$ where C is the closed curve of the region bounded by y = x and $y = x^2$. (06 Marks)
 - b. Derive Euler's equation in the form $\frac{\partial f}{\partial y} \frac{d}{dx} \left(\frac{\partial f}{\partial y^1} \right) = 0$. (05 Marks)
 - c. If $\overrightarrow{F} = xyi + yzj + zxk$ evaluate $\int_{C} \overrightarrow{F} \cdot d\overrightarrow{r}$ where C is the curve represented by x = t, $y = t^2$, $z = t^3$, $-1 \le t \le 1$. (05 Marks)

OR

- 10 a. Verify Green's theorem in the plane for $\int_C (x^2 + y^2) dx + 3x^2 y dy$ where C is the circle $x^2 + y^2 = 4$ traced in the positive sence. (06 Marks)
 - b. Evaluate $\int_C (xydx + xy^2dy)$ by Stoke's theorem C is the square in the x-y plane with the vertices (1, 0), (-1, 0), (0, 1) and (0, 1). (05 Marks)
 - c. Prove that the geodesics on a plane are straight lines. (05 Marks)