Module-1

1 a. Find the rank of the matrix,

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

By reducing it to the echelon form.

(05 Marks)

b. Solve the following system of equations by Gauss Elimination method.

$$4x + y + z = 4$$

$$x + 4y - 2z = 4$$

$$3x + 2y - 4z = 6$$

(05 Marks)

c. Find all the eigen values and the eigen vector corresponding to the least eigen value of the matix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(06 Marks)

OR

2 a. Find the rank of the matrix,

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

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By applying elementary row transformations.

(05 Marks)

b. Solve the following system of equations, by Gauss-Elimination method:

$$x + 2y + z = 3,$$

$$2x + 3y + 3z = 10$$
,

$$3x - y + 2z = 13$$

(05 Marks)

c. Using Cayley-Hamilton theorem, find the inverse of the matrix,

$$\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

(06 Marks)

Module-2

3 a. Solve:
$$(D^2 - 6D + 9)y = e^x + e^{3x}$$

(05 Marks)

b. Solve:
$$(D^2 + 3D + 2)y = 1 + 3x + x^2$$

(05 Marks)

c. Using the method of variation of parameters, solve:

$$(D^2 + 1)y = \sec x \tan x.$$

(06 Marks)

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OR

4 a. Solve:
$$(D^3 - 5D^2 + 8D - 4)y = e^{2x}$$
. (05 Marks)

b. Solve:
$$(D^2 - 2D + 4)y = e^x \cos x$$
. (05 Marks)

c. By the method of undetermined coefficients, solve: $(D^2 - D - 2)y = 10\sin x.$ (06 Marks)

Module-3

5 a. Find the Laplace transform of,

(i)
$$\sin^2 2t$$
 (ii) $e^{-t}(3\sinh 2t - 2\cosh 3t)$ (05 Marks)

b. Find
$$L\left\{\frac{\cos at - \cos bt}{t}\right\}$$
. (05 Marks)

c. If
$$f(t) = t^2$$
, $0 < t < 2$ and $f(t+2) = f(t)$ for $t > 2$. Find $\alpha \{f(t)\}$. (06 Marks)

OR

6 a. Find $L\{\sin t \sin 2t \sin 3t\}$. (05 Marks)

b. Find (i)
$$L\{te^{-t}\sin 4t\}$$
 (ii) $L\{\int_0^t e^{-t}\cos tdt\}$. (05 Marks)

c. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$ in terms of unit-step function and hence find $L\{f(t)\}$.

(06 Marks)

Module-4

7 a. Find the inverse Laplace transform of:

(i)
$$\frac{3s-4}{16-s^2}$$
 (ii) $\frac{s}{s^2-a^2}$ (06 Marks)

b. Find
$$L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$$
 (05 Marks)

c. Solve the equation, $y'' + 4y' + 3y = e^{-t}$, with y(0) = 1, y'(0) = 1, using Laplace transforms. (05 Marks)

OR

8 a. Find
$$L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$$
. (06 Marks)

b. Find
$$L^{-1} \left\{ log \left(\frac{s^2 + a^2}{s^2 + b^2} \right) \right\}$$
. (05 Marks)

c. Solve the equation $y'' + 6y' + 9y = 12t^2e^{-3t}$, with y(0) = y'(0) = 0, using Laplace transforms. (05 Marks)

Module-5

9 a. For any two events A and B, prove that

(i)
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(ii)
$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$
 (05 Marks)

b. Given
$$P(A) = 0.4$$
, $P\left(\frac{B}{A}\right) = 0.9$ and $P\left(\frac{B}{\overline{A}}\right) = 0.6$, find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{A}{\overline{B}}\right)$. (06 Marks)

c. State and prove Bayes's theorem. (05 Marks)

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OR

10 a. Let A and B be events with $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{4}$, $P(\overline{B}) = \frac{5}{8}$. Find $P(A \cap B)$, $P(\overline{A} \cap \overline{B})$, $P(\overline{A} \cup \overline{B})$ and $P(B \cap \overline{A})$. (06 Marks)

- b. In a certain engineering college, 25% of First semester students have failed in Mathematics, 15% have failed in Chemistry and 10% have failed in both Mathematics and Chemistry. A
 - student is selected at random.

 (i) If he has failed in Chemistry, what is the probability that he has failed in
 - Mathematics?

 (ii) If he has failed in Mathematics, what is the probability that he has failed in Chemistry?

 (05 Marks)
- c. Three machines A, B and C produce respectively 60%, 30%, 10% of total number of items in a factory. Percentage of defective output of these machines are respectively 2%, 3% and 4%. An item selected at random is found to be defective. Find the probability that it is produced by machine C. (05 Marks)

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