

CBCS SCHEME



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17MATDIP31

Third Semester B.E. Degree Examination, Aug./Sept. 2020 Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the modulus and amplitude of, $1 + \cos \alpha + i \sin \alpha$ (06 Marks)
- b. Express the complex number $\frac{(1+i)(2+i)}{(3+i)}$ in the form $a + ib$. (07 Marks)
- c. Find a unit vector normal to both the vectors $4i - j + 3k$ and $-2i + j - 2k$. Find also the sine of the angle between them. (07 Marks)

OR

- 2 a. Show that $\left[\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta} \right]^n = \cos n \left(\frac{\pi}{2} - \theta \right) + i \sin n \left(\frac{\pi}{2} - \theta \right)$. (06 Marks)
- b. If $\vec{A} = i - 2j - 3k$, $\vec{B} = 2i + j - k$, $\vec{C} = i + 3j - k$
find (i) $(\vec{A} \times \vec{B}) \times (\vec{B} \times \vec{C})$ (ii) $\vec{A} \times (\vec{B} \times \vec{C})$ (07 Marks)
- c. Show that $\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a} \right] = \left[\vec{a}, \vec{b}, \vec{c} \right]^2$. (07 Marks)

Module-2

- 3 a. If $y = (x^2 - 1)^n$ then prove that $(1 - x^2)y_{n+2} - 2xy_{n+1} + n(n+1)y_n = 0$. (06 Marks)
- b. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. (07 Marks)
- c. Show that the following curves intersect orthogonally $r = a(1 + \cos \theta)$, $r = b(1 - \cos \theta)$. (07 Marks)

OR

- 4 a. Show that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} \dots$ using Maclaurin's series expansion. (06 Marks)
- b. If $u = e^{ax+by} f(ax - by)$, prove that $b \frac{\partial u}{\partial x} + a \frac{\partial u}{\partial y} = 2abu$. (07 Marks)
- c. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. (07 Marks)

Module-3

- 5 a. Obtain a reduction formula for $\int \cos^n x dx$. (06 Marks)
- b. Evaluate $\int_0^2 \frac{x^4}{\sqrt{4-x^2}} dx$. (07 Marks)
- c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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OR

- 6 a. Obtain a reduction formula for $\int \sin^n x \, dx$. (06 Marks)
- b. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y \, dx dy$. (07 Marks)
- c. Evaluate $\int_{-c-b-a}^c \int_b^a \int_a^c (x^2 + y^2 + z^2) dz dy dx$. (07 Marks)

Module-4

- 7 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$.
 (i) Determine its velocity and acceleration.
 (ii) Find the components of velocity and acceleration at $t = 1$ in the direction $2i + j + 2k$. (06 Marks)
- b. Find the directional derivative of, $\phi = x^2 yz + 4xz^2$ at $(1, -2, -1)$ along $2i - j - 2k$. (07 Marks)
- c. If $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ find a, b, c such that $\text{curl } \vec{F} = 0$ and then find ϕ such that $\vec{F} = \nabla \phi$. (07 Marks)

OR

- 8 a. If $\vec{r} = xi + yj + zk$ and $r = \left| \vec{r} \right|$ prove that $\nabla(r^n) = nr^{n-2} \cdot \vec{r}$. (06 Marks)
- b. If $\vec{F} = (x + y + 1)i + j - (x + y)k$ show that $\vec{F} \cdot \text{curl } \vec{F} = 0$. (07 Marks)
- c. Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla \phi$. (07 Marks)

Module-5

- 9 a. Solve: $\frac{dy}{dx} = \frac{y-x}{y+x}$. (06 Marks)
- b. Solve: $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$. (07 Marks)
- c. Solve: $xy(1 + xy^2) \frac{dy}{dx} = 1$. (07 Marks)

OR

- 10 a. Solve: $\frac{dy}{dx} + y \cot x = \cos x$. (06 Marks)
- b. Solve: $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$. (07 Marks)
- c. Solve: $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$. (07 Marks)
