CBCS SCHEME

USN

18ELD/EIE/EVE/ECS/ESP/HCE11

First Semester M.Tech. Degree Examination, Aug./Sept. 2020
Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that :
i) A non empty subset w of a vector space V is a subspace of V over F iff $a\alpha + b\beta \in w\ \forall \alpha, \beta \in w$ and $a, b \in F$.
ii) The intersection of any two sub spaces of a vector space V over a field F is also a subspace of V. **(12 Marks)**

b. Show that the set B = {(1, 1, 0) (1, 0, 1) (0, 1, 1)} is a basis of the vector space $V_3(R)$. **(08 Marks)**

OR

2 a. Let V be a vector space of polynomials in x of degree atmost 3 with real co-efficient over the field of real numbers. Define the linear transformation $T : V \to V$ as $T(p) = \frac{dp}{dx}$ Find the matrix representation of T with respect to the following basics
i) $p_1 = 1, p_2 = x, p_3 = x^2, p_4 = x^3$
ii) $p_1^1 = 2,\ p_2^1 = 2x, p_3^1 = x^2, p_4^1 = \frac{x^3}{3}$. **(12 Marks)**

b. Prove that a mapping $T : U \to V$ from a vector space U(F) into a vector space V(F) is a linear transformation if $T(C_1\alpha + C_2\beta) = C_1T(\alpha) + C_2T(\beta) + \forall C_1, C_2 \in F, \alpha, \beta \in U$. **(08 Marks)**

Module-2

3 a. If $S = \{u_1, u_2, u_3, \text{- - -}\ u_p\}$ is an orthogonal set of non zero vectors in $\mathbb{R}^n$, thin prove that S is linearly independent and hence is a basis for the subspace spanned by S. **(08 Marks)**

b. Apply the Gram – Schmidt process to the vector $V_1 = (1, 0, 1)$, $V_2 = (1, 0, -1)$, $V_3 = (0, 3, 4)$ to obtain an orthonormal basis for $V_3(R)$ with standard inner product. **(12 Marks)**

OR

4 a. Using Given's method, find the eigen values of

$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{pmatrix}.$$ **(12 Marks)**

b. The set $S = \{u_1, u_2, u_3\}$ is an orthogonal basis for where $\mathbb{R}^3$ where

$$u_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} -1/2 \\ -2 \\ 7/2 \end{pmatrix}.$$ Express $y = \begin{pmatrix} 6 \\ 1 \\ -8 \end{pmatrix}$ as a linear combination of the vectors in S. **(08 Marks)**

Module-3

5 a. Derive Euler's formula in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(10 Marks)**

b. Find the external of $\int_{x_0}^{x_1} [y^2 - (y')^2 - 2y\sin x]dx$. **(10 Marks)**

OR

6 a. Find the external of the functional $I = \int_0^{\pi/4} (16y^2 - (y'')^2 + x^2]dx$ that satisfies the conditions $y(0) = 0,\ y(\pi/4) = 1,\ y'(0) = 2,\ y'(\pi/4) = 0$. **(12 Marks)**

b. Find a function y(x) for which $\int_0^1 [x^2 + (y')^2]dx$ is stationary, given that $\int_0^1 y^2 dx = 2$, $y(0) = 0$, $y(1) = 0$. **(08 Marks)**

Module-4

7 a. Define a probability mass function of a discrete random variable x, which follows the distribution.

x_i	0	1	2	3	4	5	6	7
$P(x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find :

i) k ii) mean iii) variance iv) $p(x \geq 6)$ v) $p(1 < x \leq 5)$. **(12 Marks)**

b. The number of breakdowns in a month of a computer is a random variable having a Poisson distribution with mean as 1.8, find the probability this computer will function for a month.
   i) Without a breakdown
   ii) With only one breakdown
   iii) With atleast one breakdown. **(08 Marks)**

OR

8 a. In an engineering examination, a student is considered to have failed, secured second class, first class and distinction according as his scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentage of students who have got first class and second class. (Given that $\phi(1.28) = 0.4$, $\phi(1.64) = 0.45$, $\phi(0.18) = 0.0714$). **(12 Marks)**

b. Define moment generating function of a discrete random variable X. If X assumes the values $\frac{1}{2}$ and $-\frac{1}{2}$ with probability $\frac{1}{2}$ each. Find the moment generating function and the first four moments about the origin. **(08 Marks)**

Module-5

9 a. Define a wide-sense stationary process with an example. Show that the process $\{X(t)\} = A \cos \lambda t + B \sin \lambda t$ where A and B are random variables, is wide – sense stationary if :

i) $E(A) = E(B) = 0$
ii) $E(A^2) = E(B^2)$
iii) $E(AB) = 0$. **(12 Marks)**

b. Given that the auto correlation function for a stationary ergodic process with no periodic component is

$$R_{xx}(\tau) = 25 + \frac{4}{1+6\pi^2}$$

Find the mean and the variance of the process $\{X(t)\}$. **(08 Marks)**

OR

10 a. If $\{X(t)\}$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1, t_2) = 16e^{-|t_1 - t_2|}$, find the probability that $X(10) \le 8$ and $|X(10) - X(6)| \le 4$. [Given that $\phi(0.5) = 0.1915$ and $\phi(0.7137) = 0.2611$]. **(12 Marks)**

b. Define auto correlation function of the random process :

i) Show that $R(\tau)$ is an even function of τ
ii) Show that $R(\tau)$ is maximum at $\tau = 0$. **(08 Marks)**

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