USN | | | | | | | | | | |

10MAT11

First Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) Maclaurin's series expansion of e^{-x} is

A) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+----$ B) $1-x+\frac{x^2}{2!}+\frac{x^3}{3!}+----$

C) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+----$ D) None of these

ii) The (n – 2) times derivative of x^{n-2} is

A) (n – 1)! B) (n – 2)! C) n! D) 0

iii) If $y = \cos^2 x$ then y_n is

A) $2^n \cos\left(2x+\frac{n\pi}{2}\right)$ B) $2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right)$

C) $2^{n-1}\cos\frac{n\pi}{2}$ D) $2^n \cos 2x$

iv) In the Rolles theorem if F′(c) = 1 then the tangent at x = c

A) parallel to x – axis B) parallel to y-axis

C) perpendicular to x-axis D) none of these

b. If $y = (\sinh^{-1} x)^2$ prove that $(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+n^2y_n = 0$. **(04 Marks)**

c. Verify Lagranges mean value theorem for the function $f(x)=\cos^2 x$ in $(0,\frac{\pi}{2})$. **(06 Marks)**

d. Expand $\mathrm{Tan}(\frac{\pi}{4}+x)$ using Maclaurin expansion up to 4[th] degree term. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The values of $Lt_{x\to 0}\mathrm{Tan}(5x)\mathrm{cat}(3x)$ is

A) $\frac{3}{5}$ B) $\frac{5}{3}$ C) 15 D) $\frac{1}{15}$

ii) The radius of curvature of the curve $2ap^2 = r^3$ is

A) $\frac{2}{5}\sqrt{2ar}$ B) $\frac{4}{5}\sqrt{2ar}$ C) $\frac{2}{3}\sqrt{2ar}$ D) none of these

iii) The angle between the curves $r = ae^{\theta}$ and $re^{\theta} = b$ is

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) 0 D) π

iv) The angle between two curves at the point P is $\frac{\pi}{2}$ iff $\mathrm{Tan}\phi_1\,\mathrm{Tan}\phi_2 = ----$

A) $\frac{\pi}{2}$ B) 0 C) –1 D) 1

b. Evaluate $Lt_{x\to\infty}\left[\frac{1^{1/x}+2^{1/x}+3^{1/x}}{3}\right]^{3x}$. **(04 Marks)**

c. Find the angle between the curves $r = 6\cos\theta$ and $r = 2(1+\cos\theta)$. **(06 Marks)**

d. Find the radius of curvature of the curves $x = a[\cos t + t\sin t]$ and $y = a[\sin t - t\cos t]$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $u = x^2 y\, \phi(y/x)$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = $ ----

A) 0 B) 2u C) 3u D) 4u

ii) If $AC - B^2 < 0$ and $A < 0$ then f(xy) at (a, b)

A) Max value B) Min value C) Saddle point D) none of these

iii) In a plane triangle the maximum value of cos A cos B cos C is

A) $\dfrac{3}{8}$ B) $\dfrac{1}{8}$ C) $\dfrac{5}{8}$ D) $\dfrac{7}{8}$

iv) If $u = f(r, s)$ where $x + y = r$, $s = x - y$ then $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y}$ is

A) $\dfrac{\partial u}{\partial r}$ B) $2\dfrac{\partial u}{\partial r}$ C) $\dfrac{\partial u}{\partial s}$ D) $\dfrac{\partial^2 u}{\partial r \partial s}$

b. If $u = f\left[\dfrac{y-x}{xy}, \dfrac{z-x}{zx}\right]$ prove that $x^2\dfrac{\partial u}{\partial x} + y^2\dfrac{\partial u}{\partial y} + z^2\dfrac{\partial u}{\partial z} = 0$. **(04 Marks)**

c. If $u = x^2 - y^2$ and $v = 2xy$ where $x = r\cos\theta$, $y = r\sin\theta$. Determine the values of $\dfrac{\partial(ur)}{\partial(r\theta)}$. **(06 Marks)**

d. Find the extreme values of $s = xy + 2yz + 2zx$ subject to condition $xyz = 32$. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) A gradient of scalar point function ϕ i.e. $\nabla\phi$ is

A) Scalar quantity B) Vector quantity

C) Zero D) None of these

ii) Directional derivatives is maximum along

A) Any unit vector B) Normal to the surface

C) coordinate axis D) none of these

iii) The divergence in orthogonal curvilinear coordinates $\nabla \cdot \vec{F}$ is

A) $\dfrac{1}{h_1 h_2 h_3}\Sigma\dfrac{\partial}{\partial u}(h_2 h_3 F_1)$ B) div $(F_2 e_2)$

C) $\dfrac{1}{h_1 h_2 h_3}\Sigma\dfrac{\partial}{\partial u}\left(\dfrac{h_2 h_1}{h}\dfrac{\partial F}{\partial u}\right)$ D) None of these

iv) If $\hat{e}_1 \hat{e}_2 \hat{e}_3$ are unit base vectors then $[e_1\, e_2\, e_3]$ is equal to

A) 1 B) e_1 C) 2 D) 0.

b. Show that $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find ϕ such that $E^2 = \nabla\phi$. **(04 Marks)**

c. Prove that $\nabla \times \left(\phi\vec{A}\right) = \nabla\phi \times \vec{A} + \phi\left(\nabla \times \vec{A}\right)$. **(06 Marks)**

d. Prove that gradient in orthogonal curvilinear coordinates is

$$\nabla\phi = \frac{1}{h_1}\frac{\partial\phi}{\partial u}\hat{e}_1 + \frac{1}{h_2}\frac{\partial\phi}{\partial v}\hat{e}_2 + \frac{1}{h_3}\frac{\partial\phi}{\partial w}\hat{e}_3.$$ **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^{\pi/8} \cos^3(4x)dx$ is

A) $\frac{1}{3}$ B) $\frac{1}{6}$ C) $\frac{\pi}{3}$ D) $\frac{1}{2}$

ii) One asymptote for the curve $x^3 + y^3 = 3axy$ is equal to

A) $x + y - a = 0$ B) $x - y + a = 0$

C) $x + y + a = 0$ D) $x - y - a = 0$

iii) The surface area of the sphere of radius 4cm is

A) $32cm^2$ B) $64cm^2$ C) $64\pi cm^2$ D) None of these

iv) The length of the arc for the curve $x = f(y)$ is

A) $s = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ B) $s = \int_{y_1}^{y_2} \sqrt{1+\left(\frac{dx}{dy}\right)^2}$

C) $\sqrt{2+\left(\frac{dy}{dx}\right)^2}$ D) none of these

b. Evaluate $\int_0^2 x^3\sqrt{2x - x^2}\,dx$. **(04 Marks)**

c. Derive the reduction formula of $\int \sin^m x \cos^n x\,dx$. **(06 Marks)**

d. Find the volume generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) The differential equation $\frac{dy}{dx} = \sin(x + y + 4)$ with $y(0) = 1$ is

A) Boundary value problem B) Initial value problem

C) General solution D) None of these

ii) The integrating factor of the differential equation $\frac{dy}{dx} + y\cot x = \cos x$

A) sin x B) cos x C) tan x D) sec x

iii) For the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ which of the following is not applicable

A) It is Bernoulli equation B) It is homogeneous

C) It is not exact D) $y = 2x^2/1 + x^3$

iv) The orthogonal Trajectory of the family $r = a(1 + \sin\theta)$ is

A) $a(1 + \cos\theta)$ B) $r = b(1 - \sin\theta)$

C) $r = b(1 - \cos\theta)$ D) None of these

b. Solve $(4x - 6y - 1)dx - (2x - 3y + 2)\,dy = 0$. **(04 Marks)**

c. Solve $(2x\log x - xy)dy + 2ydx = 0$. **(06 Marks)**

d. Find orthogonal Trajectory of the curve $r^n = a^n[1 + \cos n\theta]$. **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) If the Rank of matrix A ≠ Rank of Augment matrix [A : B]. Then the system of equations have

A) Solutions B) unique solution
C) infinite number of solutions D) In consistent

ii) The coefficient matrix is reduced by row operations to a diagonal form is

A) Gauss elimination B) Echolon method
C) Gauss Jordan method D) None of these

iii) If A is square matrix such that A A′ = I then the value of A′ A

A) A^2 B) I C) A^{-1} D) None of these

iv) A square matrix in which $a_{ij} = a_{ji}$ for all i and j then it is called

A) Unique matrix B) Symmetric matrix C) Skew matrix D) None of these

b. Find the Rank of the matrix by reducing them to the normal form

$$\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$ **(04 Marks)**

c. Solve the system of equation by using Gauss Jordan method :

$x + y + z = 9$
$2x + y - z = 0$
$2x + 5y + 7z = 52.$ **(06 Marks)**

d. Solve by gauss elimination method :

$x - 2y + 3z = 2$
$3x - y + 4z = 4$
$2x + y - 2z = 5.$ **(06 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) The eigen values of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ are

A) 6, –1 B) 2, 3 C) –6, 1 D) –6, –1

ii) The index and signature of the quadratic form $x_1^2 + 2x_2^2 - 3x_3^2$ are respectively

A) Index = 2 Signature = 1 B) Index = 1 Signature = 2
C) Index = 1 Signature =1 D) None of these

iii) If λ in an eigen value of symmetric matrix then λ is

A) Real B) Complex C) Both (A) and (B) D) None of these

iv) The matrix of the quadratic from $4x^2 - 2y^2 + z^2 - 2xy + 6xz$ is

A) $\begin{bmatrix} 4 & 1 & 3 \\ -1 & -2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 4 & -1 & 3 \\ -1 & -2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 4 & -1 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ D) None of these

b. Find inverse transformation of the following linear transformation :

$y_1 = x_1 + 2x_2 + 5x_3$
$y_2 = 2x_1 + 4x_2 + 11x_3$
$y_3 = -x_2 + 2x_3.$ **(04 Marks)**

c. Find the eigen values and eigen vector for small eigen value

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$ **(06 Marks)**

d. Reduce the quadratic form $3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2zx$ into canonical form. Find its nature. **(06 Marks**