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Third Semester B.E. Degree Examination, Aug./Sept.2020
Advanced Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a. If n is positive integer prove that :

$$(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1} \cos \frac{n\pi}{6}.$$
 (06 Marks)
 - b. Sum the series : $1 + x \cos \theta + \frac{x^2}{2!} \cos 2\theta + \frac{x^3}{3!} \cos 3\theta + \dots + \infty.$ (07 Marks)
 - c. Put the complex number $\left(\frac{2+i}{3-i}\right)^2$ into polar form. (07 Marks)

- 2 a. Find the n^{th} derivative $e^{ax} \cos(bx + c).$ (07 Marks)
 - b. Find the n^{th} derivative of $\frac{x}{(2x+1)(x-2)}.$ (06 Marks)
 - c. If $y = e^{m \sin^{-1} x}$ prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0.$ (07 Marks)

- 3 a. With usual notations prove that $\tan \phi = \frac{r d\phi}{dr}.$ (07 Marks)
 - b. Find the pedal equation : $r^m = a^m \cos m\theta.$ (06 Marks)
 - c. Expand $\log(1 + \sin^2 x)$ in powers of x as far as the term in $x^6.$ (07 Marks)

- 4 a. If $Z = e^{ax+by} f(ax - by)$ prove that $b \frac{\partial Z}{\partial x} + a \frac{\partial Z}{\partial y} = 2ab z.$ (06 Marks)
 - b. If $Z = f(x, y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$ prove that $\frac{\partial Z}{\partial u} - \frac{\partial Z}{\partial v} = x \frac{\partial Z}{\partial x} - y \frac{\partial Z}{\partial y}.$ (07 Marks)
 - c. If $u = \frac{xy}{z}, v = \frac{yz}{x}, w = \frac{zx}{y}.$ Find $\frac{\partial(x, y, z)}{\partial(u, v, w)}.$ (07 Marks)

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART - B

- 5 a. Derive the reduction formula for $I_n = \int_0^{\pi/2} \cos^n x \, dx$. (07 Marks)
- b. Evaluate $\int_0^1 x^5 \sin^{-1} x \, dx$. (06 Marks)
- c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dx \, dy$. (07 Marks)
- 6 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$. (07 Marks)
- b. Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta} \, d\theta$ in terms of gamma functions. (06 Marks)
- c. Prove that $\beta(m, 1/2) = 2^{2m-1} \beta(m, m)$. (07 Marks)
- 7 a. Solve $(x + y + 1)^2 \frac{dy}{dx} = 1$. (07 Marks)
- b. Solve $(x \tan \frac{y}{x} - y \sec^2 \frac{y}{x}) dx + x \sec^2 \frac{y}{x} dy = 0$. (06 Marks)
- c. Solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$. (07 Marks)
- 8 a. Solve $\frac{d^4 y}{dx^4} + y = \sin 2x \sin x$. (07 Marks)
- b. Solve $\frac{d^2 y}{dx^2} + 4y = x^4$. (06 Marks)
- c. Solve $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$. (07 Marks)
