USN

14MAT11

First Semester B.E. Degree Examination, Aug./Sept.2020

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least ONE question from each part.***

Module-1

1 a. If $y = e^{m\cos^{-1}x}$, prove that $(1-x^2)y_2 - xy_1 = m^2y$ and hence show that : $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. **(07 Marks)**

b. Show that the pair of curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersect each other orthogonally. **(06 Marks)**

c. Find the radius of curvature of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$. **(07 Marks)**

2 a. If $y = \sin^{-1}x$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. **(07 Marks)**

b. Find the pedal equation of the curve $r(1-\cos\theta) = 2a$. **(06 Marks)**

c. Show that the radius of curvature for the catenary of uniform strength $y = a\log(\sec(x/a))$ is $a\sec(x/a)$. **(07 Marks)**

Module-2

3 a. Expand $\tan^{-1}x$ in ascending powers of x upto the term containing x^5. **(07 Marks)**

b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$. **(06 Marks)**

c. Find the extreme values of the function f(x, y) given as $x^3 + y^3 - 63(x+y) + 12xy$. **(07 Marks)**

4 a. If $u = \sin^{-1}\left(\frac{x^8+y^8}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 7\tan u$. **(07 Marks)**

b. If $u = \frac{1}{r^2}\cos 2\theta$, prove that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. **(06 Marks)**

c. Evaluate $\underset{x\to 0}{\text{Lt}}\ \frac{xe^x - \log(1+x)}{x^2}$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve $R = (t^3-4t)i + (t^2+4t)j + (8t^2-3t^3)k$ where t denotes time. Find the magnitudes of acceleration along the tangent and normal at time t = 2. **(07 Marks)**

b. Using differentiation under integral sign, evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}dx, \alpha \geq 0$. **(06 Marks)**

c. Trace the curve $y^2(a-x) = x^2(a+x)$. **(07 Marks)**

6 a. Show that $\vec{F} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is a conservative force field. Find its scalar potential. **(07 Marks)**

b. Find the directional derivatives of the function xyz along the direction of the normal to the surface $x^2z + xy^2 + yz^2 = 3$ at (1, 1, 1). **(06 Marks)**

c. Find divF, Curl F, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x\, dx$. **(07 Marks)**

b. Solve $(2xy + y - \tan y)\, dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. **(06 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

8 a. Evaluate $\int_0^\infty \frac{x^4}{(1+x^2)^4} dx$. **(07 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(06 Marks)**

c. Find the orthogonal trajectories of the family $r^n \cos n\theta = a^n$. **(07 Marks)**

Module-5

9 a. Solve $x + y + z = 9$, $2x + y - z = 0$, $2x + 5y + 7z = 52$, by Gauss elimination method. **(07 Marks)**

b. Diagonalize the matrix, $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. **(06 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz$ into canonical form. Hence indicate its nature of rank and index. **(07 Marks)**

10 a. Solve the system of equations $27x + 6y - z = 85$, $x + y + 54z = 110$, $6x + 15y + 2z = 72$ by Gauss – Seidal method to obtain the solution correct to three decimal places. (Take 4 iterations). **(07 Marks)**

b. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation. **(06 Marks)**

c. Find the dominant eigen value and the corresponding eigen vector of the matrix

$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by power method taking the initial values $[1, 1, 1]^T$. Obtain 5 iterations. **(07 Marks)**

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