CBCS SCHEME

USN 1CR17CS187

17MAT21

Second Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Solve $y''' - y'' + 4y' - 4y = \sin h(2x+3)$. **(06 Marks)**

b. Solve $y'' + 2y' + y = 2x + x^2$. **(07 Marks)**

c. Solve $(D^2+1)y = \tan x$ by method of variation of parameter. **(07 Marks)**

OR

2 a. Solve $(D^3-1)y = 3\cos 2x$, where $D = \dfrac{d}{dx}$. **(06 Marks)**

b. Solve $y'' - 6y' + 9y = 7e^{-2x} - \log 2$. **(07 Marks)**

c. Solve $y'' - 3y' + 2y = x^2 + e^x$ by the method of un-determined coefficients. **(07 Marks)**

Module-2

3 a. Solve $x^2y'' + xy' + 9y = 3x^2 + \sin(3\log x)$. **(06 Marks)**

b. Solve $y\left(\dfrac{dy}{dx}\right)^2 + (x-y)\dfrac{dy}{dx} - x = 0$. **(07 Marks)**

c. Solve $(px-y)(py+x) = 2p$ by reducing it into Cluiraut's form by taking $X = x^2$ and $Y = y^2$. **(07 Marks)**

OR

4 a. Solve $(3x+2)^2y'' + 3(3x+2)y' - 36y = 8x^2 + 4x + 1$. **(06 Marks)**

b. Solve $p^2 + 2py\cot x - y^2 = 0$. **(07 Marks)**

c. Show that the equation $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation and find its general and singular solution. **(07 Marks)**

Module-3

5 a. Form the partial differential equation of the equation $\ell x + my + nz = \phi(x^2+y^2+z^2)$ by eliminating the arbitrary function. **(06 Marks)**

b. Solve $\dfrac{\partial^2 u}{\partial x^2} = x + y$. **(07 Marks)**

c. Derive the one dimensional heat equation $u_t = c^2.u_{xx}$ **(07 Marks)**

OR

6 a. Form the partial differential equation of the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ by eliminating arbitrary constants. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that $z = 0$ and $\frac{\partial z}{\partial y} = \sin x$ when $y = 0$. **(07 Marks)**

c. Obtain the solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables for the positive constant. **(07 Marks)**

Module-4

7 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$. **(06 Marks)**

b. Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} y^2\,dy\,dx$ by changing the order of integration. **(07 Marks)**

c. Derive the relation between Beta and Gamma function as $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ **(07 Marks)**

OR

8 a. Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}} x^3 \cdot y\,dx\,dy$ **(06 Marks)**

b. Evaluate $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ by changing into polar coordinates. **(07 Marks)**

c. Evaluate $\int_{0}^{\infty}\frac{dx}{1+x^4}$ by expressing in terms of beta function. **(07 Marks)**

Module-5

9 a. Find (i) $L[t\cos at]$ (ii) $L\left[\frac{\sin at}{t}\right]$. **(06 Marks)**

b. Find the Laplace transform of the full wave rectifier $f(t) = E\sin wt$, $0 < t < \frac{\pi}{w}$ with period $\frac{\pi}{w}$. **(07 Marks)**

c. Solve $y'' + k^2 y = 0$ given that $y(0) = 2$, $y'(0) = 0$ using Laplace transform. **(07 Marks)**

OR

10 a. Find Inverse Laplace transform of $\frac{s+2}{s^2(s+3)}$. **(06 Marks)**

b. Express the function

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t, & t > \pi \end{cases}$$

in terms of unit step function and hence find its Laplace transform. **(07 Marks)**

c. Find Inverse Laplace transform of $\frac{1}{s(s^2+a^2)}$ using convolution theorem. **(07 Marks)**

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Scheme and Solution

Sub: Engineering Mathematics - II

Sub code: 17MAT21

1) a) $y''' - y'' + 4y' - 4y = \sinh(2x+3)$

$(D^3 - D^2 + 4D - 4)y = \dfrac{e^{(2x+3)} + e^{-(2x+3)}}{2}$

A.E is $m^3 - m^2 + 4m - 4 = 0$

$m = 1, \pm 2i$

$y_{CF} = c_1 e^x + c_2 \cos 2x + c_3 \sin 2x$

$y_{P.I} = \dfrac{1}{2}\left[\dfrac{e^{(2x+3)}}{D^3 - D^2 + 4D - 4} - \dfrac{e^{-(2x+3)}}{D^3 - D^2 + 4D - 4}\right]$

$D \to (2)$ $\quad$ $D \to (-2)$

$= \dfrac{1}{2}\left[\dfrac{e^{2x+3}}{8} - \dfrac{e^{-(2x+3)}}{-24}\right]$

$y_{P.I} = \dfrac{1}{48}\left[3e^{2x+3} + e^{-(2x+3)}\right]$

$\therefore$ The complete soln is

$y = c_1 e^x + c_2 \cos 2x + c_3 \sin 2x + \dfrac{1}{48}\left[3e^{2x+3} + e^{-(2x+3)}\right]$

b) $y''' + 2y' + y = 2x + x^2$

We have $(D^2 + 2D + 1)y = 2x + x^2$

A.E is $m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$

$m = -1, -1$

$y_{CF} = (c_1 + c_2 x)e^{-x}$

$y_{P.I} = \dfrac{2x + x^2}{(D^2 + 2D + 1)}$

$$
\begin{array}{r|l}
 & x^2 - 2x + 2 \\
\hline
1 + 2D + D^2 & x^2 + 2x \\
 & x^2 + 4x + 2 \\
\hline
 & -2x - 2 \\
 & -2x - 4 \\
\hline
 & 2
\end{array}
$$

$\therefore$ The complete soln is

$y = (c_1 + c_2 x)e^{-x} + x^2 - 2x + 2$

① c) $(D^2+1)y = \tan x$ by the method of variation of Parameter

Given $(D^2+1)y = \tan x$

$\therefore$ A.E is $m^2+1=0$

$$m = \pm i$$

$$y_{CF} = c_1\cos x + c_2 \sin x$$

$$y = A(x)\cos x + B(x)\sin x$$

$$y_1 = \cos x \qquad y_2 = \sin x$$

$$y_1' = -\sin x \qquad y_2' = \cos x$$

$$W = y_1y_2' - y_2y_1' = 1, \quad Q(x) = \tan x$$

$$A(x) = \int \frac{-y_2 Q(x)}{W}dx + k_1, \quad B(x) = \int \frac{y_1 Q(x)}{W}dx + k_2$$

$$A(x) = \int \frac{-\sin x \tan x}{1}dx + k_1, \quad B(x) = \int \frac{\cos x \tan x\, dx}{1} + k_2$$

$$A(x) = \{\sin x - \log(\sec x + \tan x) + k_1\}, \quad B(x) = -\cos x + k_2$$

$$\therefore y = k_1\cos x + k_2 \sin x - \cos x \log(\sec x + \tan x).$$

2). a) $(D^3-1)y = 3\cos 2x$

A.E is $(m^3-1)=0$

$$(m-1)(m^2+m+1)=0$$

$$m=1; \quad m^2+m+1=0$$

$$m = \frac{-1}{2} \pm \frac{i\sqrt{3}}{2}$$

$$y_{cf} = c_1e^x + e^{-x/2}\left\{c_2\cos\frac{\sqrt{3}}{2}x + c_3\sin\frac{\sqrt{3}}{2}x\right\}$$

$$y_{P.I} = \frac{3\cos 2x}{D^3-1} = \frac{3\cos 2x}{(D^2.D)-1} \qquad D^2 \to -2^2 = -4$$

$$y_{P.I} = \frac{3\cos 2x}{-4D-1} = \frac{3(4D-1)\cos 2x}{-(4D+1)(4D-1)}$$

$$y_{P.I} = -\frac{3(8\sin 2x + \cos 2x)}{65} \Rightarrow y = y_{cf} + y_{PI}$$

2) b) $y'' - 6y' + 9y = 7e^{-2x} - \log 2.$

$$(D^2 - 6D + 9)y = 7e^{-2x} - \log 2$$

A.E is $m^2 - 6m + 9 = 0$

$$(m-3)^2 = 0$$

$$m = 3, 3$$

$$y_{CF} = (c_1 + c_2 x)e^{3x}$$

$$y_{PI} = \frac{7e^{-2x} - \log 2}{(D^2 - 6D + 9)}$$

$$y_{P.I} = \frac{7e^{-2x}}{(D^2 - 6D + 9)} - \frac{\log 2}{(D^2 - 6D + 9)}$$

$$y_{PI} = \frac{7e^{-2x}}{(4 + 12 + 9)} - \frac{\log 2 \cdot e^{0x}}{9}$$

$$y_{P.I} = \frac{7 \cdot e^{-2x}}{25} - \frac{\log 2}{9}$$

$$y = (c_1 + c_2 x)e^{3x} + \frac{7}{25}e^{-2x} - \frac{\log 2}{9}$$

c) $y'' - 3y' + 2y = x^2 + e^x$ by method of un-determined coefficients.

A.E is $m^2 - 3m + 2 = 0$

$$m = 1, 2$$

$$y_{CF} = c_1 e^x + c_2 e^{2x}$$

$y_{P.I} = a_0 + a_1 x + a_2 x^2 + bxe^x$ [e^x is already present in CF]

$$y' = a_1 + 2a_2 x + bxe^x + be^x$$

$$y'' = 2a_2 + bxe^x + be^x + be^x$$

$$[2a_2 + bxe^x + 2be^x] - 3a_1 - 6a_2 x - 3bxe^x - 3be^x +$$

on comparing LHS & RHS

$$\Rightarrow 2a_0 + 2a_1 x + 2a_2 x^2 + 2bxe^x = x^2 + e^x$$

$-6a_2 + 2a_1 = 0$

$+6a_2 = +2a_1$

$a_1 = 3 \times \frac{1}{2}$

$\boxed{a_1 = \frac{3}{2}}$

$\boxed{a_2 = \frac{1}{2}}$

$\cancel{b = 3b + 2b} = 1$

$\cancel{b = 0}$

$2b - 3b = 1$

$\boxed{b = -1}$

$2a_2 - 3a_1 + 2a_0 = 0$

$1 - 3a_1 + 2a_0 = 0$

$-3a_1 + 2a_0 = 1$

$\boxed{a_0 = 11/9}$

3) a) $x^2y'' + xy' + 9y = 3x^2 + \sin(3\log x)$

Take $t = \log x$ or $e^t = x$

$\therefore [D(D-1) + D + 9]y = 3e^{2t} + \sin 3t$

$(D^2+9)y = 3e^{2t} + \sin 3t$

$m^2 + 9 = 0$

$m = \pm 3i$

$y_{CF} = c_1 \cos 3t + c_2 \sin 3t$

$y_{PI} = \dfrac{3e^{2t}}{D^2+9} + \dfrac{\sin 3t}{D^2+9}$

$y_{PI} = \dfrac{3e^{2t}}{13} + \dfrac{t\cos 3t}{6}$

$\therefore y = c_1 \cos(3\log x) + c_2 \sin(3\log x) + \dfrac{3x^2}{13} - \dfrac{\log x . \cos(3\log x)}{6}$

3b) $y\left(\dfrac{dy}{dx}\right)^2 + (x-y)\dfrac{dy}{dx} - x = 0$

The given Eqn is $yp^2 + (x-y)p - x = 0$

$p = \dfrac{-(x-y) \pm \sqrt{(x-y)^2 + 4xy}}{2y}$

$p = \dfrac{(y-x) \pm (x+y)}{2y}$

$p = \dfrac{y - \cancel{x} + \cancel{x} + y}{2y}$ or $p = \dfrac{\cancel{y} - x - x - \cancel{y}}{2y}$

$p = 1$ or $p = -\dfrac{x}{y}$

$\dfrac{dy}{dx} = 1$ or $\dfrac{dy}{dx} = -\dfrac{x}{y}$

$y = x + c$ $\quad y^2 + x^2 = 2k$

$\therefore$ The General soln is $(y - x - c)(x^2 + y^2 - c) = 0$

3) c) $(px-y)(py+x)=2p$ by clairauts eqn

by taking $X=x^2$ and $Y=y^2$.

$$X=x^2 \Rightarrow \frac{dX}{dx}=2x$$

$$Y=y^2 \Rightarrow \frac{dY}{dy}=2y$$

$$p=\frac{dy}{dx}=\frac{dy}{dY}\cdot\frac{dY}{dX}\cdot\frac{dX}{dx}$$

$$p=\frac{1}{2y}\cdot P\cdot 2x$$

$$p=\frac{x}{y}P=\frac{\sqrt{X}}{\sqrt{Y}}P$$

$$(px-y)(py+x)=2p$$

$$\left[\frac{\sqrt{X}}{\sqrt{Y}}P\sqrt{X}-\sqrt{Y}\right]\left[\frac{\sqrt{X}}{\sqrt{Y}}P\sqrt{Y}+\sqrt{X}\right]=2\frac{\sqrt{X}}{\sqrt{Y}}P$$

$$(PX-Y)(P+1)=2P$$

$$Y=PX-\frac{2P}{P+1}$$

$$Y=cX-\frac{2c}{c+1}$$

$\therefore$ The g.s is $y^2=cx^2-\dfrac{2c}{c+1}$

4) a). $(3x+2)^2y''+3(3x+2)y'-36y=8x^2+4x+1$

$t=\log(3x+2)$ or $e^t=3x+2$ $\therefore x=\dfrac{(e^t-2)}{3}$

$$[9D(D-1)+9D-36]y=8\cdot\frac{1}{9}(e^t-2)^2+4\cdot\frac{1}{3}(e^t-2)+1$$

$$(D^2-4)y=\frac{1}{81}(8e^{2t}-20e^t+17)$$

$$m^2-4=0 \Rightarrow m=\pm 2$$

$$y_{CF}=c_1e^{2t}+c_2e^{-2t}$$

$$y_{P.I}=\frac{1}{81}\left[\frac{e^{2t}}{D^2-4}-\frac{20e^t}{D^2-4}+\frac{17e^{0t}}{D^2-4}\right]$$

$$y_{PI}=\frac{1}{81}\left[t\cdot 2e^{2t}+\frac{20}{3}e^t-\frac{17}{4}\right]$$

$$y=c_1(3x+2)^2+\frac{c_2}{(3x+2)^2}+\frac{1}{81}\left[2\log(3x+2)(3x+2)^2+\frac{20}{3}(3x+2)-\frac{17}{4}\right]$$

4) b). $p^2 + 2py\cot x - y^2 = 0$.

$$P = \frac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2}$$

$$P = y(-\cot x \pm \operatorname{cosec} x)$$

$P = y(-\cot x + \operatorname{cosec} x)$ or $p = y(-\cot x - \operatorname{cosec} x)$

$\frac{dy}{dx} = y(-\cot x + \operatorname{cosec} x)$ $\quad\quad$ $\frac{dy}{dx} = y(-\cot x - \operatorname{cosec} x)$

$\frac{dy}{y} = (\operatorname{cosec} x - \cot x)dx$ $\quad\quad$ $\frac{dy}{y} = (-\cot x - \operatorname{cosec} x)dx$

$y(1+\cos x) = c$ $\;|\log c = k$ $\quad\quad$ $y(1-\cos x) = c$ $\;|\log c = 5$

$\therefore$ the general soln is $\{y(1+\cos x) - c\}\{y(1-\cos x) - c\} = 0$.

4) c. Given $xp^2 + px - py + 1 - y = 0$

$$xp^2 + px + 1 = y(p+1)$$

$$y = \frac{xp(p+1)+1}{p+1}$$

$$y = px + \frac{1}{p+1} \quad \text{[Clairats form]}$$

$$y = cx + \frac{1}{c+1} \quad [\text{g.s}]$$

Diff w.r.t to c

$$0 = x - \frac{1}{(c+1)^2}$$

$$c = \frac{1}{\sqrt{x}} - 1$$

$\therefore$ g.s becomes

$$y = \left(\frac{1}{\sqrt{x}} - 1\right)x + \sqrt{x}$$

$$x + y = 2\sqrt{x}$$

$\therefore$ The singular soln is $(x+y)^2 = 4x$.

5) a) To frame the PDE by eliminating the arbitrary function.

$lx + my + nz = \phi(x^2+y^2+z^2)$

Diff w.r.t to x and y partially

$l + np = \phi'(x^2+y^2+z^2) \cdot (2x + 2zp)$ — (1)

$m + nq = \phi'(x^2+y^2+z^2) \cdot (2y + 2zq)$ — (2)

(1) ÷ (2)

$$\frac{l+np}{m+nq} = \frac{x+zp}{y+zq}$$

$(x+zp)(m+nq) = (y+zq)(l+np)$

$(mz-ny)p + (nx-lz)q = ly - mx.$

b)

$$\frac{\partial^2 u}{\partial x^2} = x+y$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = x+y$$

Integrating w.r.t to x by treating y as a constant

$$\frac{\partial u}{\partial x} = \int (x+y)\,dx + f(y)$$

$$\frac{\partial u}{\partial x} = \frac{x^2}{2} + xy + f(y)$$

Int w.r.t to x again

$$u = \frac{1}{2}\int x^2\,dx + y\int x\,dx + \int f(y)\,dx + g(y)$$

$$u = \frac{x^3}{6} + \frac{x^2y}{2} + x\,f(y) + g(y).$$

5. c) Derivation of one dimensional heat eqn $u_t = c^2 . u_{xx}$

$\therefore q_1$ the quantity of heat flowing into the cross section at a distance x in unit time is, $q_1 = -kA\left[\frac{\partial u}{\partial x}\right]_x$ per second.

-ve sign appears because heat flows in the direction of decreasing temperature.

$$\dot{q}_2 = -kA\left[\frac{\partial u}{\partial x}\right]_{x+\delta x} \text{ per second}$$

The rate of change of heat content in the segment of the rod between x and $x+\delta x$ be equal to net heat flow into this segment of the rod is

$$q - q_2 = kA\left[\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_x\right] \text{ per second} \quad \dots (1)$$

But the rate of increase of heat in the rod $\rightarrow s\rho A\,\delta x \frac{\partial u}{\partial t}$ ——— (2)

where s is the specific heat, ρ the density of the material

from (1) & (2)

$$s\rho A\delta \frac{\partial u}{\partial t} = kA\left[\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_x\right] \text{ or } s\rho \frac{\partial u}{\partial t} = k\left[\frac{\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_x}{\delta x}\right]$$

taking limit $\delta x \to 0$, we have

$$s\rho\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \text{ or } \frac{\partial u}{\partial t} = \frac{k}{s\rho}\frac{\partial^2 u}{\partial x^2} \text{ or } \frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2} \quad \dots (3) \text{ where } \frac{k}{s\rho} \text{ is}$$

- known as diffusivity constant.

Equation (3) is the one-dimensional heat equation which is second order homogeneous and parabolic type.

6) a) To frame PDE by Eliminating the arbitrary constants

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Differentiating partially wrt to x and y,

$$\frac{x}{a^2} + \frac{zp}{c^2} = 0 \text{ — (1)}, \quad \frac{y}{b^2} + \frac{zq}{c^2} = 0 \text{ — (2)}$$

(1) wrt to x

~~(2) wrt to y~~

$$\frac{1}{a^2} + \frac{1}{c^2}(zr + p^2) = 0$$

$$\therefore \frac{1}{a^2} = -\frac{zp}{c^2 x}$$

$$z\frac{\partial z}{\partial x} = xz\frac{\partial^2 z}{\partial x^2} + x\left(\frac{\partial z}{\partial x}\right)^2 \text{ is the reqd PDE}$$

6) b). $\frac{\partial^2 z}{\partial y^2} = z$, given that $z=0$ and $\frac{\partial z}{\partial y} = \sin x$ when $y=0$.

the given PDE assumes in the form of ODE,

$$\frac{d^2 z}{dy^2} = z$$

$$(D^2-1)z = 0$$

A.E is $m^2-1=0$

$$m = \pm 1$$

$$z = c_1 e^{y} + c_2 e^{-y}$$

$$z = f(x) e^{y} + g(x) e^{-y}$$

when $y=0$, $z = 0$

zero $= f(x) + g(x)$ — ①

when $y=0$, $\frac{\partial z}{\partial y} = \sin x$

$\sin x = f(x) e^{y} - e^{-y} g(x)$ | $\because y=0$

$\sin x = f(x) - g(x)$ — ②.

Solve ① & ② Simultaneously

$f(x) = \frac{\sin x}{2}$ and $g(x) = -\frac{\sin x}{2}$

$z = \sin x \sinh y$ is the reqd soln.

6c. the soln of one dimensional wave Equation

$\frac{\partial^2 u}{\partial t^2} = c^2 \cdot \frac{\partial^2 u}{\partial x^2}$ by the method of seperation of Variables

P.T.O

6(c) Solution of one-dimensional wave equation by separation of variable

One dimensional wave equation is given by $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ ---- (1)

Assume that a solution of (1) is of the form $y(x,t) = X(x)\,T(t)$

where X is a function of x and T is a function of t only

$$\frac{\partial y}{\partial x} = X'T \; ; \; \frac{\partial^2 y}{\partial t^2} = XT''$$

Also $\frac{\partial y}{\partial x} = X'T$; $\frac{\partial^2 y}{\partial x^2} = X''T$

Substituting these values in (1), we have $XT' = c^2 X''T$

By separation of variables, we get $\frac{X''}{X} = \frac{1}{c^2}\frac{T''}{T}$

Consider $\frac{X''}{X} = k \Rightarrow X'' kX \Rightarrow X'' - kX = 0$

or $\frac{\partial^2 X}{\partial x^2} - kX = 0$ or $(D^2 - k)X = 0$

Consider $\frac{1}{c^2}\frac{T''}{T} = k \Rightarrow T'' = kc^2 T \Rightarrow T'' - kc^2 T = 0$

or $\frac{d^2 T}{dt^2} - kc^2 T = 0 \Rightarrow (D^2 - kc^2)T = 0$

Thus the various possible solutions of wave equation (1) $(y = XT)$

$$y = (c_1 e^{\lambda x} + c_2 e^{-\lambda x})(c_3 e^{\lambda ct} + c_4 e^{-\lambda ct})$$

$$y = (c_5 \cos\lambda x + c_6 \sin\lambda x)(c_7 \cos\lambda ct + c_8 \sin\lambda ct)$$

$$y = (c_9 + c_{10} x)(c_{11} + c_{12} t)$$

Since the problem is that of vibration, y must be a periodic function of x

Hence the soln must involve trigonometric function. Accordingly the soln

$y = (c_5 \cos\lambda x + c_6 \sin\lambda x)(c_7 \cos\lambda ct + c_8 \sin\lambda ct)$ which can also be written in the form

$$y = (A\cos\lambda x + B\sin\lambda x)(C\cos\lambda ct + D\sin\lambda ct)$$

is the only suitable solution of the wave equation and ~~it correspond~~ corresponds to $k = -\lambda^2$.

(7).

$$I = \int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z)\,dy\,dx\,dz$$

$$I = \int_{-1}^{1}\int_{0}^{z} \left[xy + \frac{y^2}{2} + zy\right]_{x-z}^{x+z} dx\,dz$$

$$I = \int_{-1}^{1}\int_{0}^{z} (2xz + 2xz + 2z^2)\,dx\,dz$$

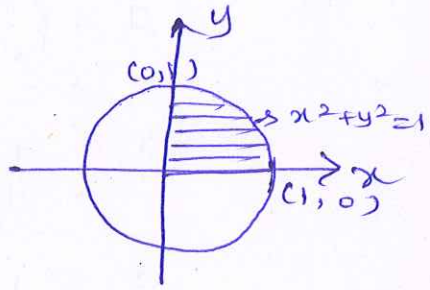
$$I = \int_{-1}^{1} (2z^3 + 2z^3)\,dz$$

$$I = [z^4]_{-1}^{1}$$

$$\boxed{I = 0}$$

7) b) $I = \int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$ by changing the order of int

$$I = \int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$$



on changing the order of int

$$I = \int_0^1 \int_0^{\sqrt{1-y^2}} y^2 \, dx \, dy$$

$$I = \int_0^1 y^2 \sqrt{1-y^2} \, dy$$

$$y = \sin\theta$$
$$dy = \cos\theta \, d\theta$$
$$\theta : 0 \to \pi/2$$

$$I = \int_0^{\pi/2} \sin^2\theta \cos^2\theta \, d\theta$$

$$\boxed{I = \frac{\pi}{16}}$$

7) c) Relationship between the Beta and gamma functions

To prove: $\beta(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma m+n}$

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$

$$\Gamma n = 2\int_0^{\infty} e^{-x^2} x^{2n-1} \, dx$$

$$\Gamma m = 2\int_0^{\infty} e^{-y^2} y^{2m-1} \, dy$$

$$\Gamma m+n = 2\int_0^{\infty} e^{-r^2} \cdot r^{2(m+n)-1} \, dr$$

$$\Gamma m \cdot \Gamma n = 4\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} x^{2n-1} y^{2m-1} \, dx \, dy$$

$x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2=r^2$, $dx\,dy = r\,dr\,d\theta$

$r : 0 \to \infty$ and $\theta : 0 \to \pi/2$

$$\Gamma m \cdot \Gamma n = \left[2\int_0^{\infty} e^{-r^2} r^{2(m+n)-1} \, dr\right]\left[2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta\right]$$

$$\beta(m,n) = \Gamma m \cdot \Gamma n \,/\, \Gamma m+n \quad //.$$

8) a) $I = \int_0^1 \int_0^{\sqrt{1-y^2}} x^3 \cdot y \, dx\,dy$

$I = \int_0^1 y \left[\frac{x^4}{4} \right]_0^{\sqrt{1-y^2}} dy$

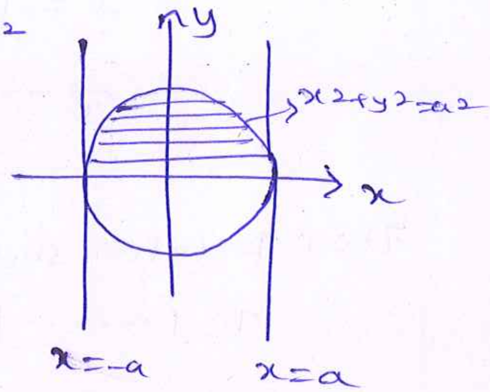
$I = \frac{1}{4} \int_0^1 y \, (1-y^2)^2 \, dy$

$I = \frac{1}{4} \int_0^1 (y - 2y^3 + y^5) \, dy$

$\boxed{I = \frac{1}{24}}$

b) $I = \int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2} \, dy\,dx$ by changing in to polars,

$x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2 = r^2$



$a^2 = r^2$

$r = a$

$\therefore r \to 0 \to a$

$dx\,dy = r\,dr\,d\theta$

$I = \int_0^{\pi} \int_0^{a} r \cdot r\,dr\,d\theta$

$I = \int_0^{\pi} \left[\frac{r^3}{3} \right]_0^a d\theta$

$\boxed{I = \frac{\pi a^3}{3}}$

c) $I = \int_0^{\infty} \frac{dx}{1+x^4}$ Take $x = \tan^{1/2}\theta$

$dx = 1/2 \tan^{-1/2}\theta \cdot \sec^2\theta \, d\theta$

$\theta : 0 \to \pi/2$

$I = \int_0^{\pi/2} \frac{\frac{1}{2}\tan^{-1/2}\theta \cdot \sec^2\theta\,d\theta}{\sec^2\theta}$

$I = \frac{1}{2} \int_0^{\pi/2} \frac{\sin^{-1/2}\theta}{\cos^{-1/2}\theta} d\theta = \frac{1}{2} \int_0^{\pi/2} \sin^{-1/2}\theta \cdot \cos^{1/2}\theta \, d\theta$

$I = \frac{1}{2} \cdot \frac{1}{2} \beta\left(\frac{-1/2+1}{2}, \frac{1/2+1}{2} \right) = \frac{1}{4} \beta\left(\frac{1}{4}, \frac{3}{4} \right) = \frac{\pi}{2\sqrt{2}}.$

9) a) $L[t\cos at] = (-1)^1 \frac{d}{ds}\left[\frac{s}{s^2+a^2}\right]$

$$L[t\cos at] = -\left\{\frac{(s^2+a^2).1 - s(2s)}{(s^2+a^2)^2}\right\}$$

$$L\{t\cos at\} = \frac{s^2 - a^2}{(s^2+a^2)^2}$$

b) $L\left[\frac{\sin at}{t}\right] = \int_s^\infty \frac{a}{s^2+a^2} ds$

$$L\left[\frac{\sin at}{t}\right] = a.\frac{1}{a}\left[\tan^{-1}(s/a)\right]_s^\infty = \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{a}\right)$$

$$L\left[\frac{\sin at}{t}\right] = \cot^{-1}\left(\frac{s}{a}\right)$$

b). $f(t) = \begin{cases} E\sin\omega t, & 0 \le t < \pi/\omega \\ 0, & \pi/\omega \le t < 2\pi/\omega \end{cases}$

the given function is periodic with $T = \frac{2\pi}{\omega}$

$$L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^{T} e^{-st} f(t)dt = \frac{1}{1-e^{-s(\frac{2\pi}{\omega})}}\int_0^{2\pi/\omega} e^{-st} f(t)dt$$

$$L\{f(t)\} = \frac{1}{1-e^{\frac{-2\pi s}{\omega}}}\int_0^{\pi/\omega} E\, e^{-st}\sin\omega t\, dt$$

$$L\{f(t)\} = \frac{E}{1-e^{-(\frac{2\pi s}{\omega})}}\left[\frac{e^{-st}}{s^2+\omega^2}\{-s\sin\omega t - \omega\cos\omega t\}\right]_0^{\pi/\omega}$$

$$= \frac{E}{1-e^{-(2\pi s/\omega)}} \cdot \frac{\omega(e^{-\pi s/\omega}+1)}{s^2+\omega^2}$$

$$\boxed{L\{f(t)\} = \frac{E\omega}{(1-e^{-\pi s/\omega})(s^2+\omega^2)}}$$

9) c). To solve $y'' + k^2 y = 0$

g.T $y(0) = 2$, $y'(0) = 0$ using L.T

$$L\{y''(t)\} + k^2 L\{y(t)\} = 0$$

$$[s^2 L y(t) - s y(0) - y'(0)] + k^2 L(y(t)) = 0$$

$$(s^2 + k^2) L(y(t)) = 2s \quad \because y'(0) = 0, \; y(0) = 2$$

$$y(t) = 2L^{-1}\left[\frac{s}{s^2+k^2}\right] = 2\cos kt$$

$$\therefore \boxed{y(t) = 2\cos kt}$$

10. a). $L^{-1}\left[\frac{s+2}{s^2(s+3)}\right]$

$$\frac{s+2}{s^2(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+3)}$$

$$s + 2 = A(s+3)(s) + B(s+3) + Cs^2$$

Put $s = -3$

$$-1 = C(9)$$

$$\boxed{C = -1/9}$$

$s = 0$

$$2 = B(3)$$

$$\boxed{B = 2/3}$$

on Expanding

$$s + 2 = As^2 + 3As + Bs + 3B + Cs^2$$

$$A + C = 0$$

$$A = -C$$

$$\boxed{A = 1/9}$$

$$\therefore L^{-1}\left[\frac{s+2}{s^2(s+3)}\right] = \frac{1}{9}L^{-1}\left[\frac{1}{s}\right] + \frac{2}{3}L^{-1}\left[\frac{1}{s^2}\right] - \frac{1}{9}L^{-1}\left[\frac{1}{s+3}\right]$$

$$L^{-1}\left[\frac{s+2}{s^2(s+3)}\right] = \frac{1}{9} + \frac{2}{3}t - \frac{1}{9}e^{-3t} \;//$$

10.b). $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \sin t & t > \pi \end{cases}$

$$f(t) = \cos t + (\sin t - \cos t)\,u(t-\pi)$$

$$L(f(t)) = L(\cos t) + L\{(\sin t - \cos t)\,u(t-\pi)\}$$

$$L\{f(t)\} = \frac{s}{s^2+1} + e^{-\pi s}\,\bar{f}(s)$$

but $f(t-\pi) = \sin t - \cos t$

$$f(t) = \sin(t+\pi) - \cos(t+\pi)$$

$$f(t) = -\sin t + \cos t$$

$$\therefore L(f(t)) = \frac{s}{s^2+1} + e^{-\pi s}\left[\frac{s}{s^2+1} - \frac{1}{s^2+1}\right] //$$

c). $L^{-1}\left[\dfrac{1}{s(s^2+a^2)}\right]$

$\bar{f}(s) = \dfrac{1}{s^2+a^2}$ and $\bar{g}(s) = \dfrac{1}{s}$

$L^{-1}[\bar{f}(s)] = \dfrac{\sin at}{a}$ $\quad L^{-1}[\bar{g}(s)] = \dfrac{1}{s}$

$f(t) = \dfrac{\sin at}{a}$ $\quad g(t) = 1$

$$L^{-1}[\bar{f}(s)\cdot\bar{g}(s)] = \int_0^t f(u)\,g(t-u)\,du$$

$$= \int_0^t \frac{1}{a}\sin au\,du \quad \Big|\; \because g(t) = 1$$

$$= \frac{1}{a}\int_0^t \sin au\,du$$

$$= \frac{1}{a}\left[\frac{-\cos au}{a}\right]_0^t$$

$$= \frac{-1}{a^2}\left[\cos at - 1\right]$$

$$L^{-1}\left[\frac{1}{s(s^2+a^2)}\right] = \frac{1}{a^2}\{1 - \cos at\} \quad //.$$