CBCS SCHEME

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18ELD11

First Semester M.Tech. Degree Examination, Jan./Feb. 2021
Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Show that the set M of all 2×2 matrices is a vector space over R. **(07 Marks)**

b. Let U and W be any two sub spaces of a vector space V, then prove that $U \cap W$ is also a subspace of V. **(07 Marks)**

c. Let $T : V \to W$ be a linear transformation, then show that range of T is a subspace of W. **(06 Marks)**

OR

2 a. Define a subspace. Show that the linear sum of two subspace of a vector space V, is also a subspace of V. **(07 Marks)**

b. Define basics vectors If, α_1, α_2, α_3 are linearly independent in $V_n(R)$ then show that $(\alpha_1 + \alpha_2)$, $(\alpha_1 + \alpha_3)$ and $(\alpha_2 + \alpha_3)$ are linearly independent in $V_n(R)$. **(07 Marks)**

c. Let $T : R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$. Find the matrix of T relative to the basis $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$. **(06 Marks)**

Module-2

3 a. Transform the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$ to tridiagonal form by Given's method. **(10 Marks)**

b. Find an orthonormal basis of a subspace of R^4 spanned by the vectors. $S = \{v_1 = (1, 1, 1, 1), v_2 = (1, 2, 4, 5), v_3 = (1, -3, -4, -2)\}$ by applying Gram-Schmidt orthogonalization process. **(10 Marks)**

OR

4 a. Use the Given's method to find the eigen values and eigen vector corresponding to the largest eigne value of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ **(10 Marks)**

b. Apply the Gram-Schmidt orthogonalization process, to find an orthonormal basis of the subspace of R^5 spanned by the vectors.

$$S = \left\{ \begin{matrix} v_1 = (1, 1, 1, 0, 1), & v_2 = (1, 0, 0, -1, 1) \\ v_3 = (3, 1, 1, -2, 3), & v_4 = (0, 2, 1, 1, -1) \end{matrix} \right\}$$ **(10 Marks)**

Module-3

5 a. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$. **(10 Marks)**

b. Find a function y(x) for which $I = \int_0^1 (x^2 + (y')^2)dx$ is stationary, given that $\int_0^1 y^2 dx = 2$, $y(0) = y(1) = 0$. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Find the extremal of the functional $v = [x, y] = \int_{x_1}^{x_2} [(y'')^2 - 2(y')^2 + y^2 - 2y\sin x]dx$. **(10 Marks)**

b. Find the shape of the curve of the given perimeter enclosing maximum area. **(10 Marks)**

Module-4

7 a. A pair of dice is thrown twice. Find the probability of scoring 7 points.
i) once ii) twice iii) atleast once. **(07 Marks)**

b. A random variable X has the density function $P(x) = Kx^2e^{-x}$, $x > 0$. Find i) K ii) Mean iii) Variance. **(07 Marks)**

c. In a city, the daily consumption of electric power can be treated as a random variable having Erlang distribution with $\lambda = \frac{1}{2}$ and $K = 3$. If the power plant has the daily capacity of 12 million Kilowatt – hours. What is the probability that this power is inadequate on any given day? **(06 Marks)**

OR

8 a. In a normal distribution 31% of the items are under 45 and 8% of the items are orver 64. Find the mean and standard deviation. [Given $\phi(1.4) = 0.42$, $\phi(0.5) = 0.19$]. **(07 Marks)**

b. A random variable X has the density function $f(x) = \begin{cases} \frac{x+1}{2} & : \ -1 \le x \le 1 \\ 0 & : \ \text{Else where} \end{cases}$. Find the first four central moments. **(07 Marks)**

c. The probability that an individual suffers a bad reaction from a certain injection is 0.001. Using Poisson distribution, find the probability that out of 2000 individuals.
i) Exactly 3 ii) More than 2 suffer a bad reaction. **(06 Marks)**

Module-5

9 a. Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide-sence stationary, If A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. **(07 Marks)**

b. The auto correlation function for a stationary Ergodic process with no periodic components is $R_{xx}(\tau) = 100 + \frac{10}{1+5\tau^2}$. Find the mean and variance of the process. **(07 Marks)**

c. Define: i) Stationary process ii) Strongly stationary process iii) Wide sence stationary process. **(06 Marks)**

OR

10 a. Define: i) Mean Erogdic process ii) Correlation Ergodic process iii) Distribution Ergodic process. **(06 Marks)**

b. Given a random variable Ω with density $f(w)$ and another random variable ϕ uniformly distributed in $(-\pi, \pi)$ and independent of Ω and $x(t) = a\cos(\Omega t + \phi)$. Prove that x(t) is a wide sense stationary process. **(07 Marks)**

c. If x(t) is a Gaussian process with $\mu(t) = 10$ and $c(t_1, t_2) = 16\ e^{-|t_1 - t_2|}$, find the probability that i) $X(10) \le 8$ ii) $|X(10) - X(6)| \le 4$. **(07 Marks)**

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