CMR
INSTITUTE OF
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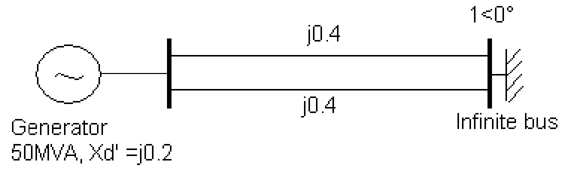
USN

Internal Assesment Test - II

Sub:	Power System Analysis II					Code:	17EE71
Date:	03/11/2020	Duration:	90 mins	Max Marks:	50	Sem: 7	Branch: EEE

Answer Any FIVE FULL Questions

		Marks	CO
1	Deduce the fast decoupled load flow model clearly stating all the assumptions made	[10]	CO2
2	In the given system, generator has a rating of 50MVA and the inertia constant H = 2.7MJ/MVA at rated speed. E = 1.05 p.u., V = 1p.u., X_d' = 0.2p.u., $X_1 = X_2$= 0.4p.u. The generator is supplying 50MW to the infinite bus. A three phase fault occurs in the middle of line 2. Plot a swing curve for a sustained fault (2 intervals) and predict about the stability of the system. The incremental time is 0.05s.	[10]	CO6



		Marks	CO
3	Derive the expression for all elements of Jacobian matrix in polar form	[10]	CO2
4	Calculate the voltages at all buses for the 3 bus system as shown in fig at the end of first iteration by NR method.	[10]	CO2

From bus	To bus	R(pu)	X(pu)	$B_c/2$
1	2	0.08	0.24	0
1	3	0.02	0.06	0
2	3	0.06	0.18	0

Bus no	P_G	Q_G	P_L	Q_L	Vsp
1	-	-	-	-	1.05
2	-	-	0.5	0.2	1.0
3	-	-	0.6	0.25	1.0

		Marks	CO
5	Solve Q.no 4 by fast decoupled method	[10]	CO2
6a	Compare NR and GS method for load flow analysis	[5]	CO2
6b	Write a short note on voltage control by generator excitation	[5]	CO2

Solutions

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7.8 DECOU

In the *NR* method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are:

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.
- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce (7.60) to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta|V|}{|V|} \end{bmatrix} \qquad ...(7.87)$$

From (7.87) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. (7.87) can be solved in two ways.

7.9 FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of Fast Decoupled Load Flow (*FDLF*) method by B. Stott in 1974. Certain assumptions are made based on observations of practical power systems. They are:

- $B_{ij} >> G_{ij}$ (Since, the *X*/*R* ratio of transmission lines is high in well designed systems)
- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i|\,|V_k|\,B_{ik}\ (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}\,|V_i|^2$$

The matrix (7.87) reduces to

$$[\Delta P] = [|V_i||V_j||B'_{ij}|][\Delta\delta] \quad \text{...(7.88a)}$$

$$[\Delta Q] = [|V_i||V_j||B''_{ij}|]\left[\frac{\Delta|V|}{|V|}\right] \quad \text{...(7.88b)}$$

where, B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (7.88) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get

$$\left[\frac{\Delta P}{|V|}\right] = [B'_{ij}][\Delta\delta] \quad \text{...(7.89a)}$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \quad \text{...(7.89b)}$$

Equations (7.89a) and (7.89b) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers.
- Setting off-nominal turns ratio of transformers to 1.0.
- In forming B'_j, omitting the effect of shunt reactors and capacitors which mainly affect reactive power.
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss less network. If further, all voltage magnitudes are assumed to be 1.0 pu, we obtain a DC power flow model. This model is acceptable where only approximate solutions are required like in planning expansions and in contingency studies. In the *FDLF* method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations. Separate convergence tests can be applied for real power and reactive power, as max $[\Delta P] \le \epsilon_P$ and max $[\Delta Q] \le \epsilon_Q$. Generally, the tolerances for power mismatch are 0.001pu.

$$P_a = m\frac{d^2\delta}{dt^2} \Rightarrow \frac{P_a}{m} = \frac{d^2\delta}{dt^2} \rightarrow \frac{\Delta\omega}{\Delta t} = \alpha$$

α_1 angular acceleration

Initial condition, acceleration = 0

$P_a = 0 \Rightarrow P_a = P_{input} - P_{output}$

(a) $0 = 1 - \frac{EV}{X}\sin\delta$ | $P_{out} = \frac{EV}{X_1}\sin\delta$

$\frac{1.05 \times 1}{0.4}\sin\delta = 1$

$\sin\delta = \frac{0.4}{1.05}$

$\delta = 22.89^\circ$

$$\frac{P_a}{m} = \frac{d^2\delta}{dt^2} = \frac{\Delta\omega}{\Delta t}$$

$$m = \frac{GH}{\pi f} = \frac{1 \times 2.7}{180 \times 50} = 3 \times 10^{-4}$$

At $t = 0 \Rightarrow P_a = \frac{P_a^{(0-)} + P_a^{(0+)}}{2} =$

$P_a(0^-) = 0$ & $P_a^{(0+)} = 1 - \frac{EV}{X_2}\sin 22.89$

$= 1 - \frac{1.05}{1}\sin 22.89 = 0.592$

$\therefore P_a = \frac{0 + 0.592}{2} = 0.296$

$\Delta\omega = \frac{P_a}{m} \times \Delta t = \frac{0.296}{3\times10^{-4}} \times 0.05 = 49.3$

$\omega_1 = \omega_0 + \Delta\omega = 0 + 49.3$

$\Delta\delta = \omega_1 \times \Delta t = 49.3 \times .05 = 2.49$

$\delta_1 = \delta_0 + \Delta\delta = 22.89 + 2.49 = 25.356$

2nd iteration

$P_a = 1 - \frac{EV}{x_2} \sin 25.356 = 1 - \frac{1.05}{1} \sin 25.356$

$= 0.551$

$\Delta\omega = \frac{P_a}{m} \times \Delta t = \frac{0.551}{3\times10^{-4}} \times .05 = 91.83$

$\omega_2 = \omega_1 + \Delta\omega = 49.3 + 91.83 = 141.13$

$\Delta\delta = \omega_2 \times \Delta t = 141.13 \times 0.05 = 7.056$

$\delta_2 = \delta_1 + \Delta\delta = 25.356 + 7.056$

$= 32.412^\circ$

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7.7 NEWTON-RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form $f(x) = 0$. Consider a set of n non-linear algebraic equations given by ...(7.26)

$$f_i(x_1, x_2 \ldots x_n) = 0 \quad i = 1, 2 \ldots n$$

Let $x_1^0, x_2^0, \ldots x_n^0$ be the initial guess of the unknown variables and $\Delta x_1^0, \Delta x_2^0, \ldots \Delta x_n^0$ be the respective corrections. Therefore, ...(7.27)

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0 \ldots x_n^0 + \Delta x_n^0) = 0 \quad i = 1, 2, \ldots n$$

(7.27) can be expanded using Taylor's series to give

$$f_i(x_1^0, x_2^0, \dots x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right)\Delta x_2^0 + \dots + \left(\frac{\partial f_i}{\partial x_n}\right)\Delta x_n^0\right]$$

$$+ \text{Higher order terms} = 0 \quad i = 1,2,\dots n \qquad \dots(7.28)$$

$\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0 \dots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2, \dots x_n$ respectively, evaluated at $(x_1^0, x_2^0, \dots x_n^0)$. If the higher order terms are neglected,(7.28) can be written in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \vdots \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & \vdots & \cdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} = 0$$

In vector form (7.29) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

$$F^0 = -J^0 \Delta X^0$$

$$\Delta X^0 = -[J^0]^{-1} F^0$$

$$X^1 = X^0 + \Delta X^0$$

to be solved are given by

$$\Delta P_i = P_{i, sp} - P_{i, cal} = 0 \quad ...(7$$

$$\Delta Q_i = Q_{i, sp} - Q_{i, cal} = 0 \quad ...(7$$

where, $P_{i, sp}$ = specified active power at bus *i*.

$Q_{i, sp}$ = specified reactive power at bus *i*.

$P_{i, cal}$ = calculated value of active power using voltage estimates.

$Q_{i, cal}$ = calculated value of reactive power using voltage estimates.

ΔP = active power residue.

ΔQ = reactive power residue.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad ...(7.35)$$

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k \quad i = 1, 2, ... n$$

The complex power S_i is given by

$$S_i = V_i I_i^*$$

$$= V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^*$$

$$= V_i \left(\sum_{k=1}^{n} Y_{ik}^* V_k^* \right)$$

Let $\quad V_i \underline{\underline{\Delta}} |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i)$

$$\delta_{ik} = \delta_i - \delta_k$$

$$Y_{ik} = G_{ik} + jB_{ik}$$

Substituting (7.7) in (7.6), we get

$$S_i = \sum_{k=1}^{n} |V_i| |V_k| (\cos \delta_{ik} + j \sin \delta_{ik}) (G_{ik} - j B_{ik})$$

$$P_i = \sum_{k=1}^{n} |V_i|\,|V_k|\,(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \qquad \text{...(7.9a)}$$

$$Q_i = \sum_{k=1}^{n} |V_i|\,|V_k|\,(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \qquad \text{...(7.9b)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form as

$$Y_{ik} = |Y_{ik}|\,\angle\theta_{ik} \qquad \text{...(7.10)}$$

Substituting (7.7a) and (7.10) in (7.6), we get

$$S_i = |V_i|\,\angle\delta_i \sum_{k=1}^{n} |Y_{ik}|\,\angle{-\theta_{ik}}\,|V_k|\,\angle{-\delta_k} \qquad \text{...(7.11)}$$

The real part of (7.11) gives P_i.

$$P_i = |V_i| \sum_{k=1}^{n} |Y_{ik}|\,|V_k|\cos(-\theta_{ik} + \delta_i - \delta_k)$$

$$= |V_i| \sum_{k=1}^{n} |Y_{ik}|\,|V_k|\cos -(\theta_{ik} - \delta_i + \delta_k)$$

Noting $\cos(-\theta) = \cos\theta$,

$$P_i = \sum_{k=1}^{n} |V_i|\,|V_k|\,|Y_{ik}|\cos(\theta_{ik} - \delta_i + \delta_k) \quad i = 1, 2, \ldots n \qquad \text{...(7.12a)}$$

Similarly, Q_i is imaginary part of (7.11) and is given by

$$Q_i = |V_i| \sum_{k=1}^{n} |Y_{ik}|\,|V_k|\sin -(\theta_{ik} - \delta_i + \delta_k)$$

Noting $\sin(-\theta) = -\sin\theta$

$$Q_i = -\sum_{k=1}^{n} |V_i|\,|V_k|\,|Y_{ik}|\sin(\theta_{ik} - \delta_i + \delta_k) \quad i = 1, 2, \ldots n \qquad \text{...(7.12b)}$$

Computation of P_{cal} and Q_{cal}

The real and reactive powers can be computed from (7.9):

$$P_{i,\,Cal} = P_i = \sum_{k=1}^{n} |V_i|\,|V_k|\,(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i|\,|V_k|\,(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$Q_{i,\,Cal} = Q_i = \sum_{k=1}^{n} |V_i|\,|V_k|\,(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i|\,|V_k|\,(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Elements of J_1

Diagonal elements

The diagonals elements of J_1 are given by $\frac{\partial P_i}{\partial \delta_i}$. From (7.36),

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1\\k\neq i}}^{n} |V_i|\,|V_k|\,\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= \sum_{\substack{k=1\\k\neq i}}^{n} -|V_i|\,|V_k|\,(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -Q_i - B_{ii}|V_i|^2$$

Off-diagonal elements

The off-diagonal elements of J_1 are $\dfrac{\partial P_i}{\partial \delta_k}$. From (7.36)

$$\frac{\partial P_i}{\partial \delta_k} = |V_i|\,|V_k|\,(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) \qquad ...(7.39)$$

(*In deriving above recollect that $\delta_{ik} = \delta_i - \delta_k$)

Recognizing $G_{ik} = |Y_{ik}| \cos \theta_{ik}$ and $B_{ik} = |Y_{ik}| \sin \theta_{ik}$, (7.39) can be written in other forms as

$$\frac{\partial P_i}{\partial \delta_k} = |V_i|\,|V_k|\,|Y_{ik}|\,(\cos \theta_{ik} \sin \delta_{ik} - \sin \theta_{ik} \cos \delta_{ik})$$

$$= |V_i|\,|V_k|\,|Y_{ik}| \sin (\delta_{ik} - \theta_{ik}) \qquad ...(7.40)$$

(7.39) can also be written in terms of real and imaginary parts of the bus voltage.

If $\qquad Y_{ik} = G_{ik} + jB_{ik}$

we define, $\qquad e_k + jf_k = |V_i|\,(\cos \delta_k + j\sin \delta_k) \qquad ...(7.41)$

and, $\qquad (a_k + jb_k) = (G_{ik} + jB_{ik})\,(e_k + jf_k)$

$$= (G_{ik} + jB_{ik})\,\{|V_k|\,(\cos \delta_k + j\sin \delta_k)\} \qquad ...(7.42)$$

From (7.42), equating real and imaginary parts we obtain,

$$a_k = |V_k|\,G_{ik} \cos \delta_k - |V_k|\,B_{ik} \sin \delta_k \qquad ...(7.43)$$

$$b_k = |V_k|\,G_{ik} \sin \delta_k + |V_k|\,B_{ik} \cos \delta_k \qquad ...(7.44)$$

Using (7.41) and (7.43), we can write

$$a_k f_i = |V_i|\,|V_k|\,G_{ik} (\sin \delta_i \cos \delta_k) - |V_i|\,|V_k|\,B_{ik} (\sin \delta_i \sin \delta_k)$$

Using (7.41) and (7.44), we get

$$b_k e_i = |V_i|\,|V_k|\,G_{ik} (\sin \delta_k \cos \delta_i) - |V_i|\,|V_k|\,B_{ik} (\cos \delta_i \cos \delta_k)$$

From (7.45) and (7.46)

$$a_k f_i - b_k e_i = |V_i|\,|V_k|\,(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik})$$

7.47) is identical to (7.39). Therefore, the off-diagonal element of J_1.

Elements of J_3

Diagonal elements

The diagonal element of J_3 is $\frac{\partial Q_i}{\partial \delta_i}$. From (7.37)

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i|\,|V_k|\,(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) = P_i - G_{ii}\,|V_i|^2$$

Off-diagonal elements

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i|\,|V_k|\,(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$$

Other forms of writing (7.49) are

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i|\,|V_k|\,|Y_{ik}| \cos (\delta_{ik} - \theta_{ik})$$

$$\frac{\partial Q_i}{\partial \delta_k} = -(a_k e_i + b_k f_i)$$

Elements of J_2

Diagonal elements

The diagonal elements are $\frac{\partial P_i}{\partial |V_i|}$. From (7.36)

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i|\,G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|\,(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) \qquad ...(7.52)$$

To obtain symmetry in the elements, (7.52) is multiplied by $|V_i|$, so that the variable on RHS can be taken as $\frac{\Delta V_i}{|V_i|}$.

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}) = P_i + |V_i|^2 G_{ii}$$

Off-diagonal elements

$$\frac{\partial P_i}{\partial |V_k|} = |V_i| (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i| |V_k| (G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$$

$$= a_k e_i + b_k f_i$$

***Elements of* J_4**

Diagonal elements

The diagonal element is $\frac{\partial Q_i}{\partial |V_i|}$ and is given by

$$\frac{\partial P_i}{\partial |V_i|} = -2|V_i| B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k| (G_{ik} \sin \delta_{ik} + B_{ik} \cos \delta_{ik}) \quad ...($$

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i| |V_k| (G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}) = Q_i - |V_i|^2 B_{ii} \quad ...$$

Off-diagonal elements

$$\frac{\partial Q_i}{\partial |V_k|} = |V_i| \left(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}\right)$$

$$\frac{\partial Q_i}{\partial |V_k|} |V_k| = |V_i|\,|V_k| \left(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}\right) = a_k f_i - b_k e_i$$

We can now rewrite (7.35) as follows:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta |V|}{|V|} \end{bmatrix}$$

The elements are summarized below

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii} |V_i|^2$

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Line impedances are converted to admittances are

$$y_{12} = 1.25 - j3.75$$
$$y_{23} = 1.667 - j5.0$$
$$y_{13} = 5.0 - j15.0$$

The voltage at Bus3 is assumed as 1+j0. The initial voltages are therefore

$$V_1^{(0)} = 1.05 + j0.0$$
$$V_2^{(0)} = 1.00 + 0.0$$
$$V_3^{(0)} = 1.00 + j0.0$$

The bus admittance matrix in polar from is

$$Y_{BUS} = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.9167 - j8.75 & -1.667 + j5.0 \\ -5 + j15 & -1.667 + j5.0 & 6.66 - j20 \end{bmatrix}$$

The bus admittance matrix in polar from is

$$Y_{BUS} = \begin{bmatrix} 19.7642\angle -71.6^\circ & 3.95285\angle 108.4^\circ & 15.8114\angle 108.4^\circ \\ 3.95285\angle 108.4^\circ & 9.22331\angle -71.6^\circ & 5.27046\angle 108.4^\circ \\ 15.8114\angle 108.4^\circ & 5.27046\angle 108.4^\circ & 21.0819\angle -71.6^\circ \end{bmatrix}$$

The real and reactive powers at bus 2 are calculated as follows:

$$P_2 = |V_2|\,|V_1|\,|Y_{21}| \cos\left(\delta_2^{(0)} - \delta_1 - \theta_{21}\right) + |V_2|^2 |Y_{22}| \cos\left(-\theta_{22}\right) + |V_2|\,|V_3|\,|Y_{23}| \cos\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$$P_2 = \left(1*1.05 * 3.95285 * \cos(-108.4)\right) + \left(1^2 * 9.22331 * \cos(71.6)\right) + \left(1*1* 5.27046 * \cos(-108.4)\right)$$

$$P_2 = -0.06228$$

$$Q_2 = |V_2||V_1||Y_{21}|\sin(\delta_2^{(0)} - \delta_1 - \theta_{21}) + |V_2|^2 |Y_{22}|\sin(-\theta_{22}) + |V_2||V_3||Y_{23}|\sin(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23})$$

$$Q_2 = (1*1.05 * 3.95285 * \sin(-108.4)) + (1^2 * 9.22331 * \sin(71.6)) + (1 * 1 * 5.27046 * \sin(-108.4))$$

$$Q_2 = -0.18754$$

The real and reactive powers at bus 3 are calculated as follows:

$$P_3 = |V_3^{(0)}||V_1||Y_{31}|\cos(\delta_3^{(0)} - \delta_1 - \theta_{31}) + |V_3^{(0)}||V_2||Y_{32}|\cos(\delta_3^{(0)} - \delta_2 - \theta_{32}) + |V_3^{(0)}|^2 |Y_{33}|\cos(-\theta_{33})$$

$$P_3 = (1*1.05 * 15.8114 * \cos(-108.4)) + (1*1* 5.27046 * \cos(-108.4)) + (1 * 1 * 21.0819 * \cos(71.6))$$

$$P_3 = -0.24953$$

$$Q_3 = |V_3^{(0)}||V_1||Y_{31}|\sin(\delta_3^{(0)} - \delta_1 - \theta_{31}) + |V_3^{(0)}||V_2||Y_{32}|\sin(\delta_3^{(0)} - \delta_2 - \theta_{32}) + |V_3^{(0)}|^2 |Y_{33}|\sin(-\theta_{33})$$

$$Q_3 = (1*1.05 * 15.8114 * \sin(-108.4)) + (1*1* 5.27046 * \sin(-108.4)) + (1 * 1 * 21.0819 * \sin(71.6))$$

$$Q_3 = -0.74328$$

The differences between scheduled and calculated power are

$$\Delta P_2^{(0)} = -0.5 - (-0.06228) = -0.43772$$

$$\Delta Q_2^{(0)} = -0.2 - (-0.18754) = -0.01246 .$$

$$\Delta P_3^{(0)} = -0.6 - (-0.24953) = -0.35047$$

$$\Delta Q_3^{(0)} = -0.25 - (-0.74328) = -0.49328$$

Iteration 1:

Elements of the Jacobian are calculated as follows:

$$\frac{\partial P_2}{\partial \delta_2} = -\left[|V_2||V_1||Y_{21}|\sin(\delta_2^{(0)} - \delta_1^{(0)} - \theta_{21}) + |V_2||V_3^{(0)}||Y_{23}|\sin(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23})\right]$$

$$\frac{\partial P_2}{\partial \delta_2} = -[(1 * 1.05 * 3.95285 * \sin(-108.4)) + (1 * 1 * 5.27046 * \sin(-108.4))]$$

$$\frac{\partial P_2}{\partial \delta_2} = 8.9393$$

$$\frac{\partial P_2}{\partial \delta_3} = -\left| V_2 V_3^{(0)} Y_{23} \right| \sin\left(\delta_2^{(0)} - \delta_3^{(0)} - \vartheta_{23} \right)$$

$$\frac{\partial P_2}{\partial \delta_3} = (1 * 1 * 5.27046 * \sin(-108.4))$$

$$\frac{\partial P_2}{\partial \delta_3} = -5.001$$

$$\frac{\partial P_2}{\partial |V_2|} = 2|V_2 Y_{22}|\cos(\vartheta_{22}) + |V_1 Y_{21}|\cos(\vartheta_{21} - \delta_2 + \delta_1) + |V_3 Y_{23}\cos(\vartheta_{23} - \delta_2 + \delta_3)|$$

$$\frac{\partial P_2}{\partial |V_2|} = (2 * 1 * 9.22331 * \cos(71.6)) + (1.05 * 3.95285 * \cos(-108.4)) + (1 * 5.27046 * \cos(-108.4))$$

$$\frac{\partial P_2}{\partial |V_2|} = 2.8541$$

$$\frac{\partial P_2}{\partial |V_3|} = |V_2 Y_{23} \cos(\vartheta_{23} - \delta_2 + \delta_3)|$$

$$\frac{\partial P_2}{\partial |V_3|} = (1 * 5.27046 * \cos(-108.4))$$

$$\frac{\partial P_2}{\partial |V_3|} = -1.6636$$

$$\frac{\partial P_3}{\partial \delta_2} = -\left| V_3^{(0)} V_2^{(0)} Y_{32} \right| \sin\left(\delta_3^{(0)} - \delta_2^{(0)} - \vartheta_{32} \right)$$

$$\frac{\partial P_3}{\partial \delta_2} = (1 * 1 * 5.27046 * \sin(-108.4))$$

$$\frac{\partial P_3}{\partial \delta_2} = -5.001$$

$$\frac{\partial P_3}{\partial \delta_3} = -\left[|V_3^{(0)}||V_1||Y_{31}| \sin\left(\delta_3^{(0)} - \delta_1^{(0)} - \theta_{31}\right) + |V_3^{0}||V_2^{(0)}||Y_{32}| \sin\left(\delta_3^{(0)} - \delta_2^{(0)} - \theta_{32}\right)\right]$$

$$\frac{\partial P_3}{\partial \delta_3} = -\left[(1 * 1.05 * 15.8114 * \sin(-108.4)) + (1 * 1 * 5.27046 * \sin(-108.4))\right]$$

$$\frac{\partial P_3}{\partial \delta_3} = 20.75$$

$$\frac{\partial P_3}{\partial |V_2|} = \left|V_3 Y_{32}\right| \cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = (1 * 5.27046 * \cos(108.4))$$

$$\frac{\partial P_3}{\partial |V_2|} = -1.6636$$

$$\frac{\partial P_3}{\partial |V_3|} = 2|V_3 Y_{33}|\cos(\theta_{33}) + |V_1 Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_2 Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_3|} = (2 * 1 * 21.0819 * \cos(-71.6)) + (1.05 * 15.8114 * \cos(108.4)) + (1 * 5.27046 * \cos(108.4))$$

$$\frac{\partial P_3}{\partial |V_3|} = 6.41$$

$$\frac{\partial Q_2}{\partial \delta_2} = \left[|V_2||V_1||Y_{21}| \cos\left(\delta_2^{(0)} - \delta_1^{(0)} - \theta_{21}\right) + |V_2||V_3^{(0)}||Y_{23}| \cos\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)\right]$$

$$\frac{\partial Q_2}{\partial \delta_2} = \left[(1 * 1.05 * 3.95285 * \cos(-108.4)) + (1 * 1 * 5.27046 * \cos(-108.4))\right]$$

$$\frac{\partial Q_2}{\partial \delta_2} = -2.9737$$

$$\frac{\partial Q_2}{\partial \delta_3} = \left|V_2 V_3^{(0)}\right| Y_{23} \cos\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$$\frac{\partial Q_2}{\partial \delta_3} = \left(1 * 1 * 5.27046 * \cos(-108.4)\right)$$

$$\frac{\partial Q_2}{\partial \delta_3} = 1.6636$$

$$\frac{\partial Q_2}{\partial |V_2|} = -2|V_2 Y_{22}| \sin(\theta_{22}) - |V_1 Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_3 Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = \left(2 * 1 * 9.22331 * \sin(71.6)\right) + \left(1.05 * 3.95285 * \sin(-108.4)\right) + \left(1 * 5.27046 * \sin(-108.4)\right)$$

$$\frac{\partial Q_2}{\partial |V_2|} = 8.5642$$

$$\frac{\partial Q_2}{\partial |V_3|} = -\left|V_2 Y_{23} \sin(\theta_{23} - \delta_2 + \delta_3)\right|$$

$$\frac{\partial Q_2}{\partial |V_3|} = -\left(1 * 5.27046 * \sin(108.4)\right)$$

$$\frac{\partial Q_2}{\partial |V_3|} = -5.001$$

$$\frac{\partial Q_3}{\partial \delta_2} = -\left|V_3^{(0)} V_2^{(0)}\right| Y_{32} \cos\left(\delta_3^{(0)} - \delta_2^{(0)} - \theta_{32}\right)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -\left(1 * 1 * 5.27046 * \cos(-108.4)\right)$$

$$\frac{\partial Q_3}{\partial \delta_2} = 1.6636$$

$$\frac{\partial Q_3}{\partial \delta_3} = \left[\left| V_3^{(0)} V_1 Y_{31} \right| \cos\left(\delta_3^{(0)} - \delta_1^{(0)} - \vartheta_{31}\right) + \left| V_3^{0} V_2^{(0)} Y_{32} \right| \cos\left(\delta_3^{(0)} - \delta_2^{(0)} - \vartheta_{32}\right) \right]$$

$$\frac{\partial Q_3}{\partial \delta_3} = \left[(1 * 1.05 * 15.8114 * \cos(-108.4)) + (1 * 1 * 5.27046 * \cos(-108.4)) \right]$$

$$\frac{\partial Q_3}{\partial \delta_3} = -6.904$$

$$\frac{\partial Q_3}{\partial |V_2|} = -\left| V_3 Y_{32} \right| \sin(\vartheta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_2|} = -(1 * 5.27046 * \sin(108.4))$$

$$\frac{\partial Q_3}{\partial |V_2|} = -5.001$$

$$\frac{\partial Q_3}{\partial |V_3|} = -2\left| V_3 Y_{33} \right| \sin(\vartheta_{33}) - \left| V_1 Y_{31} \right| \sin(\vartheta_{31} - \delta_3 + \delta_1) - \left| V_2 Y_{32} \right| \sin(\vartheta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -(2 * 1 * 21.0819 * \sin(-71.6)) - (1.05 * 15.8114 * \sin(108.4)) - (1 * 5.27046 * \sin(108.4))$$

$$\frac{\partial Q_3}{\partial |V_3|} = 19.254$$

Therefore complete Jacobian matrix is,

$$J = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix} = \begin{bmatrix} 8.9393 & -5.001 & 2.8541 & -1.6636 \\ -5.001 & 20.75 & -1.6636 & 6.41 \\ -2.9737 & 1.6636 & 8.5642 & -5.001 \\ 1.6636 & -6.904 & -5.001 & 19.254 \end{bmatrix}$$

But we known that

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

Therefore

$$\begin{bmatrix} -0.43772 \\ -0.01246 \\ -0.35047 \\ 0.49328 \end{bmatrix} = \begin{bmatrix} 8.9393 & -5.001 & 2.8541 & -1.6636 \\ -5.001 & 20.75 & -1.6636 & 6.41 \\ -2.9737 & 1.6636 & 8.5642 & -5.001 \\ 1.6636 & -6.904 & -5.001 & 19.254 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \\ \Delta|V_3| \end{bmatrix}$$

Now calculate the change in bus voltage and angle. (Take the inverse of Jacobian matrix).

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} 8.9393 & -5.001 & 2.8541 & -1.6636 \\ -5.001 & 20.75 & -1.6636 & 6.41 \\ -2.9737 & 1.6636 & 8.5642 & -5.001 \\ 1.6636 & -6.904 & -5.001 & 19.254 \end{bmatrix}^{-1} \begin{bmatrix} -0.43772 \\ -0.01246 \\ -0.35047 \\ 0.49328 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} -0.0623 \\ -0.0379 \\ -0.01057 \\ 0.0110 \end{bmatrix}$$

At the end of first iteration the bus voltages are

$V_1 = 1.05\angle 0.0°$

$V_2 = 0.98943\angle -3.569° = 0.9875 - j0.06159$

$V_3 = 1.011\angle -2.1715° = 1.0102 - j0.038307$

5

Line impedances are converted to admittances are

$y_{12} = 1.25 - j3.75$ $y_{23} = 1.667 - j5.0$ $y_{13} = 5.0 - j15.0$

The voltage at Bus3 is assumed as 1+j0. The initial voltages are therefore

$V_1^{(0)} = 1.05+j0.0$
$V_2^{(0)} = 1.00+0.0$
$V_3^{(0)} = 1.00+j0.0$

The bus admittance matrix in polar from is

$$Y_{BUS} = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.9167 - j8.75 & -1.667 + j5.0 \\ -5 + j15 & -1.667 + j5.0 & 6.66 - j20 \end{bmatrix}$$

The bus admittance matrix in polar from is

$$Y_{BUS} = \begin{bmatrix} 19.7642\angle -71.6° & 3.95285\angle 108.4° & 15.8114\angle 108.4° \\ 3.95285\angle 108.4° & 9.22331\angle -71.6° & 5.27046\angle 108.4° \\ 15.8114\angle 108.4° & 5.27046\angle 108.4° & 21.0819\angle -71.6° \end{bmatrix}$$

The real and reactive powers at bus 2 are calculated as follows:

$$P_2 = |V_2||V_1||Y_{21}|\cos\left(\delta_2^{(0)} - \delta_1 - \theta_{21}\right) + |V_2|^2|Y_{22}|\cos(-\theta_{22}) + |V_2||V_3||Y_{23}|\cos\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$$P_2 = (1*1.05 * 3.95285 * \cos(-108.4)) + \left(1^2 * 9.22331 * \cos(71.6)\right) + (1 * 1 * 5.27046 * \cos(-108.4))$$

$$P_2 = -0.06228$$

$$Q_2 = |V_2||V_1||Y_{21}|\sin\left(\delta_2^{(0)} - \delta_1 - \theta_{21}\right) + |V_2|^2|Y_{22}|\sin(-\theta_{22}) + |V_2||V_3||Y_{23}|\sin\left(\delta_2^{(0)} - \delta_3^{(0)} - \theta_{23}\right)$$

$$Q_2 = (1*1.05 * 3.95285 * \sin(-108.4)) + \left(1^2 * 9.22331 * \sin(71.6)\right) + (1 * 1 * 5.27046 * \sin(-108.4))$$

$$Q_2 = -0.18754$$

The real and reactive powers at bus 3 are calculated as follows:

$$P_3 = |V_3^{(0)}||V_1||Y_{31}|\cos\left(\delta_3^{(0)} - \delta_1 - \theta_{31}\right) + |V_3^{(0)}||V_2||Y_{32}|\cos\left(\delta_3^{(0)} - \delta_2 - \theta_{32}\right) + |V_3^{(0)}|^2|Y_{33}|\cos(-\theta_{33})$$

$$P_3 = (1*1.05 * 15.8114 * \cos(-108.4)) + (1*1* 5.27046 * \cos(-108.4)) + (1 * 1 * 21.0819 * \cos(71.6))$$

$$P_3 = -0.24953$$

$$Q_3 = |V_3^{(0)}||V_1||Y_{31}|\sin\left(\delta_3^{(0)} - \delta_1 - \theta_{31}\right) + |V_3^{(0)}||V_2||Y_{32}|\sin\left(\delta_3^{(0)} - \delta_2 - \theta_{32}\right) + |V_3^{(0)}|^2 |Y_{33}|\sin(-\theta_{33})$$

$$Q_3 = (1*1.05 * 15.8114 * \sin(-108.4)) + (1*1* 5.27046 * \sin(-108.4)) + (1 * 1 * 21.0819 * \sin(71.6))$$

$$Q_3 = -0.74328$$

The differences between scheduled and calculated power are

$P_2^{(0)} = -0.5 - (-0.06228) = -0.43772$

$Q_2^{(0)} = -0.2 - (-0.18754) = -0.01246$.

$P_3^{(0)} = -0.6 - (-0.24953) = -0.35047$

$Q_3^{(0)} = -0.25 - (-0.74328) = -0.49328$

$$\begin{bmatrix} \frac{\Delta P_2}{|V_2|} \\ \frac{\Delta P_3}{|V_3|} \end{bmatrix} = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\Delta Q_2}{|V_2|} \\ \frac{\Delta Q_3}{|V_3|} \end{bmatrix} = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{bmatrix} \begin{bmatrix} \frac{\Delta |V_2|}{|V_2|} \\ \frac{\Delta |V_3|}{|V_3|} \end{bmatrix}$$

6A

Comparison of Solution Methods

(5) Y bus is used for problem formulation.

From the computer memory requirements point of view, Polar co-ordinates are preferred for NR method and rectangular coordinates for GS method.

Time taken to perform one iteration of computation is relatively smaller in GS method as compared to NR method but the number of iterations required by GS method are greater as compared NR method. No of iterations are independent of size of system in NR method and vary b/w 3 to 5 iterations. But the no: of iterations in GS method increases with the size of the system. The convergence character of NR method are not affected by the selection of slack bus but sometimes in GS method it is seriously affected and results to poor convergence.

Main advantage of G.S method compared to NR method is the ease in programming and most efficient use of core memory.

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6.9 CONTROL OF VOLTAGE PROFILE

Control by Generators

Control of voltage at the receiving bus in the fundamental two-bus system was discussed in Section 5.10. Though the same general conclusions hold for an interconnected system, it is important to discuss this problem in greater detail.

At a bus with generation, voltage can be conveniently controlled by adjusting generator excitation. This is illustrated by means of Fig. 6.14 where the equivalent generator at the ith bus is modelled by a synchronous reactance (resistance is assumed negligible) and voltage behind synchronous reactance. It immediately follows upon application of Eqs. (5.71) and (5.73) that

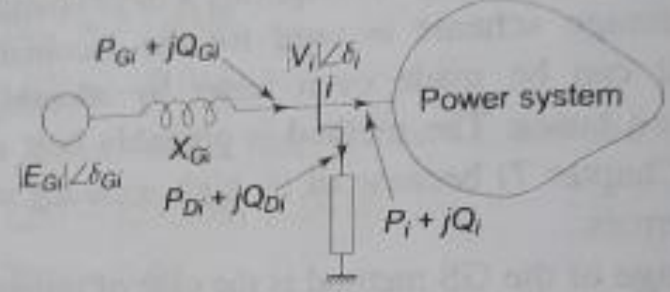

Fig. 6.14

$$P_{Gi} = \frac{|V_i||E_{Gi}|}{X_{Gi}} \sin(\delta_{Gi} - \delta_i) \tag{6.89}$$

$$Q_{Gi} = \frac{|V_i|}{X_{Gi}} (-|V_i| + |E_{Gi}|) \tag{6.90}$$

With $(P_{Gi} + jQ_{Gi})$ and $|V_i| \angle \delta_i$ given by the load flow solution, these values can be achieved at the bus by adjusting generator excitation to give $|E_{Gi}|$ as required by Eq. (6.90) and by adjusting the governor setting so that power input to generator from turbine is P_{Gi} plus losses, resulting in load angle of $(\delta_{Gi} - \delta_i)$ corresponding to Eq. (6.89). If Q_{Gi} demand exceeds the capacity of generators, VAR generators (synchronous or static capacitor) have to be used to modify the local load.

Load flow ... the voltage levels at the load buses ... (voltage control) buses. If some of the load bus voltages work out to be less than the specified lower voltage limit, it is indicative of the fact that the reactive power flow capacity of transmission lines for specified voltage limits cannot meet the reactive load demand (reactive line flow from bus i to bus k is proportional to $|\Delta V| = |V_i| - |V_k|$). This situation can be remedied by installing VAR generators at some of the load buses. These buses in the load flow analysis are then regarded as PV buses with the resulting solution giving the requisite values of VAR (jQ_C) injection at these buses.

The fact that positive VAR injection at any bus of an interconnected system would help to raise the voltage at the bus is easily demonstrated below:

Figure 6.15a shows the Thevenin equivalent circuit of the power system as seen from the ith bus. Obviously, $E_{th} = V_i$. If now jQ_C from VAR generator is injected into this bus as shown in Fig. 6.15b, we have from Eq. (5.73)

$$|\Delta V| = |E_{th}| - |V_i'| = -\frac{X_{th}}{|V_i'|} Q_C$$

or

$$|V_i'| = |E_{th}| + \frac{X_{th}}{|V_i'|} Q_C$$

$$= |V_i| + \frac{X_{th}}{|V_i'|} Q_C$$

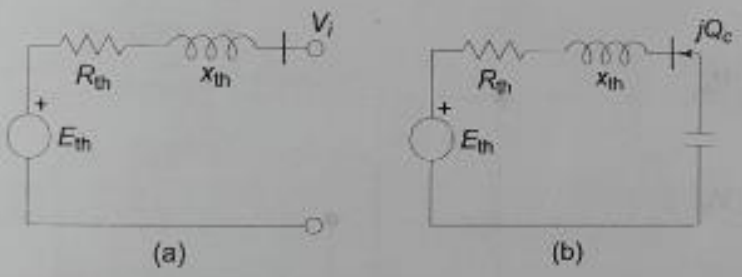

Fig. 6.15

Since we are considering a voltage rise of a few percent, $|V_i'|$ can be further approximated as

$$|V_i'| \simeq |V_i| + \frac{X_{th}}{|V_i|} Q_C \quad (6.91)$$

Thus the VAR injection of $+jQc$ causes the voltage at the ith bus to rise approximately by $(X_{th}/|V_i|)Q_C$. The voltages at other load buses will also rise owing to this injection to a varying but smaller extent.