USN | | | | | | | | | | 15MAT21

Second Semester B.E. Degree Examination, Jan./Feb.2021
Engineering Mathematics – II

Time: 3 hrs.   Max. Marks: 80

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Solve by inverse differential operator method,

$$\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = 3e^{2x} + 10.$$ (05 Marks)

   b. Solve by Inverse differential operator method,

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x.$$ (05 Marks)

   c. Solve by variation of parameters method $\frac{d^2y}{dx^2} + y = \tan x$. (06 Marks)

OR

2 a. Solve $(D^3 + 1)y = \cos x$. (05 Marks)

   b. Solve $(D^2 - 2D)y = x^2 + 2x + 1$. (05 Marks)

   c. Solve by undetermined coefficients method $(D^2 + 3D + 2)y = 2e^{-x} + x^2$. (06 Marks)

Module-2

3 a. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. (05 Marks)

   b. Solve $xy\left(\frac{dy}{dx}\right)^2 - (x^2 + y^2)\frac{dy}{dx} + xy = 0$. (05 Marks)

   c. Solve $y = 2px + \tan^{-1}(xp^2)$. (06 Marks)

OR

4 a. Find the general and singular solution of the equation,
      $\sin px \cos y = \cos px \sin y + p$. (05 Marks)

   b. Solve $p = \tan\left[x - \dfrac{p}{1+p^2}\right]$. (05 Marks)

   c. Solve the Legendre's linear equation,
      $(1 + x)^2 y'' + (1 + x)y' + y = 2\sin[\log(1 + x)]$. (06 Marks)

Module-3

5 a. Form the partial differential equation by eliminating the orbitrary functions from,
      $z = f(y + 2x) + g(y - 3x)$. (05 Marks)

   b. Solve the partial differential equation,

$$\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y),\ \text{by direct integration.}$$ (05 Marks)

   c. With usual notations derive the one dimensional wave equation as, $\dfrac{\partial^2 u}{\partial t^2} = C^2 \dfrac{\partial^2 u}{\partial x^2}$.

(06 Marks)

OR

6 a. Form the partial differential equation from $z = (x + y)f(x^2 - y^2)$. **(05 Marks)**

   b. Solve $\dfrac{\partial^2 z}{\partial x^2} + 5\dfrac{\partial z}{\partial x} + 6z = 0$ with $z = 0$ and $\dfrac{\partial z}{\partial x} = e^{-y}$ at $x = 0$. **(05 Marks)**

   c. Solve one dimensional heat equation by variable seperable method as, $\dfrac{\partial y}{\partial t} = C^2 \dfrac{\partial^2 y}{\partial x^2}$. **(06 Marks)**

Module-4

7 a. Change the order of integration and hence evaluate,
   $$\int_0^a \int_y^a \frac{x}{x^2 + y^2} \, dx \, dy.$$ **(05 Marks)**

   b. Evaluate $\displaystyle\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$. **(05 Marks)**

   c. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

OR

8 a. Evaluate by Changinginto polar co-ordinates,
   $$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) \, dx \, dy.$$ **(05 Marks)**

   b. Find the volume of the sphere, $x^2 + y^2 + z^2 = a^2$ by triple integration. **(05 Marks)**

   c. Prove that $\displaystyle\int_0^1 \frac{x^2}{\sqrt{1-x^4}} \, dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$ by using Beta gamma functions. **(06 Marks)**

Module-5

9 a. Find the Laplace transform of $(1 + te^{-t})^3$. **(05 Marks)**

   b. Find the Laplace transform of the function, $f(t) = E\sin\omega t$, $0 < t < \dfrac{\pi}{\omega}$, having the period $\dfrac{\pi}{\omega}$. **(05 Marks)**

   c. Using Laplace transform techniques, solve $\dfrac{d^2 x}{dt^2} - 2\dfrac{dx}{dt} + x = e^t$ with $x = 2$, $\dfrac{dx}{dt} = -1$ at $t = 0$. **(06 Marks)**

OR

10 a. Find the Laplace transforms of,
   (i) $t\cos t$ and   (ii) $\dfrac{\sin^2 t}{t}$. **(05 Marks)**

   b. Find $L^{-1}\left[\log\sqrt{\left(\dfrac{s^2 + b^2}{s^2 + a^2}\right)}\right]$. **(05 Marks)**

   c. Use convolution theorem to evaluate $L^{-1}\left[\dfrac{1}{(s+a)(s+b)}\right]$. **(06 Marks)**

* * * * *