CBCS SCHEME

USN

17MATDIP31

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Additional Mathematics - I

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the modulus and amplitude of $\frac{(1+i)^2}{3+i}$. (06 Marks)

b. If $x + \frac{1}{x} = 2\cos\alpha$, then prove that $x^n + \frac{1}{x^n} = 2\cos n\alpha$. (07 Marks)

c. Find the fourth roots of $1 - i\sqrt{3}$ and represent them on an argand plane. (07 Marks)

OR

2 a. If the vectors $2\hat{i}+\lambda\hat{j}+\hat{k}$ and $4\hat{i}-2\hat{j}-2\hat{k}$ are perpendicular to each other than find the value of λ. (06 Marks)

b. Find the sine of the angle between the vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+2\hat{k}$. (07 Marks)

c. Find λ such that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar. (07 Marks)

Module-2

3 a. Find the n^{th} derivative of cosx cos2x cos3x. (06 Marks)

b. With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$. (07 Marks)

c. Prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{24} + \ldots\ldots$ By using Maclaurin's expansion. (07 Marks)

OR

4 a. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. (06 Marks)

b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. (07 Marks)

c. If $u = e^x\cos y$, $v = e^x \sin y$, find $J = \frac{\partial(u,v)}{\partial(x,y)}$. (07 Marks)

Module-3

5 a. Evaluate $\int_0^{\pi} x\cos^6 x\,dx$. (06 Marks)

b. Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$. (07 Marks)

c. Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\,dx\,dy\,dz$. (07 Marks)

OR

6 a. Evaluate $\int \sin^6 x \, dx$. **(06 Marks)**

b. Evaluate $\iint_R (x^2 + y^2)\, dx\, dy$, where R is the triangle bounded by the lines $y = 0$, $y = x$ and $x = 1$. **(07 Marks)**

c. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dx\, dy\, dz$. **(07 Marks)**

Module-4

7 a. A particle moves along a curve whose position vector is given by $\vec{r} = \left(t - \frac{t^3}{3}\right)\hat{i} + t^2\hat{j} + \left(t + \frac{t^3}{3}\right)\hat{k}$. Find the velocity and acceleration at t = 3. **(06 Marks)**

b. Find the unit normal vector to the surface $xy + x + zx = 3$ at (1, 1, 1). **(07 Marks)**

c. Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

OR

8 a. A particle moves so that its position vector is given by $\vec{r} = \cos wt\ \hat{i} + \sin wt\ \hat{j}$, where w is a constant. Show that the velocity $\vec{V}$ is perpendicular to $\vec{r}$. **(06 Marks)**

b. If $\vec{F} = (x + y + 1)\ \hat{i} + \hat{j} - (x + y)\ \hat{k}$, show that $\vec{F} \cdot \text{curl}\ \vec{F} = 0$. **(07 Marks)**

c. Show that $\vec{f} = (\sin y + z)\ \hat{i} + (x \cos y - z)\ \hat{j} + (x - y)\ \hat{k}$ is irrotational. Also find ϕ such that $\vec{f} = \nabla\phi$. **(07 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + y \cot x = \sin x$. **(07 Marks)**

c. Solve $(x^2 + y)dx + (y^3 + x)\, dy = 0$. **(07 Marks)**

OR

10 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. **(06 Marks)**

b. Solve $(y \cos x + \sin y + y)\, dx + (\sin x + x \cos y + x)\, dy = 0$. **(07 Marks)**

c. Solve $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$. **(07 Marks)**

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