



Fourth Semester B.E. Degree Examination, Jan./Feb.2021

Advanced Mathematics – II

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. If  $l, m, n$  are the direction cosines of a line, then show that  $l^2 + m^2 + n^2 = 1$ . (07 Marks)
- b. A line makes an angle  $\alpha, \beta, \gamma, \delta$  with diagonals of a cube. Prove that  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$ . (07 Marks)
- c. Find the angle between the lines AB and CD where  $A = (3, 4, 5)$ ,  $B = (4, 6, 2)$ ,  $C = (-1, 2, 4)$  and  $D = (1, 0, 5)$ . (06 Marks)
- 2 a. Find the equation of a plane passes through the points  $(3, -3, 1)$  and perpendicular to the planes  $7x + y + 2z = 6$  and  $3x + 5y - 6z = 8$ . (07 Marks)
- b. Find the angle between two planes,  $x - y + 2z - 9 = 0$  and  $2x + y + z = 7$ . (07 Marks)
- c. Find the shortest distance between the lines,  $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$  and  $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ . (06 Marks)
- 3 a. Prove that  $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = 0$ . (07 Marks)
- b. Find the unit normal to both the vectors  $4\hat{i} - \hat{j} + 3\hat{k}$  and  $-2\hat{i} + \hat{j} - 2\hat{k}$ . Also find sign of the angle between them. (07 Marks)
- c. Show that the vectors  $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ ,  $\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$  and  $\vec{C} = 3\hat{i} + 4\hat{j} - \hat{k}$  are coplanar. (06 Marks)
- 4 a. Find the unit tangent vector of the space curve,  $\vec{r} = \cos t \hat{i} + \sin t \hat{j} + t \hat{k}$ . (07 Marks)
- b. A particle moves along the curve,  $x = (1 - t^3)$ ,  $y = (1 + t^2)$  and  $z = (2t - 5)$ . Determine the velocity and acceleration. (06 Marks)
- c. Find the angle between the surfaces,  $x^2 + y + z^2 = 9$  and  $x = z^2 + y^2 - 3$  at  $(2, -1, 2)$ . (07 Marks)
- 5 a. Find the directional derivative of  $x^2 y z^3$  at  $(1, 1, 1)$  in the direction of  $\hat{i} + \hat{j} + 2\hat{k}$ . (07 Marks)
- b. Prove that  $\text{F.curl} F = 0$  for  $F = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$ . (07 Marks)
- c. Show that the vectors,  $\vec{F} = (2xy + z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 2zx)\hat{k}$  is irrotational. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and/or equations written eg,  $42+8=50$ , will be treated as malpractice.

- 6 a. Find the Laplace transform of  $t^n$ , where 'n' is a positive integer. (07 Marks)  
b. Find  $L[\cos t \cos 2t \cos 3t]$ . (07 Marks)  
c. Find  $L\left[\frac{\sin^2 t}{t}\right]$ . (06 Marks)
- 7 a. Find  $L^{-1}\left[\frac{s+5}{s^2-6s+13}\right]$ . (07 Marks)  
b. Find  $L^{-1}\left[\frac{1}{(s+1)(s+2)(s+3)}\right]$ . (07 Marks)  
c. Find  $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$ . (06 Marks)
- 8 a. Using Laplace transform, solve  $y'' + 4y' + 4y = e^{-t}$ ;  $y(0) = 0$ ,  $y'(0) = 0$ . (10 Marks)  
b. By using Laplace transformations, solve the differential equation,  $y'' + 4y' + 3y = 0$ , subject to the condition  $y(0) = 0$  and  $y'(0) = 1$ . (10 Marks)



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