CBCS SCHEME

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17MAT41

Fourth Semester B.E. Degree Examination, Jan./Feb.2021
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Using Taylor's series method, compute the solution of $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$ at $x = 0.1$, correct to fourth decimal place. **(06 Marks)**

b. Using modified Euler's formula, solve the $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$ with $y(0.2) = 1.23$ at $x = 0.4$ by taking $h = 0.2$. **(07 Marks)**

c. The following table gives the solution of $\frac{dy}{dx} = x^2 + \frac{y}{2}$. Find the value of y at $x = 1.4$ by using Milne's Predictor-Corrector method.

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514

(07 Marks)

OR

2 a. Using modified Euler's method, solve $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y(20) = 5 at $x = 20.2$ by taking h = 0.2. **(06 Marks)**

b. Employ the Range-Kutta method of fourth order to solve $\frac{dy}{dx} = 3x + \frac{y}{2}$, with $y(0) = 1$ at $x = 0.1$ by taking $h = 0.1$. **(07 Marks)**

c. Using Adams-Bashforth method, find y when $x = 1.4$ given $\frac{dy}{dx} = x^2(1 + y)$, with y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979 **(07 Marks)**

Module-2

3 a. Using Runge-Kutta method of fourth order solve the differential equation, $\frac{d^2y}{dx^2} = x^3\left(y + \frac{dy}{dx}\right)$ for $x = 0.1$. Correct to four decimal places with initial conditions $y(0) = 1$, $y'(0) = 0.5$. **(06 Marks)**

b. Obtain the series solution of Legendre Differential equation leading to Pn(x). **(07 Marks)**

c. With usual notation, show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(07 Marks)**

OR

4 a. Apply Milne's method to compute y(1.4) given that $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ and

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y′	2	2.3178	2.6725	3.0657

(06 Marks)

b. State and prove Rodrigue's formula. **(07 Marks)**

c. Express $f(x) = 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

Module-3

5 a. State and prove Cauchy-Riemann equations in polar form. **(06 Marks)**

b. If $V = e^{-2y} \sin 2x$, find the analytic function f(z). **(07 Marks)**

c. Find the bilinear transformation that maps the points 0, i, ∞ onto the points $1, -i, -1$. **(07 Marks)**

OR

6 a. State and prove Cauchy's theorem on complex integration. **(06 Marks)**

b. Evaluate $\oint_C \frac{z^2+5}{(z-2)(z-3)} dz$, where $C : |z| = \frac{5}{2}$. **(07 Marks)**

c. Discuss the transformation $W = Z + \frac{1}{Z}$. **(07 Marks)**

Module-4

7 a. A box contains 100 transistors, 20 of which are defective and 10 are selected at random, find the probability that (i) all are defective (ii) at least one is defective (iii) all are good (iv) at most three are defective. **(06 Marks)**

b. Show that mean and standard deviation of exponential distribution are equal. **(07 Marks)**

c. The joint probability is,

X \ Y	0	1	2	3
0	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0

(i) Find marginal distributions of X and Y.
(ii) Also find E(X), E(Y) and E(XY). **(07 Marks)**

OR

8 a. Find the mean and variance of binomial distribution. **(06 Marks)**

b. In an examination taken by 500 candidates the average and the standard deviation of marks obtained (normally distributed) are 40% and 10%. Find approximately,
(i) How many will pass, if 50% is fixed as a minimum?
(ii) What should be the minimum if 350 candidates are to pass?
(iii) How many have scored marks above 60%? **(07 Marks)**

c. Suppose X and Y are independent random variables with the following distributions:

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	−2	5	8
$g(y_j)$	0.3	0.5	0.2

Find the joint distribution of X and Y. Also find the expectations of X and Y and covariance of X and Y. **(07 Marks)**

Module-5

9 a. The average income of persons was Rs.210 with a standard deviation of Rs.10 in sample of 100 people of a city. For another sample of 150 persons, the average income was Rs.220 with standard deviation of Rs.12. The standard deviation of the incomes of the people of the city was Rs.11. Test whether there is any significant difference between the average incomes of the localities. (Use $Z_{0.05} = 1.96$) **(06 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure : 5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05}$ for 11 d.f = 2.201). **(07 Marks)**

c. Define stochastic matrix. Find a unique fixed probability vector for the matrix $\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$. **(07 Marks)**

OR

10 a. Explain the following terms:
   (i) Type I and Type II errors.
   (ii) Null hypothesis.
   (iii) Level of significance.
   (iv) Confidence limits. **(06 Marks)**

b. Eleven school boys were given a test in mathematics carrying a maximum of 25 marks. They were given a month's extra coaching and a second test of equal difficulty was held thereafter. The following table gives the marks in two tests.

Boy	1	2	3	4	5	6	7	8	9	10	11
Marks (I test)	23	20	21	18	18	20	18	17	23	16	19
Marks (II test)	24	19	18	20	20	22	20	20	23	20	17

Do the marks give evidence that the students have benefitted by extra coaching? (Given $t_{0.05} = 2.228$ for 10 d.f) **(07 Marks)**

c. Three boys A, B and C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probabilities that after three throws (i) A has the ball, (ii) B has the ball, (iii) C has the ball. **(07 Marks)**

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