Fourth Semester B.E. Degree Examination, Jan./Feb. 2021 Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \end{bmatrix}$ by elementary applying row transformation.

(06 Marks)

- b. Solve the following system of linear equation by Gauss Elimination method x + 2y + z = 3, 2x + 3y + 3z = 10, 3x y + 2z = 13 (07 Marks)
- c. Find the inverse of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using Cayley-Hamilton theorem. (07 Marks)

OR

- 2 a. Reduce the matrix $\begin{vmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{vmatrix}$ into its echelon form and hence find its rank. (06 Marks)
 - b. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. (07 Marks)
 - c. Solve the following system of linear equation by Gauss Elimination method x + y + z = 9, x 2y + 3z = 8, 2x + y z = 3. (07 Marks)

Module-2

- 3 a. Solve $\frac{d^2y}{dx^2} 6\frac{dy}{dx} + 9y = 6e^{3x}$ (06 Marks)
 - b. Solve $\frac{d^2y}{dx^2} 2\frac{dy}{dx} + y = \cos 3x$ (07 Marks)
 - c. Solve $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameters. (07 Marks)

OR

- 4 a. Solve $\frac{d^2y}{dx^2} + 4y = x^2$ (06 Marks)
 - b. Solve $\frac{d^2y}{dx^2} 3\frac{dy}{dx} + 2y = \frac{e^x + e^{-x}}{2}$ (07 Marks)
 - c. Solve $\frac{d^2y}{dx^2} 3\frac{dy}{dx} + 2y = 4e^{3x}$ by the method of undetermined coefficients. (07 Marks)

2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice. Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.

Module-3

5 a. Prove that L[Cosh at] = $\frac{s}{s^2 - a^2}$

(06 Marks)

b. Find the Laplace transform of cost cos2t cos3t

(07 Marks)

c. Find the Laplace transform of $f(t) = \begin{cases} t & 0 \le t \le a \\ 2a - t & a < t \le 2a \end{cases}$ where f(t + 2a) = f(t) (07 Marks)

OR

6 a. Find the Laplace transform of sint sin2t sin3t.

(06 Marks)

b. Find the Laplace transform of t²sin at.

(07 Marks)

c. Express $f(t) = \begin{cases} t^2 & 1 < t \le 2 \\ 4t & t > 2 \end{cases}$ interms of unit step function and hence find $L\{f(t)\}$. (07 Marks)

Module-4

7 a. Find the inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$

(06 Marks)

b. Find the inverse Laplace transform of $\log \frac{(s^2 + 1)}{s(s+1)}$

(07 Marks)

c. Using Laplace transform, solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ under the initial condition y(0) = 1y'(0) = 0. (07 Marks)

OR

8 a. Find the inverse Laplace transform of $log\left(\frac{s+a}{s+b}\right)$

(06 Marks)

- b. Find the inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$. (07 Marks)
- c. Solve by using Laplace transform $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$ under the initial condition y(0) = 0, y'(0) = 0. (07 Marks)

Module-5

9 a. Prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(06 Marks)

- b. Find the probability that a leap year selected at random will contain 53 Sundays. (07 Marks)
- c. An office has 4 secretaries handling 20%, 60%, 15%, 5% respectively of the files of certain reports. The probabilities that they misfile such reports are respectively 0.05, 0.1, 0.1 and 0.05. Find the probability that a misfiled report is caused by the first secretary. (07 Marks)

OR

10 a. State and prove Baye's theorem.

(06 Marks)

- b. A problem is given to four students A, B, C, D whose chances of solving it are 1/2, 1/3, 1/4, 1/5 respectively. Find the probability that the problem is solved. (07 Marks)
- c. Three machines A, B, C produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3%, 4% and 5% respectively. If an item is selected at random. What is the probability that it is defective? If a selected item is defective, what is the probability that it is from machine A?

(07 Marks)