CBCS SCHEME

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15CS36

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define the following terms with an example : i) Conjunction ii) Tautology iii) Quantifiers iv) Proposition v) Conditional or Implication vi) Dual of statement. **(06 Marks)**

b. Prove the validity of the following arguments :

i) $$\begin{array}{l} p \to r \\ \neg p \to q \\ q \to s \\ \hline \therefore\ \neg r \to s \end{array}$$

ii) $$\begin{array}{l} (\neg p \vee \neg q) \to (r \wedge s) \\ r \to t \\ \neg t \\ \hline \therefore\ p \end{array}$$

(06 Marks)

c. Find the negation of each of the following quantified statements :

i) $\forall x, \forall y\ [(x > y) \to ((x - y) > 0)]$

ii) $\forall x, \exists y\ [(p(x, y) \wedge q(x, y)) \to r(x, y)]$. **(04 Marks)**

OR

2 a. Prove that the following compound propositions are tautologies :

i) $[p \wedge (p \to q)] \to q$ ii) $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$. **(05 Marks)**

b. Prove the following by using laws of logic :

i) $[(p \to q) \wedge (\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$

ii) $[p \vee q \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$. **(05 Marks)**

c. Show that "If n is an odd integer then n + 11 is an even integer" by i) Direct proof ii) An indirect proof iii) Proof by contradiction. **(06 Marks)**

Module-2

3 a. Prove the following by Mathematical Induction :

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$ **(05 Marks)**

b. i) How many arrangements are possible for all the letters in the word SOCIOLOGICAL?

ii) In how many of these arrangements A & G are adjacent?

iii) In how many of these arrangements all the vowels are adjacent? **(06 Marks)**

c. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. Find a_n in explicit form. **(05 Marks)**

OR

4 a. If Fi's are the Fibonacci numbers and Li's are the Lucas numbers, prove that $L_{n+4} - L_n = 5 F_{n+2}$ for all integers $n \geq 0$. **(06 Marks)**

b. A certain college question paper contains 3 parts A, B and C with 4 questions in part A, 5 questions in part B & 6 questions in part C. It is required to answer 7 questions by selecting atleast 2 questions from each part. In how many different ways can a student solve the question paper? **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

c. Find the coefficient of :
i) $x^2 y^2 z^3$ in the expansion of $(x + y + z)^7$.
ii) $v^2 w^3 x^2 y^5 z^4$ in the expansion of $(v + w + x + y + z)^{16}$. **(04 Marks)**

Module-3

5 a. Let A = {a, b, c, d} and B = {2, 4, 5, 7}. Determine the following :
i) 1 A × B1.
ii) Number of relations from A to B.
iii) Number of relations from A to B that contain (a, 4) and (c, 7).
iv) Number of relations from A to B that contain exactly six ordered pairs.
v) Number of binary relations on A that contain atleast Fourteen ordered pairs. **(06 Marks)**

b. Let f, g, h be functions from z to z, define by $f(x) = x^2$, $g(x) = x + 5$ and $h(x) = \sqrt{x^2 + 2}$. Determine (h o (g o f)) (x) and ((h o g) o f) (x). Verify that h o (g o f) = (h o g) o f. **(05 Marks)**

c. Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 3), (1, 3), (4, 1), (1, 4), (4, 4)}. Is R is an equivalence relation? Find the corresponding partition on A. **(05 Marks)**

OR

6 a. Prove that if f : A → B, g : B → C are invertible functions, the g o f : A → C is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. **(05 Marks)**

b. Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 3), (3, 4)}, S = {(3, 1), (4, 4), (2, 4), (1, 4)} be relations on A. Determine the relations R o S, S o R, R^2 and S^2. Write down their matrices. **(05 Marks)**

c. Consider the Hasse diagram of a POSET (A, R) given in Fig. Q6(c).
i) Determine the relation matrix R ii) Construct the digraph for R
iii) Write maximal, minimal, greatest and least elements. **(06 Marks)**

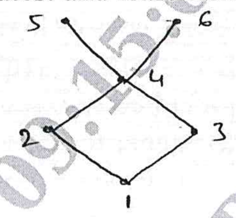

Fig. Q6(c)

Module-4

7 a. How many integers between 1 and 300 (inclusive) are i) divisible by atleast one of 5, 6, 8? ii) divisible by none of 5, 6, 8? **(05 Marks)**

b. Determine in how many ways can the letters in the work ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters. **(06 Marks)**

c. Find the Rook polynomial for 3 × 3 board by using the expansion formula. **(05 Marks)**

OR

8 a. A person invests Rs 100,000 at 12% interest compounded annually : i) Find the amount at the end of 1st, 2nd, 3rd year ii) Write the general explicit formula iii) How long will it take to double the investment. **(06 Marks)**

b. Solve the recurrence relation $a_{n+2} - 8a_{n+1} + 16a_n = 8(5^n) + 6(4^n)$, where $n \geq 0$ and $a_0 = 12$, $a_1 = 5$. **(05 Marks)**

c. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 and B_4. The Boys B_1 and B_2 do not wish to have apple, B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(05 Marks)**

Module-5

9 a. Define the following : i) Complete Graph ii) Bipartite Graph iii) Isolated Vertex iv) Regular Graph v) Subgraph. **(05 Marks)**

b. Let G = (V, E) be simple graph of order $|V| = n$ and size $|E| = m$. If G is a bipartite graph, prove that $4m \leq n^2$. **(05 Marks)**

c. Construct an optimal prefix code for the symbols a , o, p, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(06 Marks)**

OR

10 a. Prove the following :

i) A path with n vertices is of length n-1.
ii) If a cycle has n vertices, it has n edges.
iii) The degree of every vertex in a cycle is two. **(06 Marks)**

b. Define Isomorphism. Verify the two graphs are Isomorphic. (Refer Fig. 10(b(i),(ii))

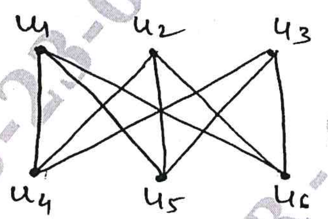

Fig. Q10(b (i))

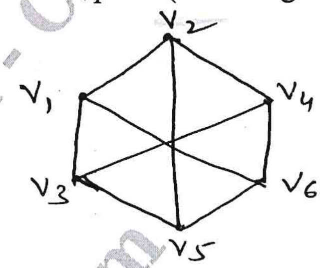

Fig. Q10(b (ii))

(04 Marks)

c. List the vertices in the tree given in Fig. Q10(c), when they are visited in :
i) Preorder ii) Postorder iii) Inorder Traversal. **(06 Marks)**

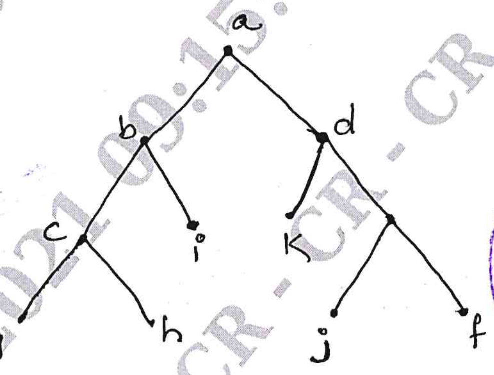

Fig. Q10(c)

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